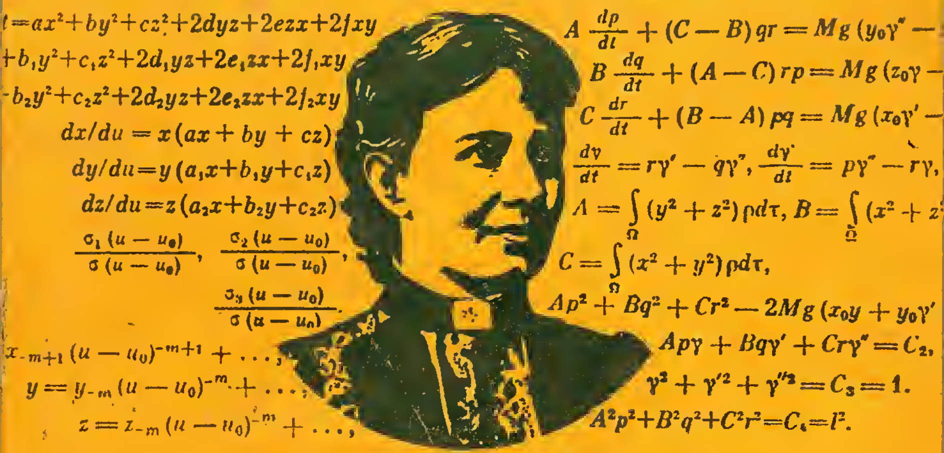
Pelageya Kochina

LOVE and MATHEMATICS:



Sofya KOVALEVSKAYA

Mir Publishers Moscow

LOVE AND MATHEMATICS: SOFYA KOVALEVSKAYA

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LOVE and MATHEMATICS: Sofya KOVALEVSKAYA

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FROM THE AUTHOR

The sheer personality of Sofya Kovalevskaya, the renowned Russian woman mathematician, was so remarkable, multi-faceted, and interesting that the great Norwegian playwright Henrik Ibsen said to write her biography would need a poem. My aim in writing this book is different and somewhat more modest: it is to present the basic information about her life from the numerous sources available.

S. Ya. Shtraikh was the first person to collect Sofya Kovalevskaya's vast correspondence as well as that of her husband Vladimir Kovalevsky and his brother Aleksandr. Shtraikh wrote several biographies: *S. Kovalevskaya*, *The Korvin-Krukovsky Sisters*, *The Kovalevsky Family*, and he prepared for publication *Memoirs and Letters by S. V. Kovalevskaya*. These books are a rich source for a new biography of Sofya Kovalevskaya.

After the Second World War, the USSR Academy of Sciences received photocopies of Kovalevskaya's correspondence from the Archive of G. Mittag-Leffler in Stockholm. The then President of the Academy, S. I. Vavilov, gave me the chance to work on these copies, and step by step, I published portions of this correspondence. It included new data on the years of Kovalevskaya's scientific activity as contained in letters from outstanding scientists. The letters from Charles Hermite and Karl Weierstrass were published separately. The correspondence between G. Mittag-Leffler and Kovalevskaya, the most numerous in the Archive, was published in 1984. These letters have been thoroughly studied, and I used them to write three small essays on Kovalevskaya's life and activity and some articles commemorat-

ing her anniversaries. L. A. Vorontsova [86] used all these materials to write a fictional biography about Sofya Kovalevskaya.

My intention was to put together the most significant and interesting facts from all my sources and to deal with the mathematical problems in considerable detail.

Kovalevskaya's biographers have written about her private life at length. But there still are people who consider Kovalevskaya a "bluestocking". No, she was not! She was in fact a woman with a great desire for happiness; she experienced all the joys and woes that may befall a woman, while her life was a noble struggle to blaze open trails for women. And she won through with flying colours, making her name in mathematics and winning hearts by her literary work. I cannot pay much attention in this book to Kovalevskaya's private life though at the same time I cannot avoid it since every admirer of the first woman professor is interested in her life.

I have relied extensively on the Mittag-Leffler Archive. Some of the letters from the Archive have been published in Russian.

I consider it my pleasant duty to express my deep gratitude to E. P. Ozhigova, who gave me valuable advice on many occasions and who was a source of bibliographic information. My heartful thanks go also to Sergei Stel'makh, who read my manuscript and gave me much useful advice.

My special gratitude goes to Mrs. Silvia Ljungqvist-Carleson of the Mittag-Leffler Institute in Sweden, who sent me many valuable materials.

The Department of Scientific Information of the Institute of Problems of Mechanics (USSR Academy of Sciences), headed by I. A. Viktorova, was always very helpful. I am sincerely grateful to everyone who was kind enough to help in the preparation of the book.

I feel that my destiny is to serve the truth, that is, science, and to blaze the trail for women, because that means to serve justice.

S. V. Kovalevskaya

Chapter 1

CHILDHOOD

Parents and Relatives

In the history of science there are few women of world renown and of whom every educated person should be aware. Sofya Vasilievna Kovalevskaya was and is indeed world famous. She was one of the outstanding mathematicians of the 19th century, the first ever woman Corresponding Member of the Petersburg Academy of Sciences in Russia, a professor of the University of Stockholm, a writer, and a progressive social activist.

Sofya (Sonya, Sofa) Kovalevskaya was born on January 3(15)*, 1850 in Moscow. Her parents were Vasily Vasilievich and Elizaveta Fedorovna Kryukovskóy (with the stress on the last syllable). Later their surname was more frequently pronounced Krukóvsky, with the stress on the second syllable. Sofya was registered at birth as Krukóvskaya, but she wrote in *Memories of Childhood*** that a sexton had once advised her:

> "Look there, my little miss, there's a *kryuk*, a hook, hanging on the gate. When you forget your surname all you have to do is think, 'There's a kryuk on Kryukovskóy's gate,' and then you'll remember it right away" [68, p. 48].

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* The date given in brackets is the one according to the calendar then current in the West and adopted for use in the Soviet Union in 1918.

** First published in English as *A Russian Childhood* [68] (with her autobiographical sketch and my analysis of her mathematics).

There is a record in the 1850 birth register of the church of the Sign (over St. Peter's gate in Moscow) which goes:

> "A girl born 3 January, christened Sofya 17 January. Parents: artillery colonel Vasily Vasilievich, son of Krukovskoy, and his wedded wife Elizaveta Fedorovna; the husband is Orthodox and the wife is Lutheran. Godparents: retired second lieutenant of artillery Semen Vasilievich, son of Krukovskoy, and the spinster Anna Vasilievna, daughter of quartermaster Vasily Semenovich, son of Krukovskoy. The sacrament of baptism was performed by the local priest Pavel Krylov, Deacon Pavel Popov, and Sacristan Aleksandr Speransky." (Signed by the priest, deacon, sexton, and sacristan.)

Documents from the State Central Historical Archive in Leningrad* show that in the early 1840s there were several people with the surname Krykovskóy, from the Pskov and Poltava provinces, and they applied to the Senate for their name to be entered into the sixth section of the Genealogical Record for the Nobility**. The Senate refused, the reason being that there was no documentation that their ancestors owned land and serfs.

When in 1858 the requisite documents were presented, Vasily Vasilievich and a number of his relatives were entered into the Genealogical Record and given the surname Korvin-Krukóvsky.

But even now inhabitants of the village of Polibino (Kovalevskaya wrote 'Palibino'), formerly the ancestral estate of the Korvin-Krukovskys, still call them the Kryukovskóys, with the stress on the last syllable.

In the 1850's many members of the gentry endeavoured to register their family trees, following a Polish fashion or in view of greater privileges after the coming abolition of serfdom.

Vasily Korvin-Krukovsky's service record shows that he was born in 1800 and became a cadet in the Army in 1817. He rose to command the Moscow artillery garrison and arsenal by 1848, and was a colonel of field artillery. From 1828

* Tsentralnyi Gosudarstvennyi istoricheskii arkhiv (hereafter cited as *TsGIA*), f. 1343, op. 23, No. 6550.

** *Ibid.*

to 1830 he fought in the Balkan campaigns during the Turkish war and was awarded the Orders of St. Ann of the second and third degrees with bows.*

He married Elizaveta Fedorovna Schubert on January 17(29), 1843, and they settled in Moscow, where Vasily was serving. In the early 1850s the Krukovskys moved to the city of Kaluga, because Vasily Vasilievich had been transferred there.

We have a newspaper announcement for 1858 that on April 21 Major-General Vasily Vasilievich, Mikhail Semenovich, and Fedor Vasilievich Korvin-Krukovskys were granted certificates of nobility.

In 1858, Vasily V. Korvin-Krukovsky, now a lieutenant-general of artillery, retired to his estate in the village of Palibino in the Nevel district of the Vitebsk province. He was elected the province's Marshal of Nobility for 1863 and several subsequent years.

It was a cherished tradition of the family of General Korvin-Krukovsky that his blood was of mixed origin, and it was said that Korvin was the nickname of the Hungarian king Matthias Hunyadi, a valiant knight, a patron of the arts and literature, and a bibliophile. One of the king's daughters fell in love with the Polish knight Krukovsky, and they married, settled in Lithuania, and established the Korvin-Krukovsky line.

Sofya Kovalevskaya recalls in her memoirs that her sister Anna (Anyuta) was at one time consumed by tales of chivarly. She would sit in the tower of their Palibino manor**, which was built somewhat in the style of a medieval castle, and embroider the family crest of King Matthias Corvinus in beads on canvas. The coat of arms granted to the Korvin-Krukovskys is a raven (*Corvus corax*) surrounded by ostrich feathers against a shield.

According to Sofya, the Korvin-Krukovskys married Russians and Lithuanians for centuries, and legend has it that one married a Gipsy girl.

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* *Ibid.*

** Later Kovalevskaya described the manor-house at Pabilino. It has well been built by serfs. During the Second World War it was destroyed by fascist raiders. The house was restored after the war (but not the whole of it). There was a boarding school there, and now Korvin-Krukovsky sisters' museum is being organized in it. An orphanage named after S. V. Kovalevskaya is nearby [84].

Sofya's mother Elizaveta Fedorovna was the grand-daughter of the astronomer Fedor Ivanovich Schubert, who was a Member of the St. Petersburg Academy of Sciences, and the daughter of the geodesist Fedor Fedorovich Schubert, who was an Honoured Member of the Academy.

The Schuberts came from Germany. Johann Ernst Schubert, Sofya's great-great-grandfather, was a Lutheran theologian who was compelled to move from Braunschweig to Pomerania during the Seven-Year War. His family inherited a large library.

There were eleven children in the Schubert family, six sons and five daughters. The youngest son Theodor (Fedor Ivanovich in Russia) and Sofya's great-grandfather had studied oriental languages and theology at Cöttingen University. He could not find a good job in Germany and so moved to Russia to become a land-surveyor in Revel (now Tallinn). He loved mathematics and astronomy passionately, and although he studied them on his own he succeeded immensely; eventually being elected a Member of the St. Petersburg Academy of Sciences several years after he came to Russia. He wrote a textbook on theoretical astronomy and another book popularising astronomy. He managed the Kunstkamera, the observatory, and the library of the Academy. His correspondence with Laplace, Gauss, Bessel, and other mathematicians was published in Russian in Volume 1 of *Scientific Heritage* [87].

Sofya's grandfather Fedor Fedorovich Schubert was a prominent scientist and military leader. He published books and papers on geodesy and geographical maps of Russia. His biography [88] is interesting. He was taught at home until he was eleven, and was well educated and loved mathematics, music, and languages. He had a chance to read books both from library at home and from one at the Academy of Sciences, which was managed by Fedor Ivanovich Schubert, his father.

Fedor Fedorovich began to work after three years at school, at the age of 14. He was given a job on the General Staff on request of his father, where he acquired his liking for topography and geodesy.

When he was 17, he joined the Army and showed great valour in the battles against Napoleon. He suffered all kinds of deprivations, was severely wounded and fell ill

with typhus, but he always took the first chance to return to the front. It is conjectured that F. F. Schubert was connected to the Decembrists [64, p. 461].

He was active in military geodesy and science for more than 60 years, supervising vast cartographic studies in many parts of Russia, and overseeing the triangulation of the Baltic. He participated in a chronometric expedition sponsored by the governments of Russia, Prussia, Sweden, and Denmark. F. F. Schubert was elected an Honoured Member of the St. Petersburg Academy of Sciences in 1827.

In 1858 F. F. Schubert published a large book on astronomy and geodesy in Russia (in French since he wanted the public abroad to know the work in Russia too).

A book by F. F. Schubert (Friedrich von Schubert) [89], his recollections until 1814, was published in 1962 in Stuttgart. It is of great interest for military history, as well as for the history of Russian science and culture. Here is the story of the publication. Alexandrina, a daughter of F. F. Schubert, married N. F. Adelung, the son of Petersburg historian and specialist in literature F. P. Adelung. N. F. Adelung was a secretary of grand duchess Olga Nikolaevna. The latter became the queen of Würtemberg and the Adelungs moved to Stuttgart, Germany. Sofya Adelung, a daughter of theirs, became friends with Sofya Kovalevskaya and later wrote recollections of her [102]. Sofya Adelung and her sister Olga were the keepers of the reminiscences of their grandfather who had died in Stuttgart. The memoirs were passed from the Adelungs to their grand-nephew, also called Friedrich von Schubert, who published them.

Fedor Fedorovich Schubert had one son Fedor, and three daughters, Elizaveta, Alexandrina, and Sofya. Sofya Schubert, the third one, remained unmarried. She was the last mistress of the Schubert house in Petersburg, 12/14 Line 1, Vasilievsky Island. In 1887 Sofya Schubert invested 10 550 roubles to endow a biannual prize named after her father F. F. Schubert and awarded for a paper or book in theoretical astronomy. A prize of 1000 roubles was given on February 2, 1889, the 100th anniversary of F. F. Schubert's birth to Hugo Gyldén for his *Studies of the Convergence of Series Applied to Determine the Coordinates of Planets* [89, p. 14].

The death of Sofya Fedorovna marked the end of a line

started by Sofya Kovalevskaya's great-grandfather Academician Fedor Ivanovich Schubert. But his five brothers, one of whom settled in Estland, the others in Sweden and Germany, continued the Schubert family.

Vasily Vasilievich, Sofya Kovalevskaya's father, was 20 years older than his wife Elizaveta Fedorovna. He treated his wife as a child, and this relationship was maintained until the end of their life together. He had his own life and his wife was not admitted into it at all for the first few years of their marriage. Consequently there were many lonely and distressing evenings at the beginning, when Elizaveta suffered from jealousy.

Elizaveta Fedorovna's Diaries for the years 1843 to 1851 and 1863 to 1864* have been found in the archives. To begin with she wrote almost entirely of her joys and torments, but later she started to describe some of the events in her life.

The short remarks give us some idea about the people Elizaveta Fedorovna met during the first years of her married life. These included Osip Ivanovich Senkovsky, a well-known journalist who had married one of Elizaveta Fedorovna's aunts, Petr Lavrovich Lavrov, a mathematician by education (he later became a revolutionary and emigrated), Fedor Antonovich Moller, an artist and a friend of Gogol, and Nikolai Ivanovich Pirogoff, the famous surgeon who was then a Professor of the Medical-Surgical Academy. Here is a passage from her diary for December 22, 1847:

> "It was so boring at the Filippovs' dinner. But I have made an impression. However, the dinner was followed by a very nice evening with Moller and then with Pirogoff, who came later. What entertaining conversations, what interesting stories. But my rake husband spoiled the treat by his absence; how I curse his club. We talked about everything: religion, love, family, etc. Nothing was left out. Pirogoff amazed me with the novelty of his views. How many reminiscences he has awaken in me!" (*Ibid.*, p. 40)

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* Leningradskoe Otdelenie Arkhiva AN SSSR (Leningrad Division of the Archive of the USSR Acad. Sc., hereafter cited as *LOA AN*), f. 768, op. 1, No. 29.

Elizaveta Fedorovna was acquainted with the ideas of the French philosopher Jean Jacques Rousseau. But she did not attempt to implement any principles in bringing up her children who grew up freely, even too freely, as we shall see later. According to Sofya Kovalevskaya, her mother did not possess a tough character or the capacity to manage anything at home except the drawing-room. She was a nice kind woman who liked the company, and people liked her for her quick nature and appreciable musical talent. Before she was married, she had lived with her sisters Sofya and Alexandrina and their brother Fedor as a "German" family. Music was always in the air and Liza (Elizaveta) amused the company, consisting mainly of her numerous aunts, by playing the piano. In Leningrad, at the State Russian Museum, there is a charming water-colour of young Elizaveta Fedorovna painted by A. Bryullov, an architect and artist who married one of her aunts [67, p. 65].

It is interesting that in her diaries for 1843-1851 Elizaveta Fedorovna does not mention her own children even once, thoungh Anyuta was born in 1844 and Sofya in 1850.

When the Krukovskys lived in Kaluga, Elizaveta Fedorovna entered society and performed in concerts. A local newspaper described a charity concert. The concert was given on September 20, 1855, at 8 p.m. in the hall of the Nobility Assembly.

"This time the talents of several of the performers were already known so the hall was filled to capacity," the newspaper reported. E. F. Krukovskaya, E. A. Delyanova, A. Ya. Bilibina, N. V. Aklicheev, and K. O. Vedichek participated in the concert. The first two women were pianists and played together including allegros by Hummel and duets by Weber. These performers were highly praised, especially Elizaveta Fedorovna, of whom the newspaper said,

> "Mrs. Krukovskaya was so wrapt in her performance that it seemed she did not care for manners; all her movements were involuntary expressions of feelings she could barely contain, communicating them in sound".

After the Korvin-Krukovskys moved to Palibino in 1858, Elizaveta Fedorovna had one more diversion: home theatricals that she loved both as a spectator and actor, her acting

being reportedly good.

The diary of Elizaveta Fedorovna shows us the life of a landed gentry family, for which a subordinate position of the woman was typical. The woman was very limited in her activities, the head of the family being a despot who disregarded his wife and the wife living a life that was separate from his life. E. I. Druzhinina in this connection asks a question,

> "Having submitted to her husband, Elizaveta Fedorovna locked herself within the walls of her house, but suffered immensely from this isolation. Was it this dissatisfaction and concealed, suppressed protest that showed her daughters Anna and Sofya a life that must not be lived and made them leave home?" [90, p. 218]

But V. V. Korvin-Krukovsky was far from being the worst representative of his class: he was educated, he managed his large estate efficiently, and he tried to bring up his children well and to give them good education. He was a gentry serf-owner, but gradually his views became milder under the influence of his children and by the end of his life he reconciled himself to the democratic attitudes of his daughters.

E. F. Litvinova [91], the first Russian biographer of Sofya Kovalevskaya and who knew the Korvin-Krukovsky family well, wrote that

> "ardent passions lived in the soul of Vasily Vasilievich, he was a man of reason, character, and heart, who handed all these down to his younger daughter".

Fedor Vasilievich Korvin-Krukovsky, Sofya Kovalevskaya's brother, wrote,

> "Sofya was by nature the image of her father, but she took the power of her reason from her great-grandfather and grandfather the Schuberts, of whom the former was a famous astronomer and an outstanding scientist of his time and the latter a well-known Russian general." [92, p. 635]

This was the genealogy of Sofya Kovalevskaya. According to Ellen Key, a Swedish writer, Sofya described her relationship to her ancestors to her Stockholm friends in a way

that defined her spiritual and intellectual development

"I inherited my passion for science from my ancestor the Hungarian King Matthias Corvinus; my love for mathematics, music, and poetry from my mother's grandfather, the astronomer Schubert; my love for freedom from Poland; my love for wandering and my inability to obey the accepted tradition from my Gipsy great-grandmother; and all the rest comes from Russia." [64, p. 409]

Early Childhood

As I write about Sofya's first years I cannot avoid citing her own memories of *A Russian Childhood*, in which the dawn of her life is described in interesting detail. She was not always exact in the chronology, and there are some discrepancies between her recollections and the memoirs of Iosif Malevich, her tutor, and those of her cousin, Sofya Adelung, but they can be accounted for as time wearing out and transforming anyone's perception of events. M. I. Semevsky [93], a historian and writer who knew the people and situation in Palibino about 1863, asserts that Sofya described them accurately.

Her parents wanted a son, so the appearance of Sofya (Sonya) was unwelcome. They had already had a six-year-old daughter Anyuta and they had a son Fedor (Fedya) five years later, which consoled the father.*

Sonya was convinced that she was not loved by her parents. This conviction was supported by her Nanny, who loved and pitied Sofa, as the girl was called in the family. Anyuta, a pretty and intelligent blond girl, was pampered as the first child, and Fedya was spoilt as the son and the youngest. The dark-complexioned Sofa was in the middle and drew less attention. She was a sympathetic, bashful child, and somewhat unsociable in the company of adults. It was painful for the girl to feel her parents did not love her much, but she did not retire into herself. Her unsociability smooth-

* E. Amburger, introducing recollections of Kovalevskaya's grandfather Fedor Fedorovich (Friedrich von Schubert) [89], writes that the latter had four grandchildren born by his daughter Elizaveta. True, Elizaveta Fedorovna and Vasily Vasilievich Korvin-Krukovsky had another son called Vasily who died early.

ed out and she became more cheerful and playful over the years. Possibly, her self-isolation promoted the development of childish fantasies, for the girl had a vivid imagination and could immerse herself in her own world.

Semevsky described twelve-year-old Sofa as a frisky, often laughing girl, who would run with a ball to her sister, kiss her, and run away [93, p. 714].

When the family was in Kaluga, the young mother gave the children over to be brought up by their Nanny and the other servants. This life under the wing of Nanny during their early childhood had its positive aspects for as P. D. Boborykin pointed out, the house-serfs had been the link between the gentry chidren and the peasants and had been the first "developers" of the children [94, p. 52].

The love of a simple caring woman and the relative freedom contributed to the development of Sonya's individuality. She was obliged to Nanny for her knowledge of Russian, otherwise her first language would have been French. Nanny's tales benefited Sonya but much of what Nanny recounted, the terrifying fairy tales especially, was inappropriate for a nervous and sensitive girl like Sonya.

One incident shows how wild Sonya could sometimes be. Her mother's brother, Fedor Fedorovich Schubert, was Sonya's favourite uncle. He was a jolly young man and told Sonya interesting stories about infusoria and algae, and what not. She would listen to everything sitting in her uncle's lap. Once a little girl called Olya, a guest at Palibino, happened to be sitting on the uncle Fedor's knees instead of Sonya. Sonya was stunned.

> "I looked at her, I looked," wrote Sofya, "and then, suddenly—even now, I swear, I don't understand how it came about—something awful happened. It was as though someone else was pushing me. Without a conscious thought of what I was doing, I suddenly, to my own amazement, sank my teeth into her bare, chubby little arm, slightly above the elbow, and I bit her till the blood came." [68, p. 131]

Kovalevskaya drew some pictures of her early childhood which clearly showed a situation unfavourable for upbringing.

It is morning and the nursery is stuffy and unventilated.

The governess enters the room, puts a handkerchief to her nose, and tells Nanny to open the transom.

> "What's she got into her head now, the heathen?" Nanny grumbles as the governess leaves, "I can just see myself opening up the windows so the master's children can catch cold!" [68, p. 50]

Still in bed, unwashed and uncombed, the children get their coffee with cream and rich fat buns, and sometimes after breakfast, tired out by all the pranks, the children fall asleep again. Morning toilet is accomplished with great speed.

> "Nanny wipes our faces and hands with a wet towel, passes a comb once or twice through our tousled manes, dresses us in dresses which are often missing a few buttons, and there it is we're ready!" [68, p. 51]

Now "not constrained by our presence, Nanny sweeps the floor with a brush, raising a cloud of dust," Sofya continues. The children were seldom taken out for walks unless the weather was especially fine or it was a major holiday, when Nanny would take them to church.

Some of the fairy tales Nanny recounted seem to have made such a strong impression on Sonya that even when she had grown up she would dream about them occasionally, and sometimes "the black death", "the werewolf", or "the twelve-headed dragon" would reappear.

> "And these dreams," Sofya wrote, "always call up in me the same unaccountable, breath-stopping horror I felt when I was five years old listening to Nanny's fairy tales." [68, p. 57]

She began to show other symptoms of extreme sensitivity such as a revulsion bordering on horror for any kind of physical deformity, for example.

> "Altogether, I was well on the way to becoming a morbid, neurotic child. Soon afterward, however, my whole environment changed, and what had gone before came to an end". [68, p. 58]

Quoting Fedor Korvin-Krukovsky [64] and Malevich [95], Ann Koblitz [284, p. 17] writes:

"When Sofa was about six, on her own initiative she embarked on what her brother considered to be "the first instance of her fully independent activity". According to Fedor, the grownups did not believe in teaching children to read at an early age, and Sofa desperately wanted to learn. She would examine newspapers for hours, trying to impress the indecipherable symbols on her mind. Then she would catch some adult, perhaps one of her father's unmarried sisters, and ask the woman to tell her what just one letter was. Sometimes she would approach the governess while she was preoccupied teaching Anyuta, and dance around her until finally, to get rid of Sofa, the exasperated woman would tell her the letter. In this way asking questions when she could not sound something (Russian has a largely phonetic system of spelling), Sofa soon taught herself to read and came proudly to her father to show off her skill.

At first, the general could not believe his daughter had done so much on her own. He thought someone had coached her to memorize a particular passage in his newspaper as a joke. When the indignant Sofa proved that she could read anything that was set before her, he was proud. This was the type of intellectual achievement and initiative which Vasily Vasilievich found much to his taste. Sofa's character resembled his own, and General Korvin-Krukovsky was pleased about this."

Soon after the move to Palibino,

"Father all at once made the unlooked-for discovery that his children were far from being the exemplary, beautifully brought-up children he had assumed they were."

There were many servants but the supervision over the children was lax in the extreme. Once the girls ran away to the forest and got lost. It turned out that Anyuta at twelve lacked a systematic education and didn't even know how to spell. A "bad thing" came to light about the French governess. The general resolved upon a radical transformation in the manor, which, as Sofya remarked, had been the seat, "for ages and generations with gentry customs, slovenliness and laxity" [68, p. 80].

He dismissed the governess, removed Nanny from the nursery and employed new people to bring up his chidren.

Miss Smith, the Governess

Two new personages were taken into the household: an English governess, Miss Margaret (Margarita Frantsevna) Smith, and a resident tutor, Iosif Ignatievich Malevich. Miss Smith was determined to bring up physically hardened, punctual, well-bred young ladies out of the Russian girls. She could not manage Anyuta, who became a "free Cossak", so she spent more effort on Sonya.

Sonya had now to get up early, when it was still dark in winter, and when she was still sleepy. Every morning, cold water was poured over her.

> "One second of a biting, breath-stopping cold, then a sensation like boiling water coursing through my veins, and finally, what remained was astonishingly a pleasant feeling of springiness and well being", wrote Sofya Kovalevskaya [68, p. 99]. There was a music lesson after breakfast, "accompanied by the monotonous striking of the stick with which the governess beat time" and that "subdued once and for all the feeling of exuberance" in Sonya. Then she was "faced with a dismal hour and a half together with my governess walking back and forth along the path".

The girl was glad when the weather was bad and she was "sent upstairs to get my exercise by playing ball in the drawing room" alone. She managed to deceive the English governess so that by bouncing the ball from time to time, she could read all sorts of books from the library near the hall.

Sonya loved writing verses and by the time she was twelve she was convinced that she was going to become a poetess. The two pieces of poetry she was proudest of were "The Bedoin's Salutation to His Horse" and "The Sensations of a Pearldiver". She also dedicated a poem to the Panama Canal, a project then being discussed in the press. Miss Smith tried to suppress Sonya's attempts to write verses. By contrast Malevich seemed proud of this trend in his pupil. He wrote in his memoirs that he especially remem-

bered a poem called the "Swimmer" (possibly, it was the "Pearldiver") that began "I throw myself into the water" and ended with "I've lost my spirit... and I'm dying" [95, p. 643].

The stern upbringing instituted by Miss Smith had its disadvantages but it undoubtedly had a positive effect on the girl since it encouraged her to work systematically and persistently from childhood. However, Sofya was by nature purposeful and insistent on reaching her goals, and I can quote her own expression, "intensity is my essence" [67, p. 546]. Professor Mittag-Leffler once said that if Sofya wanted to achieve something, her energy exceeded everything.

Iosif Malevich, the Resident Tutor

Iosif Ignatievich Malevich, Sofya's first teacher was a son of a petty gentry landowner. He was born in 1813 in Kreslavka, a small town in the Vitebsk province, and finished a six-year school there. He made up his mind to devote himself to teaching, passed the required examinations for the title of a resident tutor, and started bringing up and educating children in landowners' families. For instance, one by one, he had taught the six sons of Ivan Egorovich Semevsky, a Pskov landowner, one of whom, Mikhail, later became the publisher of the magazine *Russkaya Starina* (Russian Old Times).

Malevich stayed with the Korvin-Krukovskys from 1858 until early 1868. First, he taught Anyuta and Sonya, and then, when Fedya was eight, he taught him until Fedya entered a Petersburg high school. Then Malevich moved to the household of N. I. Evreinov, the Marshal of Nobility in the Nevel district.

> "After my retirement," wrote Malevich, "from the service as a resident tutor with the Ministry of Public Education, I moved to live in peace at Palibino, accepting General V. V. Korvin-Krukovsky's invitation." [95, p. 646]

When the general died, Malevich remained with the family; thus, he was living at Palibino in 1877 supervising the construction of a monument on the general's grave.

In 1890, when Malevich was writing his recollections of his famous pupil, he was 77 years old and complained of an eye disease that was making his work difficult.

It appears that Malevich was more interested in the humanities than in the natural sciences and mathematics; at the beginning he was more delighted by the literary talent of his pupil. He writes,

> "I was amazed and carried away by her truthful, capable, and well-worded viewpoint, against which even the best teacher of literature could not dissent. When I returned to my room after the lesson, I thought for a long time, and not only about the unusual progress of my talented pupil. I thought about what might become of a rich girl from a good family. What if her destiny were different? What if she did not have an excess of worldly goods and were only given the means for a higher education, which is, alas, unavailable for women at our universities? Then, oh, then, I was sure, my talented pupil would win a high position in the world of literature!" [95, p. 626]

Listing the other subjects that he taught her, Malevich describes his method of teaching and the textbooks he gave to his pupils.

In connection with mathematics he wrote:

> "When I first met my talented pupil in October 1858, I saw an eight-year-old girl, rather sturdily built, and nice and attractive. Her brown eyes shone with a perceptive intellect and cordial kindness. The first lessons revealed that she was uncommonly attentive, grasped things rapidly, was perfectly manageable, accurately performed what was needed, and invariably knew her lessons. In developing her abilities according to the method I described above, I could not discern any special talent during our first lessons in arithmetic. Things were going as they had been with my former pupils, and I was even embarrassed later, when once, at lunch, the general asked his beloved daughter whether she liked arithmetic. 'No, Papa', she replied. 'Then you shall like it, and you shall like it more than any other subject,' I said with some fervour. Less than four months later Sofya answered

almost the same question with, 'Yes, Papa, I like arithmetic, it gives me pleasure'.

"Her father smiled and seemed glad to hear this from his daughter. My pupil learned the elementary subjects well, and I became convinced that I could teach her further easily.

"Three or four years later our lessons continued successfully and without incident. But then we came in geometry to the ratio of the circumference to the diameter, which I presented with all the proofs and inferences, and I was amazed when my pupil made her presentation of the material at the next lesson, coming to the same conclusion but in her own way and using special combinations. I demanded she repeat the lesson, thinking that she had not quite remembered my presentation correctly. I told her that although her conclusions were correct, she should not have resorted to such a roundabout way, and therefore required her to present the material as I had given it. I do not know whether she was embarrassed by my unexpected demand or maybe I offended her self-esteem. However, she blushed deeply, cast down her eyes, and started weeping.

"I tried to calm her down with tender words, encouraged her, brought her to reason, repeated the material, and put it off until the next lesson. These were the first and last tears I saw from my pupil during lessons throughout all my nine years of teaching her. The same day I recounted the incident to the general, himself an old mathematician, and when I explained what had happened, he praised the ingenuity of his daughter and said, 'Well done, Sofa! This is nothing like my times. I would have been glad to have learnt a lesson at least a bit, and there she is, a chit of a girl, and all by herself she finds another way'. He was very glad and warmly shook my hand." [95, p. 640]

The desire to earn her father's praise and win his love was a great stimulus for Sonya during her lessons, especially in mathematics.

Fedor Korvin-Krukovsky, Sofya's brother, wrote in his memoirs about his sister that she was given an upbringing and education in agreement with the outlook of her class,

i.e. Sofya was destined to become a young lady, a woman of the world. She was managed in a routine manner, but "this manner could not satisfy her fervid and impressionable nature" [92, p. 632], and she had to fight for the freedom of her education and by her success to acquire the right to further her education.

According to Malevich, Sonya learned arithmetic for two and a half years, and then she studied a course of algebra written by Bourdon (in two volumes) [97];

> "she started geometry after completing half the algebra course; and finished plane and solid geometries by the seventh year of her studies" [95, p. 631].

There is an interesting reminiscence in which Sofya described an episode when she and her cousin Michel were given maths lessons by I. I. Malevich. Michel was the only son of a landowner's widow who dreamed of her son becoming an artist and studying the arts while travelling in Europe. The mother tried to talk him into entering high school to get general education and so the young man was preparing to enter the seventh grade, an action to which he agreed very reluctantly.

When he and his mother came to stay for a summer to Palibino, it was decided that Malevich, an experienced teacher, would give him lessons in mathematics. Whenever Malevich would present a theorem using his fine learned expressions, Michel would listen to it condescendingly, make remarks on the proof, and finally would ask, "Do you want me to prove the reverse right away?" and he would start talking nonsense, thus stumping Malevich.

"Evidently, Michel does not want to understand," Malevich said at last to Michel's mother, "and there is no purpose in his further studies." The mother appeared at the lessons, but to no avail. She entreated the teacher to think up something to make the lad to study.

"'Michel needs company,' decided Malevich, and he chose me for want of something better," wrote Sofya [67, p. 339]. She had perceived her role: it was necessary to prove that even a girl can easily understand the things Michel could not understand. Then the youth changed his tactics immediately. It was shameful for him to lag behind a girl, and a girl who was a year and a half younger. "Who cannot

understand such trivialities", he would then answer the questions. Sonya too spared no effort to be equal to her mission. The result was that appreciable portions of algebra and geometry were studied during that summer.

Petr Korvin-Krukovsky, Sofya's Uncle

Sofya Kovalevskaya believed that the first glimmer of her interest in mathematics appeared during her talks with her favourite uncle Petr Vasilievich Korvin-Krukovsky.

He was a tall white-haired old man who was considered to be a man "not of this world". He had handed over to his only son his entire estate, leaving for himself only a very small monthly pension [68, p. 112]. His wife had been a harsh serf-owner, and she had been murdered by her house-serfs.

Petr Vasilievich was only in the military service for a short time and retired with the rank of lieutenant. Having become a lonely man, he often stayed with the family of his brother. He was very well-read, and Sonya, who always listened avidly to the stories he told about what he had read, was his favourite.

He once arrived at Palibino for Christmas. But Elizaveta Fedorovna and her daughters were in Petersburg, from where they sent him their greetings. Petr Vasilievich answered Sonya on January 5, 1867.

"My priceless Sofa, I really wanted to spend your birthday with your father. We were very sad during the holidays as we had no news from you, but on January 3* the letters, your nice letter among them, were handed to us just after we had awoken and I was so glad to read it.

"Kiss the hands of your Mama and sister on my behalf and convey to them my cordial wishes that you, my priceless ones, should enjoy your life, and that not a single cloud should shadow your horizon.

"Be happy and healthy, and the Lord grant all your heartfelt wishes; but wish reasonably, and wish only what is possible.

Your truly devoted,
old Uncle
Petr Krukovsky."**

* Sofya Kovalevskaya's birthday.
** *LOA AN*, f. 768, op. 1, No. 53.

Her uncle's exhortation was in vain for Sofya was unable to wish for what was reasonable and wished for much what was possible.

Sofya herself tells us of the influence of Petr Vasilievich:

> "Although he had never studied mathematics he had the most profound respect for that branch of learning. From various books he had accumulated some smattering of mathematical knowledge and loved to philosophize about it, and then he often reflected aloud in my presence. It was from him for example, that I heard for the first time about squaring the circle, about the asymptote, toward which a curve approaches constantly without ever reaching it, about many other matters of similar nature. The meaning of these concepts I naturally could not yet grasp, but they acted on my imagination, instilling in me a reverence for mathematics as an exalted and mysterious science which can open up to the initiates a new world of wonders, inaccessible to ordinary mortals" [68, p. 122].

Sofya recalled an incident in her life at Palibino that encouraged her to pay attention to mathematics.

> "As I speak of these, my first contacts with mathematics, I cannot help mentioning a curious circumstance which also helped to arouse my interest in the field.
>
> "When we moved permanently to the country, the whole house had to be redecorated and all the rooms had to be freshly wallpapered. But since there were many rooms, there wasn't enough wallpaper for one of the nursery rooms. Because ordering wallpaper was a very complicated business, and it really wasn't worth while to go through all of that for just one room. It was all waiting for a propitious occasion, and in expectation of this the maltreated room just stood there for many years with one of its walls covered with ordinary paper. But by happy chance, the paper for this preparatory covering consisted of the lithographed lectures of Professor Ostrogradsky on differential and integral calculus, which my father had acquired as young man.
>
> "These sheets, all speckled over with strange, unintelligible formulas, soon attracted my attention. I remem-

ber as a child standing for hours on end in front of this mysterious wall, trying to figure out at least some isolated sentences and to find the sequence in which the sheets should follow one another. From this protracted daily contemplation, the outer appearance of many of these formulas imprinted themselves in my memory; indeed, their very text left a deep trace in my brain, although they were incomprehensible to me while I was reading them". [68, p. 122]

An Episode from Childhood

In her diaries for 1863-1864, Elizaveta Fedorovna Korvin-Krukovskaya wrote some notes about events that were somehow connected with the Polish rebellion of 1862-1863. In April 1863 Elizaveta Fedorovna wrote,

"We had but a few guests for a whole month; Vasily had been at a community conference and on April 30 he went to Vitebsk for the nobility elections."*

She wrote then that her husband had met the new provincial governor.

"We now are awaiting the decision as to whether Vasily will be appointed the province's Marshal of Nobility... This is a very troublesome post, bearing in mind the involved Polish affairs."

However, she was glad because it would mean she would have more chance to visit Vitebsk. Vasily Vasilievich held moderately conservative views, and he was the right figure for the Marshal of Nobility.

In her *Memoirs of Childhood from the Times of the Polish Rebellion* [67, pp. 342-357], Sofya described an episode that characterized both herself as a girl of about fifteen, and the public opinion of the surrounding society.

Neighbouring landowners, many of whom were old Poles (the young ones had either perished in the rebellion of 1862-1863, or had been exiled to Siberia, or had emigrated) came to a family holiday, Elizaveta's name-day, on September 5, 1865. There also was an uninvited and unwanted guest, Colonel Yakovlev, who had been sent to the province by

* *LOA AN*, f. 768, op. 1, No. 29, p. 51.

tsar's Governor-General Muraviev; Muraviev was the one who had dismissed the civil administration and replaced it with military officers.

There was a rumour that many high-ranking officers had resigned after they had been ordered to go to Poland. Others who had taken some part in armed suppression of the rebellion declined the role of executioner. But Colonel Yakovlev had become the military commander of the Vitebsk province and incurred everybody's hatred.

Sonya wholeheartedly sided with the Poles and even started clandestine lessons of Polish with Malevich. She admired the young and handsome Polish landowner Buinitsky, who was paying attention to the girl who was sympathizing with the rebels. He wrote in her album a verse in Polish that read,

> "Child, if I never see you again, I shall cherish a fond memory of you forever.
>
> "How happy I would be if I could see the blossom of the bud that is about to burst.
>
> "But my destiny does not grant me this happiness, and I can only bow down before its beauty at parting!" [67, p. 349]

Kovalevskaya wrote that she had been engulfed in joy and sorrow at the same time while reading these lines:

> "What was the meaning of this verse? I was happy and proud that he had devoted it to me, but my heart ached with sad anticipation".

Several days passed, and Buinitsky disappeared. Maybe he went into the forest to join the rebels, maybe he emigrated, maybe he perished or was exiled to Siberia. Sonya was convinced that he had been sent to the Siberian mines and she dreamed about going to Siberia, finding, and liberating him!

And so when she learned that Colonel Yakovlev was going to be present at dinner, she began to indulge in fantasies,

> "Tomorrow, just as he sits down at the table, I'll take a large knife and pierce him through the heart while crying, 'This is for Poland!' Of course, I'll be arrested, put in irons and exiled to Siberia where I shall meet Buinitsky!" [67, p. 352]

However, things turned out somewhat differently. Yakovlev let himself go and, wanting to show his talents, wished to draw a picture in Sonya's album. The grownups told Sonya to bring her coveted album. She had glued up Buinitsky's verse so that only her pictures could be seen. Nonetheless Yakovlev's picture seemed to her to be a blasphemy. All of a sudden the girl snatched the album out of Yakovlev's hands, tore the page with his picture into smithereens and trampled on them. Naturally the governess grabbed her by her arm and pushed her into a dark closet. The girl was punished, but only it seemed to her for the sake of appearances while the sympathy of the other guests was on her side. It was somehow explained to Yakovlev that Sonya misbehaved because she envied his excellent drawing.

Anyuta, Sofya's Sister

Sonya's spiritual development was greatly influenced by her elder sister Anyuta (Anna). Anna was an exceptional girl. Bored in the backwoods province of Vitebsk, she began to devour English novels, tales of chivalry, and then she started writing long stories herself, revealing an extraordinary talent. She had a lively imagination and identified herself with the heroines of the novels she was reading. She too was capable of casting her thoughts into poetic form. Sonya admired her sister and called Anyuta her spiritual mother. When Anyuta disclosed to Sonya that her story "The Dream" [99] was to be printed in the *Epokha* (Epoch), a magazine published by Dostoevsky, Sonya was utterly thrilled. Her sister was a writer!

But General Korvin-Krukovsky looked at it differently. He did not like women writers and considered it shameful that his daughter was an authoress. But after the story had been read to him, he softened his attitude and then even agreed to Anyuta and her mother's request that they meet Dostoevsky in Petersburg.

Fedor Mikhailovich Dostoevsky fell in love with Anna Vasilievna (Anyuta), who was very pretty,

> "tall, slender, with a flawless complexion, and a mass of fair hair, she was virtually a raving beauty; and besides that she had a great deal of her own special charm" [68, p. 135].

But when sometimes Dostoevsky was angry or wanted to tease her, he would compare Anyuta's face with Sonya's dark complexion. Sofya remembered her childish love for Dostoevsky. She suffered immensely when she knew that he wanted to marry Anyuta and was amazed when her sister refused him. Sofya ably portrayed her grief in her *Memories of Childhood*. On one occasion she had, to surprise Dostoevsky, learned Beethoven's *Pathétique*, his favourite, even though the sonata was too advanced for her. She played it once with enthusiasm for him, but when she had finished, there was only an unexpected silence. It turned out that Dostoevsky had in the meanwhile been telling Anyuta of his love for her in his "passionate, spasmodic whisper" [68, p. 192]. One can imagine how hurt poor Sofya had been!

Anna's youth coincided with the period characterized by Lenin as the first revolutionary situation period. Anna went through a deep intellectual crisis when the ideas of enlightenment that were flourishing in the Russian society of the 1860's reached the backwater province of Vitebsk.

As Sofya relates in *Memories of Childhood*, the signs of new times had begun to be noticeable in the immediate vicinity of Palibino. Aleksei Filippovich, a son of the parish priest, refused pointblank to enter the priesthood after completing his course at the seminary.

> "He preferred to go off to Petersburg to enter the University and study the natural sciences as a paying student, dedicating himself to four years on tea and a crust of dry bread."

During his very first holiday he came home and

> "started talking rubbish to the effect that man, allegedly, is descended from monkeys, and that Professor Sechenov had proved, allegedly, that there is no soul, only reflexes. Poor Father Filipp was so taken aback that he grabbed his holy water basin and started to sprinkle his son from it." [68, p. 148]

Anyuta obtained through Aleksei Filippovich such periodicals as *Sovremennik* (The Contemporary) and *Russkoe Slovo* (The Russian Word), the organs of the revolutionary democrats and the "raznochinets"* intelligentsia,

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* A 19th century Russian intellectual not of gentle birth.

"each issue of which was regarded as the event of the day by the young people of that time. On one occasion he brought her an issue of Alexander Herzen's illegal publication *Kolokol* (The Bell)." [68, p. 151]

Anyuta developed very rapidly under the influence of her discussions with the young man and her reading of the books he brought her, and she changed with each passing day.

"She changed even outwardly. She began to dress very simply in black dresses with plain collars, to wear her hair pulled back and covered by a net. Now she spoke contemptuously of balls and social visits. In the mornings she would call the servants' children together and teach them to read, and when she met peasant women out walking, she would stop them and have long conversations with them." [68, p. 152]

But the most remarkable thing of all was that Anyuta, who had formerly loathed her studies, now displayed a passion for learning. When Sonya was very little, she had great pleasure when she could be present at her sister's lessons. She listened with attention and sometimes, when fourteen-year-old Anyuta failed, eight-year-old Sonya would triumphantly prompt her. Sonya was amused but her admiration for her elder sister was unshakable. Now Anyuta would spend her pocket money on serious books, order whole boxes of books bearing such sage titles as *The Physiology of Life*, *The History of Civilization*, and so forth (*Ibid.*).

Elizaveta Fedorovna relates in a letter of this period that Anyuta would sit for whole days indoors poring over Aristotle or Leibniz and filling sheets of paper with excerpts and comments.

As to Sofya, she was often called later a "nihilist", because she joined the social movement of the 1860's.

M. V. Nechkina wrote, "The historical science of Marxism has long since revealed the positive social content of the concept of 'nihilism'. It was objectively a stage in the revolutionary-democratic raznochinets movement, which sprang up after the revolutionary situation of 1859-1861. Headed by the vanguard publicists, the 'people of the sixties', who lived on the ideas of Chernyshevsky, 'nihilism',

which was best expressed by Pisarev, Chernyshevsky's follower. The movement acquired its slighting title from the enemies of progress and it daringly took up the challenge. The Latin 'nihil' meaning 'nothing' seemed to the young protesters to be most suitable for their militant social character. Subjectively, the nihilists did not want to take anything from the old order; they strove to overthrow the foundations of feudalism, fought against the old social structures, gentry aesthetics, and the moribund morales of the privileged exploiting classes. 'Nihilism' swears on the name of Chernyshevsky, and a 'nihilist' believes himself to be his follower who has gone further along his path." [100, p. 493]

The designation "nihilist" would stick to Kovalevskaya to the end of her life. Naturally, the Russian administration understood the word "nihilist" in quite a different manner. The censor Nikitenko cited an example of such peculiar understanding,

> "The Nizhny Novgorod provincial governor identifies as nihilists all women who 'wear round hats, blue glasses, hoods that conceal their short hair and do not wear crinolines'; he orders the police to arrest them, make them take off all these garments, and if they resist, to exile them from the province." [101, p. 581]

Sofya Adelung, Anyuta's and Sonya's cousin, wrote in her memoirs about the sisters who stayed, together with their mother, with the Adelungs in Stuttgart en route to Switzerland in 1866-1867.

When the family conversation touched on politics, Anyuta also participated. Adelung wrote:

> "A deep furrow would appear on her forehead when she spoke, and how well she could talk! She has already absorbed many political and social ideas... Her soul dreamed of reforms. Her pretty head was full of thoughts and almost completed projects and plans. Should someone happened to utter the word 'gendarme', she would become furious, and one could talk of nothing else with her for a long time. She would often sit quietly and silently for hours on end, but when she was inspired she talked with extraordinary eloquence, and words would fly from her

while she was walking hurriedly back and forth in the room, trembling with indignation and burning with wrath or plunging deep in sorrow." [102, p. 402]

Sofya Adelung, who was the same age as Sofya Kovalevskaya, was far from being interested in social problems then. Although understanding Anyuta's oratory, she

"was glad to admire her cousin... and believed her to be the wisest, most intelligent, and beautiful being she had ever met." [102, p. 403]

Adelung remembered that she loved Anyuta's soft mezzo-soprano, especially when she sang Lermontov's "A Cossack Lullaby".

The reason for Anyuta's feelings and in particular her indignation towards the gendarmes is simple. Following D. V. Karakozov's unsuccessful attempt upon the life of Tsar Alexander II in 1866, the reaction in Russia worsened, not only were those implicated in the attempt arrested but so were those merely suspected of holding radical views. No doubt, there were victims among the acquaintances of Anyuta and Sonya. For instance, P. L. Lavrov, an old friend of their family, was arrested in spring 1866 and V. A. Sleptsov was imprisoned during the same period. Sleptsov's commune had been organized in 1863 and "was loud in the chronicles of the epoch" (see K. Chukovsky's Introduction to V. A. Sleptsov's book *Difficult Time*) [103, p. 12].

Sofya Kovalevskaya knew of Sleptsov although we do not know when they met. That the prominent revolutionary democrat influenced Sonya deeply is beyond all question. This can be seen in the first Swedish edition of the *Memories of Childhood* [43], in which Kovalevskaya called Sleptsov one of the inspirers of the minds of Russian youth, along with Dobrolyubov and Chernyshevsky. There is a short description of Sleptsov in the extant fragment of Kovalevskaya's novella about Chernyshevsky. If the Korvin-Krukovsky sisters had not met Sleptsov by 1866, then they had at least heard of him.

During the time the sisters were abroad (they stayed at Montreux for a while), the congresses of the First International (International Workingmen's Association) were

held in Switzerland: the first congress was convened in Geneva on September 3-8, 1866, and the second met in Lausanne on September 2-8, 1867. A congress of the League for Peace and Freedom was held in Switzerland during the same period. The activities of the congresses were widely covered in the newspapers of the time. There were also many Russian émigrés in Switzerland. Therefore we can agree with I. S. Knizhnik-Vetrov [104] who surmised that there were additional influences on Anyuta when she was abroad. Moreover, it may be assumed that in Switzerland Anyuta had conceived the idea, two years later the conviction, to go to Paris and join in its intense social life.

But back to the memoirs of Sofya Adelung. According to her, Sonya was the shadow of her big sister and therefore it was no surprise that Sonya had the same viewpoints as Anyuta. But Adelung notes that Sonya could also thrill others with the

> "inspiration that shone in her eyes and sounded in her words". Sonya "was always ready to go through fire, to die like a martyr for her lofty ideals, for humanity" [102, p. 400].

Noting the "feverish state that was seizing a part of the Russian youth", Adelung offered the opinion that Sofya Vasilievna came to marry Vladimir Kovalevsky because she was

> "under pressure from the others who carried her away with their political and social catechism, and the new ideas and outlooks of the period. Anyuta was in the first ranks of this movement, she was older and more experienced and drew her younger sister into all the exciting events in which she was herself absorbed and from which she could not disentangle herself throughout her life". [102, p. 404]

What events has Sofya Adelung had in mind? Anyuta went abroad with Sonya in 1869 and settled alone in Paris. She worked as a type-setter for some time, then she met Charles Victor Jaclard, a French revolutionary, and married him. Both of them became prominent activists in the Paris Commune. On March 18, 1871, Victor Jaclard led the armed forces in the area of Monmartre; he was elected member of

the Central Committee of the National Guard and commanded its 17th legion; he exhibited great courage in the battles with the Versailles soldiers. Anyuta was a strong supporter of the Paris Commune and an active participant; she helped organize its hospitals and took part in commissions on the "women's question" and education. Her signature can be found under many of the Commune's proclamations.

After the defeat of the Paris Commune the Jaclards could not live in France, and so they emigrated first to Switzerland and then, in 1874, with their year-old son to Russia.

Sonya loved her elder sister dearly all her life.

> "Darling Anyuta, you are so nice, so fine, so great that I am ready to bow before you to the earth. Anyuta, my best, my purest, most sensible ideal...,"

wrote eighteen-year-old Sonya to her sister on December 28, 1868 [105, p. 87].

Chapter 2

YOUTH

First Lessons in Advanced Mathematics

The Korvin-Krukovsky family was visited at one time by Petr Lavrovich Lavrov, a professor of mathematics, and Nikolai Nikanorovich Tyrtov, a Professor of physics of the Naval Academy. Tyrtov had written a textbook on physics that was popular at the period [106], and he presented a copy of the book to General Korvin-Krukovsky. He was greatly surprised when Sofya, who was then about fourteen years old, became interested in the text and began to study it on her own.

In the section on optics she encountered the sine and tangent functions, which she had not known yet. Possibly, Malevich could not satisfy her curiosity for in his memoirs he wrote that he was only going to do trigonometry with her (probably because of her interest in physics). But events interfered with this intention: in early September 1866, because the elder sister was ill, Sofya went with her mother and Anyuta to Switzerland to winter there. Although Malevich went abroad the next year to be with his pupil, the lessons were only given in snatches. They all came back to Russia in late 1867.

But Sofya had begun to think over the enigmatic formulas herself and she deciphered them when she took the chord of a small angle for its sine (see [67, p. 370]).

Tyrtov was amazed when he visited the Korvin-Krukovskys next time and saw that Sofya had recreated on her own the primary theorems of trigonometry. He praised the girl in front of her father, calling her a "new Pascal" and advising the general to give his daughter a chance to study mathematics seriously.

It was decided that Sofya would start these studies in Autumn 1867, when she was going to St. Petersburg with her mother and sister, and she would continue her lessons during later visits to the capital.*

Tyrtov recommended to General Korvin-Krukovsky to take on Aleksandr Nikolaevich Strannolyubsky, a student at the Naval Academy and a lieutenant in the Navy, as his daughter's teacher.

Aleksandr Strannolyubsky was born on the Kamchatka Peninsula, where his father was in charge of the district. Aleksandr studied at the Naval Cadet Corps in Petersburg and graduated in 1856 as a midshipman [107, 108].

Fascinated by the ideas of Chernyshevsky, Pisarev, and Dobrolyubov, he joined the enlightenment movement of the "people of the sixties" and participated in a student circle that organized the free of charge school on Vasilievsky Island. Strannolyubsky was elected the first inspector of the school where the standards of teaching were very high.

In 1866 the school was closed like all the other free and Sunday schools. Strannolyubsky graduated from the Naval Academy in the same year, and the Academy sent him abroad to study the theory and practice of navigation. A paper of his was published later in the *Morskoi Sbornik* (Marine Collection) [109].

Sofya's lessons with Strannolyubsky came to an end in 1868. In the same year Strannolyubsky became a permanent teacher of mathematics at the Marine School and worked there for about 30 years. Aleksei Nikolaevich Krylov, an outstanding specialist in mechanics and ship-buidling, was one of his students. Strannolyubsky contributed greatly to the development of higher education for women in Russia [110].

The first Higher Courses for Women were opened in Moscow in 1872 (the courses headed by Vladimir Ivanovich Gerie) while the Women's Doctor Courses began in Petersburg at the same time. In 1876 Professor Sorokin established Higher Courses for Women in Kazan. In 1878 the courses that became known as the Bestuzhev Courses in honour of their official founder K. N. Bestuzhev-Ryumin [111-113] opened in Petersburg.

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* St. Petersburg (now Leningrad) was then capital of Russia.

But the establishment of these courses was not a final solution of the problem of higher education for women. For instance, the Bestuzhev Courses were closed for new students in 1886 and opened again after a certain transformation in 1889. For 14 years Strannolyubsky was a member of the Fundraising Committee for these Courses. He taught algebra at the Alarchin Courses, which preceded the Bestuzhev ones, and compiled a textbook on algebra, published a number of articles on the education of the people, and wrote reviews for several textbooks: more than 60 articles. In 1891 he wrote his memoirs of his pupil Sofya Kovalevskaya [114].

A. N. Strannolyubsky died in 1903. His obituary said,

> "A. N. Strannolyubsky was an educated and noble representative of the brilliant galaxy of the educators of the 60s. He was fair, firm, and held deep principles, a man who never compromised his conscience in any deed. His noble carriage, forceful intellect, universal education, rare humanism, and grace won him sincerely warm sympathy and the deep respect of everyone he met." [115, pp. 89-90]

This was the man who gave Kovalevskaya her first lessons in advanced mathematics.

Sofya admired Strannolyubsky. She later remembered,

> "I took my first lesson in differential calculus from the eminent Petersburg Professor Aleksandr Nikolaevich Strannolyubsky. He was amazed at the speed with which I grasped and assimilated the concepts of limit and of derivatives, 'exactly as if you knew them in advance'. I recall that he expressed himself in just those words. And, as a matter of fact, at the moment when he was explaining these concepts I suddenly had a vivid memory of all this, written on the memorable sheets of Ostrogradsky; and the concept of limit appeared to me as an old friend." [68, p. 123]

In a letter to Anyuta Sofya wrote:

> "Strannolyubsky was with us the whole evening. He was not at all angry when I told him that besides mathematics I was going in for physiology, anatomy, physics,

and chemistry; on the contrary, he agreed that mathematics alone is too dull and advised me not to dedicate myself to science exclusively but engage in practical activities as well." [105, p. 220]

Besides mathematics, Sofya took physics lessons from Fedor Ivanovich Shvědov, who held a magister's degree from Petersburg University. Strannolyubsky approved of her desire to get a broad education. And in general he encouraged her noble social aspirations and introduced her to the ideas of the "people of the sixties".

There are two known letters of Strannolyubsky to Sofya Kovalevskaya. One of them (undated) was sent when Sofya was taking mathematics lessons from him [75, p. 104]. Strannolyubsky wrote that he could not give the lesson because he was not well and would call on Sofya the next Monday, when he would be going from his lesson to Petrova, and until that time he asked Sofya to review her algebra.

The second letter, dated November 2, 1880 [107, p. 126], reveals that Sofya Kovalevskaya was a member of the Fund-raising Committee for the Bestuzhev Higher Courses for Women and that Strannolyubsky was the secretary of the committee.

Vladimir Onufrievich Kovalevsky

During the 1860s the best part of Russian society strove for enlightenment: the desire was to be of benefit to the people in its fight against the gloom of ignorance and the oppression of the tsarist government. Chernyshevsky's ideas on the reconstruction of the society, and the emancipation and equal rights for women were widely accepted. A movement to open schools and universities for women gained ground, but no such institutions had been established by that time and some Russian women had gone abroad, especially to universities in Switzerland, mainly to get a medical education.

When Anyuta Korvin-Krukovskaya told her father that she wanted to go to Petersburg to study, he was enraged. But Sofya also wanted to broaden her education. She dreamed of opening a new road for women and becoming a scientist. To achieve this, it was necessary to go to lectures in a uni-

versity. The sisters began to think of going abroad for that. There were other girls who wanted to join them, but it was evident that they would encounter strong resistance from their relatives.

Fictitious marriages were then a common practice. Young men, who held progressive views on women's education and who wanted to help the women, married girls thus freeing them of their parents' rule. The newly-wed young man then gave complete freedom to the girl, and she went abroad alone or with the young man. Not uncommonly these fictitious marriages became genuine ones.

L. F. Panteleev in his book [123] informs us there was a man who wanted to help Sofya to go abroad. This was Ivan Rozhdestvensky*, a priest's son, who was a typical democrat and nihilist, and always ready to provide comradely assistance. He was brave enough to come to General Korvin-Krukovsky and ask for the hand of his daughter. When the general asked him about his social status, he answered, "I am engaged in free pedagogics." The general thanked him for the honour of his proposal but flatly refused saying that his daughter was far too young to get married [64, p. 492].

Anyuta and her friend Anna Mikhailovna (Zhanna) Evreinova zealously began to look for young men who would be eligible fictitious husbands; they called them "preserves" in their correspondence. One such was Vladimir Onufrievich Kovalevsky, a publisher then [116].

Vladimir Kovalevsky was born on August 2(14), 1842, in the family of a petty gentry landowner in the Vitebsk province. His father was a collegiate registrar Onufry Iosifovich Kovalevsky and his mother Polina Petrovna. Their estate was at the village of Shustyanka and was not far from Palibino. Vladimir Kovalevsky first studied at the Megin boarding school in Petersburg and then, when he was twelve, at the aristocracy's School of Law.

When his mother died, his father's family proved to be hard-pressed for money, so Vladimir Kovalevsky took the

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* I. Rozhdestvensky participated in the student movement of 1861 for which he was imprisoned and exiled from the capital. Later he founded a school in his village but was banished by his father who did not sympathize with his son's ideas.

initiative and applied to be kept at public cost. The application was approved. He was good at foreign languages and managed to make money on the side by translating books while he was in the senior grades.

During his last year at the School of Law Kovalevsky went to meetings of the circles of revolutionary students. P. D. Boborykin wrote that

> "he was amazing, in comparison with the students, in his inquisitiveness, his easy assimilation of all the sciences, his unbelievable memory, the sharpness of his dialectics (even though his voice was still that of a child), and his unusual tendency to participate in any movement. That was the time he started his publishing activities by translating textbooks." [94, p. 230]

In 1861 Kovalevsky obtained a job in the Department of Heraldy at the Senate, but he soon applied for a four-month leave to go abroad for medical treatment. He went to Heidelberg, Tübingen, Paris, and Nice, where he met his friends from the student circles. He did not return to his work but settled at the end of the year in London, where he met Herzen and started to give lessons to Herzen's daughter Olga. Kovalevsky struck up a friendship with Herzen and often saw the Russian émigrés in London. He was followed by agents of tsar's gendarmerie.

In 1863, the émigrés he knew persuaded Kovalevsky to go to Poland to take part in the Polish uprising, which was soon defeated.

Upon his return to Petersburg, Kovalevsky started a lively business translating textbooks, scientific books, and books on science. In 1866 he also published Herzen's novel *Who Is to Blame?* without naming the author. A second edition was printed, but all the copies were burned by order of the tsar's censors. This incurred heavy losses on Kovalevsky.

Vladimir Kovalevsky was a brisk, energetic, and passionate man. He did not strive to make a profit out of his publishing affairs, his aim was to propagate a certain ideology. He was not really a businessman and lacked punctuality. For instance, he could leave a book cover unsigned for weeks on end, and the book was greatly delayed; he often owned money to his translators. To begin with, his books were

always sold out, but then the enthusiasm of the broad public for Western natural scientists subsided, and the demand slumped. At the time he met the Korvin-Krukovskys, Kovalevsky owed about twenty thousand roubles, and had a hundred thousand roubles' worth of unsold books in his warehouse.

In 1866 Kovalevsky took part in Garibaldi's campaign to free Italy. He was in the leader's camp, among his closest assistants, and sent reports from the battlefields to the *Sankt-Petersburgskie Vedomosti* (St. Petersburg Bulletin) [117].

In Petersburg, before his departure to join Garibaldi, Kovalevsky became good friends with the family of the outstanding social activist N. V. Shelgunov. He fell in love with Mariya Petrovna Mikhaelis, the younger sister of Shelgunov's wife. She was an enthusiast of the social movement of the "people of the sixties" and was later arrested many times and finally exiled. When she was twenty, she was prosecuted for demonstrating during the "civil execution"* of Chernyshevsky: she was in the crowd at the site of execution and threw a bunch of flowers to Chernyshevsky when he was returning from the scaffold.

Vladimir Kovalevsky and Mariya Mikhaelis were to be married in September 1865. But two hours before the wedding ceremony, the bride and groom announced that they were not going to marry each other. L. P. Shelgunova, Mariya's sister wrote that they had argued over something principal. The two young people separated but remained friendly.

Now Kovalevsky gave his consent to become the fictitious husband of the younger Korvin-Krukovskaya sister because he too hoped to be of benefit to science. He wrote to his brother:

> "Though she is only eighteen, the little sparrow** is well educated, knows languages as well as she knows her native tongue, and she is at present learning math-

* Civil execution was a disgracing punishment for the gentry in 18th-19th century Russia. The executed person was tied to a pillory and a sword was broken over his head as a sign of deprivation of the basic rights, rank, etc. Chernyshevsky was thus executed in 1864 and sent to Siberia for hard labour.

** Sofya was short.

ematics, and poring over spherical trigonometry and integrals; she is as busy as an ant, from morning till night, but she is also lively, sweet, and has very good looks. On the whole, this is a happiness that has come like a bolt from the blue." [83, p. 59]

General Korvin-Krukovsky did not like Vladimir Kovalevsky at all. He did not have a solid financial position and could not, in the opinion of the father, bring happiness to his daughter. However, Sofya made every effort to marry Kovalevsky, though she kept secret from the parents the true reasons for the marriage. She was so sure that fictitious marriages were rational for the general well-being that she also made a great deal of effort to provide them for other girls of her circle.

When Vladimir Kovalevsky became friends with Sofya, it was a great stimulus for him to study science. He threw up his own subject, jurisprudence, and started to learn the natural sciences, an interest which was also helped by his elder brother, Aleksandr Onufrievich Kovalevsky, being a prominent zoologist.

Vladimir Kovalevsky's publishing business helped him broaden his knowledge. He read the books he published, sometimes translating and writing the introductions himself. These books were, for instance, *The Antiquity of Man* and *Principles of Geology* by Lyell and *Zoological Essays* by Vogt. Kovalevsky wrote two articles for a book called *Natural Historical Reader*. In one of them he described the ideal of popularising science:

"The sciences in general and the natural sciences in particular have the goal of improving and raising the level of the life of man and his society." [74, p. 197]

By Autumn 1869 when Kovalevsky and Sofya arrived in Heidelberg to attend lectures on physics, geology, and mineralogy, Vladimir was not an ordinary student, but someone seeking his path in science. He was especially interested in paleontology. After Heidelberg he visited many other German cities, met paleontologists, examined paleontological collections, and collected many fossils himself. It is amazing, but he even managed to study mollusks and then fossil mammals while he was in Paris during the heat

of the war between France and Prussia, and then during the period of the Paris Commune.

Kovalevsky was interested in how the different species of animals emerged, how each evolved, and how they acquired their modern appearance. "How can such a beast as, say, the hippopotamus come about?", he asked [74, p. 202].

He studied the evolution of the horse from the anchitherium, one of its ancestors, on the basis of Darwin's teaching. Vladimir Kovalevsky wrote to his brother from Jena on December 27, 1871, that he was getting ready to take the examination for a doctor's degree and was preparing part of his work on the anchitherium for his dissertation. He added that his work was

> "the only and so far the first... true application and verification of evolution; the whole work is a detailed comparison of four animals, of which the forefather was the *Paleotherium Medium* in the early Tertiary formation, the great-grandfather (my anchitherium) lived in the early part of middle Tertiary (Miocene)." The next form, the grandfather, existed, as Kovalevsky assumed, in America. "The father is the hipparion from the upper part of Miocene, and then comes the horse in the Pliocene. But the transition is so amazing that I am simply thrilled." [116, p. 139; 74, p. 203]

In 1872 Vladimir Kovalevsky passed his examination in Jena and received a Ph. D. in biological sciences. The following year his investigation "On the anchitherium and the paleontological history of the horse" was published by the Petersburg Academy of Sciences in French (see [118, 119]).

For a short period after the examination, Kovalevsky visited a number of cities in Germany, Switzerland, France, and Italy thinking over new research on fossils.

In his second important work, this time on fossil ungulates, the anthracotheriides, he advanced a theory that is now called "Vladimir Kovalevsky's law" and which Kovalevsky himself called "the double method of development". He made a distinction between the adaptive and inadaptive simplifications in the development of organisms causing their evolution.

The Kovalevsky's third and largest work was dedicated

to the anthracotherium*. The wonderful thing is that all these investigations were undertaken in such a short and troubled period, between 1870 and 1874. He was going through a number of unfavourable situations: he was anxious over his publishing business, he suffered a slanderous attack against him, etc. This though was a period of creative inspiration in the life of a prominent scientist, a period that never came again.

Before his articles and monographs appeared in print, Kovalevsky was known to many Western scientists via personal contacts. Russian sciѐntists understood the importance of his works and they were quickly printed in Russia. Charles Darwin praised his research, which supported the theory of evolution.

Kovalevsky kept up a correspondence with Charles Darwin and in 1867, when Kovalevsky was publishing Darwin's *The Variation of Animals and Plants Under Domestication*, he received from Darwin the page proofs, which were immediately translated, so that the Russian edition was printed earlier than the English original. Kovalevsky corresponded with Darwin later and visited him more than once. The second volume of this book was translated by Vladimir Kovalevsky and Sofya together. Sofya Kovalevskaya edited the Russian translation of Darwin, corrected five quires of it, and translated five quires.

Fictitious Marriage

Sonya was optimistic in the Summer of 1868 when her marriage to Vladimir Kovalevsky was agreed upon in principle. In August she wrote to her sister in Petersburg of how she imagined her future life. It was full of self-sacrifice, renunciation, and entirely dedicated to work. She wrote that she was studying hard and regularly without her sister and there was not a minute of tedium.

> "At night, when I was tired of studying and started walking about my room, I even had some moments of delight that flooded me. But strange as it may seem, although personally for me all seems well and right,

* A genus of Tertiary artiodactyl mammals of Asia and Europe related to the pigs but sometimes as large as the rhinoceros.

I have never before felt so strongly our ominous fatum and the need for asceticism...

"When I think of asceticism, I always imagine a small and very poor room in Heidelberg, very difficult and serious work, and no society, I live alone (it is not asceticism with brother*, but happiness; asceticism is when I am alone); twice a week I get letters from Anyuta who is very busy, but next winter she too is going to go to Heidelberg, because it is only in summer that she needs to be in Russia. She will bring with her several other young ladies whom she developed and freed." [105, p. 234]

It is not hard to imagine a young man who has made up his mind to dedicate himself to science and believes that he is going to become an ascetic, because science demands utter devotion. Such may have been the dreams of many young men who later became prominent scientists and scholars. But Sonya's dreams are also related to the current social mood requiring complete renunciation of everything personal. She went on with her dreams in her letter:

"I am getting ready for my examination, writing my dissertation. Anyuta puts her travel notes in order; later I study on my own, still later we together set a colony, and I go to Siberia. I find a mass of difficulties and disillusions there, but I can manage to be of help there nonetheless. Anyuta writes a wonderful paper; I succeed in making a discovery; we establish a high school for girls and another one for boys; I have a physics laboratory of my own. Now I do not go in for medicine, I am engaged in physics and the application of mathematics to political economy and statistics (this is *ad libitum*). And there is a whole family of our protégés around us.

"At the time when I am making my discoveries, and Anyuta writes her wonderful works, we are actually younger than the youngest of our pupils. This life is indeed bliss, although it should be as ascetic as I can imagine, and it depends exclusively on the two of us; it is on purpose that in my dreams I estrange even Zhanna and our darling, nice, and fine brother; what a life it will become when they join us!" [105, p. 237].

* Vladimir Kovalevsky was called "brother".

It is possible that Anyuta's mood of the time also brought about these dreams of ascetism. Young Sonya was sure that this was the only thing to dream of. But she felt that such a life was going to be hard for her, and she added,

> "It is only hard for me to live alone, I just have to have somebody to admire every day, you know what a doggy I am.
>
> "My darling, darling dear, I am so carried away while I am writing to you that I forgot the depression that was gripping me when I started to write. I had a gloomy disposition yesterday, misgivings, but they were dispelled...
>
> "Well, I shall be patient for a week, and maybe the square root of —1 will overcome even my frustration." *(Ibid.)*

She was frustrated because Vladimir Kovalevsky and her mother had been delayed; the carriage they were to travel in had burned. And although Sonya knew that they were unable to arrive in time, she was startled by a rustle and could not find anything interesting in her usual studies.

The feelings of a young and inexperienced girl were complicated; she was getting ready to embark on a road that was quite unusual for a general's daughter.

Sonya was engaged in a number of things and wrote of them in August to Kovalevsky:

> "I have also understood the working principle of the ophthalmoscope, and by the time you arrive I shall know the eye well; but I really study chemistry and physiology too little, only some two hours a day... There are many sick people in the neighbourhood, as usual in autumn; every day sometimes up to ten people come to me to get medicine; I read a book of home cures and get cross that I am not a doctor yet. I think that I am going to develop a passion for treating people.
>
> "The pike, my comforter, has passed away and, alas, it was most deplorable and ungainly; a mole-cricket lives here now, but it is a small comfort.
>
> "I am translating Darwin diligently..." [105, p. 219]

Obviously, the interests of Vladimir Kovalevsky were becoming the interests of Sonya. He in his turn tried to

engage himself in his science, and Sonya advised him to use his time to the maximum, but he was not to worry, she would overtake him.

Vladimir wrote to his brother,

> "What with all my experience, book-reading and push, I cannot grasp and comprehend political and economic problems half as rapidly as she does, and rest assured, she is not just carried away, this is a sober analysis.
>
> "I think that she will make of me a decent person, that I shall drop my publishing business and start research, although I cannot conceal from myself that her nature is a thousand times better, more intelligent and talented than mine, to say nothing of her diligence: they say she works in her country house for twelve hours a day without a stop, and as far as I can see for myself here, she can work like I never could.
>
> "On the whole, she is a young phenomenon, and why I have got it I can't imagine." [71, p. 67]

It looked as if both of them, bride and bridegroom, were preparing for a creative life.

The wedding ceremony of Vladimir Kovalevsky and Sofya Korvin-Krukovskaya took place on September 15(27), 1868, in Palibino. To overcome her father's resistance, Sofya had to "compromise" herself: she ran away to her fiance's residence and sent a note to mother she would not come back. Her parents had to consent.

One of Sonya's aunts wrote about the wedding,

> "At last, Sonya appeared; she was fresh, beautiful, and radiating with happiness, as only a bride can be." In her simple dress "she, however, looked charming... Not a single gem, no adornment. But she was so nice that all present claimed that they had never seen such an enchanting bride. The gleaming expression did not leave her face for a minute during the entire ceremony; and this was not an expression of a superficial excitement, but a deep sense of true happiness." [102, p. 413]

The aunt was mistaken in her explanation of Sonya's mood, but Sonya was indeed convinced that she was moving towards true happiness.

St. Petersburg

The Kovalevskys went to Petersburg immediately after the wedding. They started attending lectures given by I. M. Sechenov on physiology and lectures by V. L. Gruber on anatomy in the Medical-Surgical Academy. Ivan Mikhailovich Sechenov had taken the responsibility and had suggested that Sofya attend the lectures. He also gave the Kovalevskys practical lessons. Sonya went into the lecture hall accompanied by Petr Ivanovich Bokov, Vladimir Kovalevsky, and her uncle Petr Krukovsky: they helped her avoid the curiosity of others.

Sofya Kovalevskaya was surprised by the attention Sechenov, the great Russian physiologist, paid to her studies and she was uneasy because he spent so much time on the practical lessons with her and Vladimir Kovalevsky. On December 8, 1905, in a speech dedicated to I. M. Sechenov, the physiologist Professor I. R. Tarkhanov explained this in the following words,

> "I recall the manner in which he cordially greeted in his laboratory a talented, quite young mathematician, the famous Sofya Kovalevskaya, before she went abroad. He saw great scientific potential in her and opened the door of his laboratory for her to study." [120]

The range of Kovalevskaya's knowledge was wide, she recognized the increased interest of the Russian society in the natural sciences and could be enthusiastic about the lectures Sechenov gave. But as to the subject taught by Gruber, she wrote her sister, "Anatomy is such a boring thing!"

Her basic interest was mathematics. She was well prepared by Strannolyubsky and was by now capable of studying at university. But women were not allowed there.

Sonya did not sleep enough while she worked steadily and immensely to grasp various branches of knowledge. But she understood that her dreams of comprehensive education were unattainable, there was a need to narrow the scope of her studies. In late 1868 she wrote to her sister:

> "I am studying rather hard, but I am engaged in practically the same subjects as in Palibino, i e., mainly mathematics. You know, my incomparable Anyuta, I all

but made up my mind not to take a course of medicine, but to enter a physico-mathematical department. Don't you think it will be better this way? I am convinced now that I do not have the heart for medicine, no practical activity attracts me. I am only happy when I am immersed in my meditations; and if now, in my best years, I do not study my favourite subjects exclusively, then maybe I shall waste time that I shall never be able to recover. I have become convinced that an encyclopaedic education is no good and my whole life will hardly be enough to achieve what I think I can on the road I took." [105, p. 88]

The Kovalevskys had a growing intention to go abroad. The last straw was probably the closing of the Medical-Surgical Academy in the Spring of 1869 following a students' revolt.

In Petersburg Sofya Kovalevskaya started to feel her false position. She was uneasy in the large furnished apartment rented by Vladimir Kovalevsky. In her letter dated September 29, 1868, she wrote to her sister:

"In my present life, despite its seeming logic and completeness, there is a certain false note that I cannot determine, but which I feel nonetheless; I can explain it by your absence: you won't believe me, Anyuta, how lonely I am, in spite of all my happiness and all my friends. I feel that I can't be good without you, Anyuta. I am becoming completely sober after my former illusions, and I wish you knew, Anyuta, how I love you.

"The point is that we have complicated matters unnecessarily and forgotten that we are not the same people as the rest of the mankind, and God forbid that we become like them." [105, p. 223]

Having noticed that the sentence was awkward, she added:

"I am sorry for all this rubbish. Soon you will understand what I really want to say and how immaterial and elusive the things are I am complaining of.

"Brother is very dear, nice, and good, and I am sincerely attached to him, although naturally my friendship has lost its naive enthusiasm.

"You won't believe how he cares about me, courts me,

and is ready to submit his every desire and habit to mine. I am awfully ashamed to be so much obliged to him; I love him truly with all my heart, but as I would my younger brother." [105, p. 224]

Pafnuty Chebyshev

Pafnuty Lvovich Chebyshev was a Member of the Academy of Sciences and then the head of the Russian mathematical school and one of the most important persons in the world of science [122].

Once a week Chebyshev received at home all those who wished to consult him on mathematical problems. He wrote in a letter of October 3, 1889,

> "More than 20 years ago S[ofya] V[asilievna] (she was not married then yet) came to me for advice about her studies of mathematics, and all that happened to her later I know well."*

If Petersburg University had been open for women, there is no doubt that Sofya Kovalevskaya would have been a student of the great Russian scientist. But the situation in the universities for Russian women was difficult. In 1860, when the universities in Russia received a certain autonomy, women students began to appear at lectures given by certain professors. Everybody was allowed to attend lectures at universities in 1861, and military officers, clergymen, writers, and teachers could all be found. F. P. Tolstoy, a vice-president of the Academy of Arts, used to go to N. I. Kostomarov's lectures with his wife and children. The number of women who were permitted to attend university courses without having the formal status of student increased (although the question of their passing the examinations had not then been raised). Most of the professors then only considered the struggle of women for a higher education to be a fashion and they did not believe it heralded the beginning of a strong movement.

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* AN SSSR Archive (hereafter cited as *AAN*), f. 2, op. 1, No. 7, pp. 2-3 (also in [121]) (without an address, possibly to the permanent secretary of the Petersburg Academy of Sciences, K. S. Veselovsky).

L. F. Panteleev [123] writes that the professors of Moscow University discussed the admission of women to lectures and rejected the proposal overwhelmingly (only two voted in favour).

An end was put to all of the liberties of Petersburg University in 1862 when it was closed in connection with student unrest. When it was reopened in 1863, there was no place for women.

N. P. Suslova and M. A. Bokova (later she became Sechenov's wife), who were the first women doctors in Russia, were educated at the University of Zürich. A host of Russian women went to the universities in Switzerland, and this eventually promoted the opening of medical courses in Russia.

Consequently, by the time Sofya had become acquainted with Chebyshev, there was no chance for a woman to be admitted to Petersburg University.

Going ahead of myself, I can mention a message dated April 28 (May 10), 1874, which was sent to Sofya Kovalevskaya when she lived abroad. It is clear from this message that Chebyshev had been following her progress.

The message is unsigned and written in the hand of an old man. It is probably an accompanying note to a letter written by Petr Vasilievich Korvin-Krukovsky, Sonya's uncle. It says:

> "Chebyshev instructed me to send you, dear Sofa, the enclosed dissertation* for a Doctor's degree; he gave it to me during the defence of his dissertation. I believe it will be interesting for you: first, because, as far as I could understand from your letter, the subject of your dissertation is also concerned with the development of Abelian functions, and second, with respect to the work of Weierstrass.
>
> "Write to me in as much detail as possible about the progress of your dissertation."

Chebyshev showed interest in Kovalevskaya's advances throughout her scientific career; as we shall see, his interest increased as time passed.

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* Apparently it was by Yu. V. Sokhotsky, "On the definite integrals and functions utilized in expansions in series" (St. Petersburg, 1873).

Chapter 3

STUDENT YEARS

Lectures at Heidelberg

Having decided to go abroad, Sofya Kovalevskaya wanted to help other girls to do the same so that they might obtain specialized education. She heard that Yuliya Vsevolodovna Lermontova, the daughter of the head of the Moscow Cadet Corps, was interested in chemistry, and Sofya started a correspondence with her to persuade her to go abroad with Sofya. The problem was to convince both Yuliya's parents and Yuliya herself to make this step, a step that seemed both risky and frightening. A biography of Yu. V. Lermontova was written by Yu. S. Musabekov [124].

Vladimir Kovalevsky, Sonya, and Anyuta went abroad first. In a letter of April 16(28), 1869, to Yuliya Lermontova, Sonya described the first steps they took and the efforts they had undertaken to be admitted to the lectures.

They arrived first in Vienna and were permitted to attend lectures on physics. But Sofya made up her mind to try her luck in Heidelberg, where professors of mathematics were better than those in Vienna (she also indicated that another reason was that life in Vienna was too expensive).

Sofya left for Heidelberg with her sister. She went to the physicist Kirchhoff.

> "He proved to be a little old man on crutches," wrote Sonya to Yuliya, he "was amazed that women should have such an uncommon wish and announced that it did not entirely depend on him to admit me, and that I had to apply for permission to Kopp, the prorector of the university." [64, p. 237]

She had to go from one professor to another and at last a special commission was summoned. Information about

Sofya was collected in the meanwhile and one lady told Kopp that Sofya was a widow. This contradiction with what Sofya had said was considered important. Fortunately, Vladimir Kovalevsky arrived and confirmed that Sofya was right.

Admitting women to lectures was a new thing for the university, so the commission did not grant Kovalevskaya the right to attend lectures, but it allowed individual professors the right to give the permission should she apply to them. This is clear from a letter Yuliya Lermontova received from the commission, dated October 21, 1869, in reply to a similar application:

> "In accordance with the decision of the admissions commission concerning the precedent set by Mrs. Kovalevskaya, you may not attend every lecture; at the present time it is left entirely to the discretion of the professors, in which case they will consider whether you may attend chosen lectures, provided it does not cause any complication." [124, p. 34]

In other words, Kovalevskaya and Lermontova were not admitted as full-fledged students. Yuliya Lermontova was only permitted to study chemistry owing to the efforts of Kovalevskaya. This may be seen from a letter Weierstrass sent Sofya dated September 21, 1874. Informing her that he had been to Heidelberg, where he visited Koenigsberger and met Bunsen, Weierstrass wrote:

> "Bunsen, I think, did not know that you had become my student for he called you a 'dangerous woman'... He substantiated this by something which I would like to know whether it was related to me as it had truly happened. He had vowed not to take women, especially Russian ones, into his laboratories. Thus, he did not want to take Miss Lermontova and did not even want to hear of her. Then, allegedly, you came to him and began asking him so tenderly that he could not resist and changed his mind." [125, p. 190]

At the beginning, Sofya attended 18 lectures a week, and then 22, of which 16 were on mathematics. It is known that she went to Koenigsberger's and Du Bois-Reymond's lectures on mathematics, Kirchhoff's on physics, and Helmholtz's

on physiology; Kirchhoff and Koenigsberger allowed her to attend both their lectures and seminars on physics and mathematics.

Recalling later her life in Heidelberg, Yuliya Lermontova said that Sofya immediately drew the attention of the professors with her uncommon talent, and rumours of the amazing Russian girl student spread throughout the small town. Sometimes people stopped in the street to look at Sofya. Once a mother pointed her out to her child and said, "Here comes a girl that is very diligent at school." [64, p. 383]

Sofya kept aloof from the professors and students and was very shy. Once during some studies

> "her attention was arrested by a mistake that one of the professors or students had made in calculations written on the blackboard. The poor man was worried over the problem and could not understand where the trouble was. Sofya hesitated for a long time, then finally made up her mind and although her heart was beating strongly she stood up, went to the blackboard, and clarified the point." (*Ibid.*)

At the time it was not only Russian women who went abroad to study. Many Russian men (writers, scientists, and public figures) travelled abroad both because they wanted to and because they were compelled to emigrate.

Klimenty Arkadievich Timiryazev writes in his memoirs that when he lived in Heidelberg it was always in the company of the Kovalevskys: Vladimir Onufrievich ("famous geologist") and Sofya Vasilievna ("eminent mathematician") [126].

Vladimir Kovalevsky stayed for some time in Heidelberg and studied subjects related to geology.

Anna Vasilievna (Anyuta) finally got to Paris, but only by keeping it a secret from her parents, who sent their letters to Heidelberg. Sofya had to forward them to Paris. When father became aware that Anyuta had taken her destiny into her own hands he stopped sending her an allowance. Anyuta therefore started working as a type-setter. In August 1869 Vladimir Kovalevsky mentioned in his letter to brother that in four weeks Anna would be getting 120 francs a month. Sofya had to send about 300 roubles to

sister out of her allowance of 1000 roubles a year.

The Kovalevskys did not return to Russia during the summer holidays of 1869. Sofya studied hard, and according to Vladimir Kovalevsky, she finished a section of mechanics intending to take the doctorate examination in mathematics and mechanics a year and a half later. In September they both went to London which was where Vladimir Kovalevsky wanted to go due to his scientific interests and where he met Darwin.

On October 5, 1869, they visited Miss M. Evans who was known by her pen-name as George Eliot. Miss Evans induced Sofya to discuss the "women's question" with the famous philosopher G. Spencer (without mentioning his name to the Kovalevskys), and Sofya, young and zealous, argued for equal rights for women [67, p. 238].

On their way back the Kovalevskys were delayed in Paris (partly because of lack of money). From there Vladimir Kovalevsky was going to go, money permitting, to Vienna, where various prominent geologists resided, and Sonya wanted to go back to Heidelberg.

During their first year abroad, Vladimir Kovalevsky revealed his extraordinary liking for travel. At the beginning Sofya Kovalevskaya did not like their travelling, but later her life required her to do a lot.

The Women in the Footsteps

Yuliya Lermontova was the first girl to be directly influenced by Kovalevskaya. She was born on December 21, 1846 (January 2, 1847) into the family of a general who was a second cousin of the poet M. Yu. Lermontov, and who was very proud of this kinship.

Yuliya received an elementary education at home. She learnt the major European languages, read constantly and, what was rare for a girl at the time, was interested in chemistry. Her father employed good teachers to give her private lessons.

Sofya Kovalevskaya took to heart the organization of Yuliya's study and research. In 1874 Yuliya Lermontova received a doctorate for her research in chemistry at the same time that Sofya Kovalevskaya received her doctorate. When she returned to Russia, she worked in Petersburg for

two years in Butlerov's laboratory and then moved to Moscow where her parents lived.

Sofya's daughter, Sofya Vladimirovna Kovalevskaya, wrote some memoirs called "Life in Sweden in 1885-1891 with my mother". These notes remain unpublished but they contain an account of Yuliya Lermontova:

> "She... refused to marry and became engaged in the management of her small estate, wanting as far as possible to make it a model economy and to apply the knowledge of chemistry she obtained while studying abroad. After her return she first did some research in the laboratory of Professor Markovnikov, but something happened which she did not like to talk about and which made her leave the laboratory suddenly, and leaving behind all her chemical equipment which she had got abroad; she never returned. When I asked her why she had stopped her chemical studies, she replied, 'Well, I was not really gifted enough, I had no true love for chemistry, otherwise I would have overcome all the obstacles and attained something.' She was very much attached to us—to my mother, father, and me. After the catastrophe that had befallen father—he took his life in a fit of melancholy because he was tormented by pangs of conscience following an unlucky money affair—she devoted herself entirely to my mother and me."*

Two more girls, Zhanna Evreinova and Natasha Armfeldt, went from Russia to Heidelberg under the influence of Sofya Kovalevskaya.

Anna (Zhanna) Mikhailovna Evreinova was an adjutant-general's daughter. She dreamed of getting a higher education in law. Sonya vigorously helped Zhanna, and she ran away from home. She walked across the border on November 10, 1869 without a passport and under the fire of the guards. In Heidelberg Zhanna Evreinova attended lectures at the law department. In 1872 she went to Leipzig, but she was not admitted to the university to begin with. However, the King of Saxony John took her part and she passed her examination in his presence with flying colours.

Evreinova was the first Russian woman lawyer. She trav-

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elled across France, England, Italy, and East Europe, and studied the law of the southern Slavs using documents extant in the monasteries on the Adriatic coast. She published articles on law, mainly concerning equal rights for women.

When the magazine *Otechestvennye Zapiski* (Domestic Notes) was closed, Evreinova published a magazine *Severny Vestnik* (Northern Messenger) from 1885 to 1889, to which many eminent writers and columnists contributed.

Natalya Aleksandrovna Armfeldt was a cousin of Sofya's and the same age. She was the daughter of a professor of medicine at Moscow University. She ran away from her parents to study mathematics but was soon engaged in revolutionary and propagandist activity. Armfeldt was arrested several times and in 1879 was sentenced to hard labour in Siberia, on the Kara River, where the conditions for convicts were especially stringent. Selflessly she took care of patients and helped the poor there. She married an exiled revolutionary. In 1887 Natalya Armfeldt died of consumption.

Elizaveta Fedorovna Litvinova, née Ivashkina (with the stress on the last syllable), later wrote that Sofya Kovalevskaya was the bright star that guided girls who wanted to study. Kovalevskaya certainly was the lodestar for Elizaveta Litvinova who passed the university entrance examinations in Russia and was prepared in mathematics by Strannolyubsky. In 1872 Litvinova went to Zürich to continue her mathematical education.

The University of Zürich and the Zürich Polytechnical Institute were the only higher education establishments where women were admitted and could get their graduate without hindrance. However, in 1873 the tsarist government issued an "announcement" concerning Russian women students: they were ordered to stop attending lectures in Switzerland and return to Russia under the threat of being stripped of citizenship. The tsarist administration regarded the women's movement for education as a potential source of nihilism and socialism.

Many women students went back to Russia and started to fight the reactionary Russian government. The "going to the people" movement evolved: women of gentle birth went to work as weavers, spinners, feldshers, or teachers, and became revolutionary propagandists. This was the road

Natalya Armfeldt had taken.

Litvinova stayed on to study mathematics. First she went to Professor Mequet's lectures, and from 1873 to lectures given by Professor H. Schwarz. She graduated from Zürich University with a diploma as a teacher for boys' high schools (the diploma was invalid in Russia). In 1877 Litvinova presented in the University of Bern (to Professor Schläfli) her papers on the theory of complex variables and, having passed the examinations, received the degree of Doctor of Mathematics, Philosophy, and Mineralogy.

She only returned to Russia in 1878 and therefore had not complied with the government's decree. As a consequence, Litvinova was for the rest of her life restricted in her service and pension rights, and her requests to take the magister's examinations and teach at the Higher Courses for Women refused [127].

Litvinova began to teach mathematics at the A. A. Obolenskaya high school in Petersburg. First she taught the beginners grades, but in 1880 she became the ever first woman in Russia to receive the right to teach the senior grades.

Nadezhda Konstantinovna Krupskaya was one of Litvinova's students, and she praised Litvinova as a teacher:

> "In 1921, in the course of a discussion on trade unions, Lenin was telling that elementary schools should teach logical thinking, and I recalled the way Litvinova taught us that... Nobody told us that we were learning logical thinking, but that is what it was. Litvinova taught us another thing: to make generalizations... We started with typical mathematical problems. Litvinova asked us to look for similar ones in various books of problems. This was awfully fascinating for us. We began to think up problems of our own... She taught us todeduce rules ourselves." [128, p. 263]

N. K. Krupskaya became the wife and coworker of V. I. Lenin. When she finished high school, she was unable to study at the Bestuzhev Higher Courses for Women, because admittance had been suspended from 1886 to 1889. Officially, the courses were being reorganized, but Krupskaya wrote that it was done "on the orders of the tsaritsa, who belived that women should not study but should rather

stay at home looking after their husbands and children" [128, p. 262]. In 1889, when the Bestuzhev Courses were reopened, Krupskaya entered the Department of Mathematics where she "attended Imshenetsky's lectures with delight" (*Ibid.*).

Litvinova was a writer and columnist. She published more than a hundred articles on various aspects of education and ten biographies of outstanding people in the "F. F. Pavlenkov Library". These included biographies of N. I. Lobachevsky, V. Ya. Struve, S. V. Kovalevskaya, L. Euler, P. S. de Laplace, Aristotle, F. Bacon, J. Locke, d'Alembert, and Condorcet. The biography of Kovalevskaya was the first one printed in Russia.

In addition, Litvinova wrote a number of essays under the general heading "Rulers and Thinkers" [129] about La Harpe and Alexander I, about Descartes and Queen Christina, etc. The censors removed 75 pages.

These were the first women whose destinies were greatly influenced by Sofya Kovalevskaya.

Karl Weierstrass

Karl Weierstrass, Professor of Mathematics at Berlin University, was famous as a great scientist, Leo Koenigsberger being one of his students.

Weierstrass was a giant of thinking and has left an indelible mark on mathematics. His name is familiar to everyone who studied the theory of complex variables. Proceeding from the theory of real numbers he had developed he provided a logical substantiation for mathematical analysis, while his theory of analytic functions is very significant.

The name Weierstrass is deeply ingrained in many sections and theorems of mathematics: the Bolzano-Weierstrass theorem, the Weierstrass theory of elliptic functions; the study of sufficient conditions for the maximum of an integral (in the calculus of variations); the geodesic lines and minimal surfaces in differential geometry; the theory of elementary divisors in linear algebra; the application of series in the theory of analytic functions (in 1841, when Weierstrass was twenty-six, he knew of the theorem Laurent published in 1843); the theory of analytic continuation; the example

of a continuous function with no derivatives in any point; etc.

His father, Wilhelm Weierstrass, was a school teacher when he was nineteen and later became a civil official. He became a secretary of the burgomaster in Ostenfeld, and then moved to Westernkotten and worked there as an officer at the saltern.

Karl Weierstrass's mother, née von der Vorst, died in 1827 leaving four children: twelve-year-old Karl, seven-year-old Peter, four-year-old Klara, and Elisa who was barely one.

His father was an educated man: he knew physics and the phylological sciences, and he gave his children good knowledge of languages. In later years they corresponded with each other not only in German, but also in French and English. Peter Weierstrass also knew Latin well.

Karl Theodor Wilhelm Weierstrass was born on October 31, 1815, in Ostenfeld, Westphalia.

There were no good schools in Ostenfeld, and therefore Karl was sent to Paderborn, a town not far from Münster (the principal city of Westphalia). Karl studied at the high school in Paderborn from 1829 to 1834.

The French order of stimulating pupils through their ambition prevailed at the school. When a pupil had the highest marks in three subjects, a choir would perform a piece in his honour and a new piece for each of the other subjects in which the pupil had also gained the highest mark. Karl was usually honoured with four pieces, because he would have six highest marks (and once he had even seven). The subjects invariably included the German language, but mathematics alternated with other subjects. The only subject he could not master was handwriting.

Karl started studying mathematics with a schoolfriend, but the schoolfriend was awed by Karl's superiority, dropped mathematics, and switched to another subject that seemed more interesting.

Weierstrass entered at the sixth grade of the high school (in this school pupils usually began at the eighth grade and finished in the first); he studied in the fifth and fourth grades, but skipped the third grade. He finished the high school in 1834 when he was nineteen.

Western Prussia had universities in Münster (though it

was called an academy) and in Bonn. The latter was considered one of the better ones in Prussia. In 1834 Weierstrass entered the Law Department of Bonn University.

When he became an old man Weierstrass liked to recall his years at university. He was a zealous member of the students' corporation "Saxonia" (and even received the rank of Fuchsmajor): he never missed a single merry-meeting. His brother noted with pride that Karl, who was nimble in fencing, had never been wounded at a duel.

Weierstrass does not seem to have become interested in law and, apparently, he did not pass any examination, though once he happened to be an active opponent during a defence of his friend's dissertation.

At the same time Weierstrass studied mathematics on his own. *Celestial Mechanics* by P.S. de Laplace and *Fundamenta Nova* by K. G. Jacobi were his main books. He attended Plucker's lectures for a semester. Christoph Guderman, a professor in Münster, gave lectures on the theory of elliptic functions, and somehow Karl got a copy of the lectures.

In 1838 Weierstrass returned home without a graduation certificate from the university. He was twenty three and then made up his mind to complete his mathematical education and obtain a diploma of school teacher. He went to Münster and registered for lectures in the Academy from Summer until Autumn 1839; he attended only one course of lectures announced by Guderman. The course started with 13 auditors but soon only one remained, Weierstrass.

In 1841 Weierstrass passed a *facultatae docendi* examination (to give lectures) at Münster. He had had to work on either of three topics, out of which he chose one himself; he had worked on it since 1840. E. Lampe remarks [131] that even then Weierstrass was an able mathematician: he laid down the foundation of the theory of power series and analytic functions. This theory became the basis of his further work. Weierstrass's initial studies were published in 1894 in the first collection of his works.

In 1841-42 Weierstrass was in Münster for his "test year" in order to obtain the rank of a high school teacher. By the way, he wrote in his application for the examination that for half a year (from Autumn 1838 until Spring 1839) he had "suffered in body and spirit". Evidently, his life as a student was not as simple and serene as he later recalled.

Possibly, there were some feelings, or it can be explained by the beginning of the disease that troubled Weierstrass when he was overstrained or had some worries.

Following his "test year" in Münster, Weierstrass began work as a teacher in the Catholic school in Deutsch-Krone (a town to the north of Poznan) and worked there for six years.

As Weierstrass later recollected, this was a period when he "helped" the censor of the local newspaper: he admitted "The Song of Freedom" by Georg Herwegh, who was believed to be "the bard of the dawn of the 1848 German revolution"; the poet was then in the zenith of his fame but was not favoured by the royal court.

In 1848 Weierstrass became a teacher in a Catholic high school in the city of Braunsberg. The first years of Weierstrass's life and work are associated with two cities where Catholicism was strong. His father was converted from Protestantism to Catholicism, and all his children were Catholics, "although without fanaticism" [132, p. 2]. Weierstrass worked in Braunsberg for eight years and therefore he was a teacher for fourteen years.

During these years Weierstrass was thoroughly engaged in mathematical research. The director of the high school in Braunsberg regarded his studies with respect. Once Weierstrass failed to attend his own lesson in the morning, and his pupils created turmoil in the class. The director went to where Weierstrass was living and found that Weierstrass had been thinking about his mathematics all night, and had not noticed it was morning time; his lamp was still burning as he sat thinking over a problem.

The result was an article on the theory of Abelian functions, which was published in *Journal für die reine und angewandte Mathematik* (*Journal of Pure and Applied Mathematics*) now known as *Crelle's Journal*, founded and edited by August Leopold Crelle. It was dated September 11, 1853 (Weierstrass was then 38), and bylined Westernkotten. His father was living there at the time and his son came to him for holidays. It appears that this was the first article that Weierstrass published in a large mathematical journal. F. Klein noted that Weierstrass had been giving his earliest results in the weekly papers of the schools in Deutsch-Krone (1843) and Braunsberg (1849).

The article in the journal, which was edited by Crelle, made a great impression in mathematical circles and was recognized as the best work in the field since the studies of Jacobi. In 1854 Weierstrass was granted the degree of Doctor *honoris causa* by the University of Königsberg where Richelot, under the influence of Jacobi, was working on the theory of elliptic functions. Richelot himself visited Weierstrass in Braunsberg, and was followed by Borchardt from Berlin (Borchardt succeeded Crelle as the editor of the *Journal of Pure and Applied Mathematics*). He wanted to pay tribute to the new mathematical luminary. Borchardt and Weierstrass became close friends and remained so for twenty years until Borchardt died.

In 1856 Weierstrass went to Berlin. The chair of pure mathematics in the Industrial Institute of Berlin had become vacant, and Weierstrass had been nominated to take it. The Minister of Commerce approved the appointment, and the new professor was solemnly presented to the meeting of the faculty on June 16, 1856. The Minister of Education confirmed Weierstrass's tenure as extraordinary professor of Berlin University on November 11 of the same year. Almost at the same time, on November 19, he was elected a Member of the Berlin Academy of Sciences.

According to established tradition, Weierstrass gave his inauguration speech in the Academy of Sciences on July 3, 1857, the birthday of Leibniz. The academician-secretary, the astronomer Encke, cordially greeted the new academician, and the recent school teacher was accepted into the ranks of the most eminent scientists. In the expression of Lampe, it was Kummer, Weierstrass, Borchardt, and Kronecker who made up the "excellent round table" [131, p. 51].

Weierstrass settled in Berlin when he was 41, and he lived there for the remaining 40 years of his life, leaving the city only to go to the country or to a resort. Once he stayed in Göttingen for 24 hours and remarked that the trip spared him writing a dozen letters. He lived in the company of his two unmarried sisters, and his father stayed with him for the last years of his life. A biographer points out that Karl Weierstrass and his brother remained bachelors for financial reasons [132, p. 8].

Several years later Weierstrass gave up the chair in

the Industrial Institute and concentrated his activities in the University. In early 1864 he became ordinary professor of mathematics. This was a chair that was specially established for him, the third one in the University. The first two chairs were headed by Martin Ohm and Ernst Kummer. Weierstrass occupied this chair to the end of his life.

Weierstrass was greatly respected among the faculty owing to his open nature and good disposition. In 1873 he was nominated Rector of the University, and he took the post for a year.

Given Poincaré's introduction of the division of mathematicians into logicians and geometricians, Weierstrass belonged to the former. He believed that the function theory should develop logically, without the support of demonstrative representation. It was forbidden to make inferences from geometrical figures. Mathematical understanding was assumed to be in the absolute completeness of its substantiation. Weierstrass paid tribute to the genius of Riemann, but he could not apply his geometrical theories himself [133, p. 41].

Lampe, Mittag-Leffler, and Iris Runge (Carl Runge's daughter) marked that Weierstrass was an excellent teacher, friend, and advisor to everybody who worked with him. However, Felix Klein noted that it was difficult to be a student of Weierstrass, because a student was more dominated by his authority than stimulated to independent thinking [134, p. 327]. This was perhaps true for some of his students when they were not sure of themselves.

Klein says that Weierstrass enjoyed an absolute and indisputable authority, and that he avoided Weierstrass's lectures out of his spirit of rebellion but he was sorry later. Runge explains this feeling of rebellion by saying that Klein was intrinsically a geometrician [133, p. 28].

Typically, Weierstrass took a long time before he published his works. He always wanted to proceed from a single general idea and impart to his mathematical investigations the greatest possible harmony and completeness, but he rarely held this to be attained. Numerous auditors came from all over Europe to listen to his lectures on the theory of functions of a complex variable. In 1883-84 there were so many students that he had to give his lectures

on the theory of elliptic functions in the large hall of the Chemical Institute. The manuscripts of Weierstrass's lectures circulated among mathematicians. Later some mathematicians published whole books presenting his ideas, often without truly understanding and sometimes, as Weierstrass complains in one of his letters to Sofya Kovalevskaya, without reference to their source [125, p. 196].

In 1882 a French mathematician J. Molk sent Kovalevskaya the Weierstrass's proof of the transcendence of π and wrote,

> "Do you find as I in the methods of Mr. Weierstrass both extraordinary clarity and the flavour of strictness that is rare in the numerous memoirs published daily?" [85, p. 14]

Nothing human was alien to Weierstrass. He had no musical ear (unlike his sisters and especially his brother), but when he was 35 or 36 he tried to study music, without much success in the opinion of his younger sister. He liked poetry and sometimes quoted verse in his letters to Kovalevskaya. He wrote poems himself. Occasionally, Weierstrass went to theatre with Sofya Kovalevskaya and Yuliya Lermontova or with Sofya and the physicist Gustav Hansemann, when the latter persistently invited him.

Weierstrass was sympathetic and kind and this was why his friend Borchardt gave him the guardianship of his six children in his will. When Borchardt passed away Weierstrass helped his widow in the legal problems of inheritance.

A review of Weierstrass's scientific activity makes one think over why no one suggested he go into research after he passed his examinations in Münster. It is no secret that Gudermann esteemed the abilities of his student very highly and wrote that Weierstrass "with his work entered on equal footing into the ranks of glorious researchers" [135, p. 195], and expressed the hope that he might become an academic docent.

A rather interesting investigation by Professor Biermann [135] revealed that Diellenberg, the chairman of the examination commission, noted the praise given by Gudermann, but the latter for some reason did not consider it necessary to convey it to his student. Although this had serious con-

sequences for his entire career, Weierstrass only saw the Gudermann's conclusion in 1853, after his death. Later Weierstrass told Hermann Schwarz that if he had known earlier about his high assessment (K.-R. Biermann noted that Weierstrass had criticized Gudermann's methods, but nevertheless Gudermann looked upon this favourably), he would have tried to publish his results sooner and striven for a post at the university.

Without the good opinion of his first and only teacher in advanced mathematics, Weierstrass passed his most creative years isolated from the scientific world. His health was poor, and possibly his disorder was due to strain: he combined intensive scientific effort with strenuous teaching.

There were vacancies in mathematics in the 1850s in some of the German universities, but the number of contenders was much greater. Therefore for Weierstrass the support of such major scientists as Crelle, Dirichlet and Alexander von Humboldt, who was the patron of many talented people, was significant.

Sofya Kovalevskaya and Weierstrass

In 1870 a new stream flowed into the life of Weierstrass: Sofya Kovalevskaya became his student.

On October 3, 1870, twenty-year-old Kovalevskaya went to fifty-five-year-old Weierstrass and said that she wanted to attend his lectures. Weierstrass enquired and received a good reference of Kovalevskaya from Koenigsberger and he wrote that he would readily take her on. His audience had diminished because of the war between Prussia and France. Instead of 50 auditors he had had the previous year, he had then only 20. But the university council did not permit it, so Weierstrass started giving Kovalevskaya private lessons.

Weierstrass's students recalled that he was their good friend and counsellor; these qualities were greatly pronounced with respect to Kovalevskaya. The teacher called Sofya, his pupil, his only true friend, and shared with her his thoughts and hesitations.

The courses given by Weierstrass were a complete cycle which was the foundation of his mathematics from bottom to top, starting from the concept of number and terminating

with the theory of Abelian functions, step by step through several semesters.

Carl Runge was a student at the University in the Winter of 1877-78; when he approached Weierstrass in his reception hours to seek advice about whether it was worthwhile attending his course on Abelian functions, the professor asked him in detail about his previous experience and answered negatively. Runge was very sorry, but when Weierstrass started a new cycle the next semester, Runge began attending his lectures and completed the full course given by Weierstrass. Adolf Hurwitz was among the auditors, and Runge became friends with him; during their walks together they could talk endlessly about Weierstrass's mathematics, admiring it and discussing its various niceties [133, p. 35].

Very soon Kovalevskaya became Weierstrass's favourite pupil.

Yuliya Lermontova studied chemistry in a private laboratory. They found a place to live not far from Weierstrass. Usually Sofya went to lessons twice a week, and once a week Weierstrass would visit her. At the lessons he repeated the lectures given to his students, talked about his investigations, and discussed the news in science. He considered with Sofya the problem of non-Euclidean geometries whose special cases were the geometries of Riemann and Lobachevsky. He told her of his geometry of a finite world for his fellow mathematicians were not interested in it (very few details of it can be found).

There are 88 extant letters of Weierstrass to Kovalevskaya. Most were published by Mittag-Leffler [136], and the entire set was published in German and Russian in 1973 [125]. The first 44 letters belong to the period when Kovalevskaya was Weierstrass's student, i.e., 1871-74 [125]. The correspondence started on March 11, 1871, with a note saying that Weierstrass could not visit Kovalevskaya the day before, but that he would wait for her at his place the day after [125].

Anna, Kovalevskaya's sister, was in Paris in March of that year. The news of Anna and Victor Jaclard becoming activists of the Paris Commune came to the Kovalevskys. Sofya began to worry about her sister who was in the middle of the revolutionary events in besieged Paris, and Sofya

wanted to see her. Vladimir Kovalevsky decided to accompany Sofya and soon found an excuse for his trip: he wanted to see the works of Paris paleontologists.

The Kovalevskys stayed in Paris for 38 days in the heat of the revolution, from April 4 to May 12. Soon after they returned to Berlin they heard of the defeat of the Paris Commune and hurried again to Paris to help the Jaclards. Anna had managed to leave the city and avoid arrest, but Victor Jaclard was arrested and the Kovalevskys summoned General Korvin-Krukovsky to Paris. There are different versions of the way Victor Jaclard was freed. One version is that Jaclard escaped when he was being taken from one prison to another. Another version is that he was freed at the command of Thiers who was approached by General Korvin-Krukovsky.

In June Sofya Kovalevskaya returned to Berlin to continue her studies in mathematics. Weierstrass notified his student in a message dated June 3, 1871 (Saturday), that he would be at home after lunch and would be glad to see her.

Sofya Kovalevskaya went back to her studies with Weierstrass in the autumn, after spending the summer holidays in Russia.

A letter from Weierstrass dated November 6, 1871 contained a note to the head of the university library with a request to include Kovalevskaya in the list of subscribers. A letter dated November 14 of the same year was supplemented by a copy of *The Minimal Surfaces* by H. Schwarz [125, p. 152].

In 1872 Weierstrass began in his letters to touch and sometimes to discuss in detail the mathematical questions that he considered in his lessons with Kovalevskaya. Thus, in a message of March 25, 1872 he asked Sofya to note down the proof of the theorem they had considered the day before, because he wanted to give it to a friend [125, p. 153].

In a letter of June 14, 1872 Weierstrass informed Sofya of an indisposition that would prevent him from receiving her the next day and sent sketchy notes concerning theta functions.

I would like to quote the whole letter that Weierstrass sent on the morning of October 26, 1872 [125, p. 154]:

"My dear Sofya,

"I have just found in my papers several old notes concerning the simplest problem of the calculus of variations. Despite a different method of notation, I think you would be able to use them for your home work, and therefore I am sending them, assuming that you have not yet begun your working day.

"I thought very much about you last night; it could not be otherwise: my thoughts wandered in different directions but always returned to the point of which I shall have to talk to you today.

"Do not be afraid that I shall touch on questions that we have agreed not to tackle at least now.

"What I am going to tell you is related mainly to your scientific aspirations, but I am not sure whether you, because of your nice modesty with which you judge what you have attained so far, would be inclined to agree to my plan. But it is better to discuss all this in person.

"Although only several hours have passed since our last meeting which drew us so much closer together, I am asking your permission to visit you again today before lunch for an hour to speak my mind.

Sending my cordial regards,
Yours, Weierstrass."

Mittag-Leffler explained this letter by pointing out that the day before Sofya had revealed to her teacher the secret of her fictitious marriage [136, p. 137].

Probably, someone had already related to Weierstrass the rumours about the unusual relationship between Vladimir and Sofya Kovalevskys. On the other hand, it appears that he was not unaware of Sofya being uncommonly depressed. The point was that she had separated from Vladimir and the separation from her fictitious husband was hard for her. The separation lasted for more than a year. In Spring 1873 they renewed their correspondence. (There are six letters from that period from Sofya to Vladimir in the Archive of the USSR Academy of Sciences*.) It resulted in their reconciliation which transformed their union into a genuine marriage. Sofya guessed that Vladimir had been

* *AAN*, f. 603, op. 1, No. 11.

so jealous of her friends and her teacher that his self-esteem had begun to suffer (he started to talk about a divorce), etc. Sofya could sincerely tell him that his suspicions were ungrounded; she had laughed at his previous attempts at reconciliation: he had sent her shoes from London.

Weierstrass wanted to direct the thoughts of his student to a definite goal: to obtain a Doctor's diploma. Sofya and Vladimir had already aimed at getting such diplomas. The problem was only time and the theme. Apparently, Weierstrass had thought over the selection of problems for Sonya.

In his later letters, when Weierstrass was ill, he became more intimate with Sofya. The first of these letters, dated November 4, 1872 [125, p. 154] begins as follows:

> "My dear friend,
>
> "I am sending you a sheet of paper that you forgot to take with you yesterday, and I have to draw your attention to certain points."

He said that he was sending her two theorems concerning linear differential equations. Then he presented his ideas on the variation of definite integrals; the next day Weierstrass sent a correction of his previous presentation,

> "Yesterday, dear Sonya, I made a mistake at one point, and I am hurrying to correct it, so that you would not get involved in erroneous inferences." [125, p. 156]

The next letter was written in the morning of November 22, 1872. Weierstrass explained why he had not answered her letter with mathematical questions at once,

> "But you will excuse me when you hear that because of a cold I did not feel well either yesterday or the day before, and yesterday I did not quite feel mathematically inclined.
>
> "Today I feel much better and can send you in writing the desired information." [125, p. 159]

He explained some of the properties of theta functions, but remarked that she would not have to use them because he had thought up a new method of investigation. He wanted to present it in person and asked Sonya to visit him. He added

> "Do not worry that you will be a nuisance, this can never happen to my sweet friend." (*Ibid.*)

In a message of December 27, 1872 during the Christmas holidays, Weierstrass invited Sofya Kovalevskaya and Yuliya Lermontova to a concert. Yuliya later recalled that Weierstrass and his sisters held a Christmas-tree party for the two Russian girls.

In a letter of April 25, 1873, Weierstrass wrote to Sofya,

> "However much you can and wish to learn from me, you will stay my student in the best sense of this word; I would not have pointed this out had there not been a place in your letter which did not direct me to it." [125, p. 164]

Sofya had said that she only burdened him, but Weierstrass objected,

> "Rest assured that I shall never forget that it is the gratitude of my student that I am obliged to by having not only my best, but also my only true friend. Therefore, if you will retain the disposition to me you have so far revealed, you can firmly count that I shall always faithfully support you in your scientific aspirations." (*Ibid.*)

This letter refers to a number of letters that were written when both Weierstrass and Sofya were ill, in April 1873, and during a later trip by Sofya to Switzerland to rest and visit her sister.

Weierstrass advised his student to stop studying until she was strong again and reminded her the words of a physician to the effect that

> "besides camomile tea, there is only one medicine that is firmly known to be beneficial, and that is fresh soft air!" [125, p. 161].

Hermann Schwarz

In Autumn 1873 Sofya Kovalevskaya arrived in Zürich where her sister was living at the time. Elizaveta Fedorovna and General Korvin-Krukovsky also arrived to see their grandson Yuri, Anna and Victor's son, who was born on March 18.

This was the period in the life of Sofya Kovalevskaya Elizaveta Litvinova wrote about using her diary notes [137].

Just before that Litvinova had decided to turn to the well-known German mathematician Hermann A. Schwarz, who lived in Zürich, with a request to help her get acquainted with his works. Anna Jaclard had advised her to do this as she had heard about the Zürich mathematician from Sofya. Sofya knew his theory of minimal surfaces.

Schwarz received Litvinova very warmly; when he heard the name Kovalevskaya, he livened up and said,

> "Oh, she is a wonderful woman; our common great teacher Professor Weierstrass writes to me so much about her studies. Recently he sent me her compilation of his lectures on Abelian functions. You will not have been able to study them yet, they are the most difficult subject in mathematics, and few men dare tackle them." [137, p. 37]

Sofya Kovalevskaya told Litvinova that she had already had a paper suitable for a Doctor's degree, but she was in no hurry because she did not see a professorship waiting for her. A certain work by Sofya was mentioned: Weierstrass had wanted to send it to a mathematical journal, but then he received from Schwarz an investigation of the same problem that had already been published.

When Sofya met Schwarz, she felt an urge to collaborate with him; they happened to have common scientific interests.

Schwarz and his wife visited the Korvin-Krukovsky sisters, and later he talked with Sofya about mathematics several times. Without intention, each time he aroused in Sofya a desire to stay in Zürich. She learned from Schwarz that Weierstrass had been appointed Rector of Berlin University, and she thought that he would probably not have time enough for her.

> "As if in answer to her thoughts wise Weierstrass wrote to her that he would in any case have time for her", wrote Litvinova. [137, p. 40]

There is a later letter, of August 20, 1873, in which Weierstrass wrote to Sofya.

> "If it seems to you that this winter you will see me very rarely, you misunderstood me: in any event we shall not give up our Sundays, and I shall still be able to find an hour that I can devote to my sweet friend." [125, p. 166]

He asked Sofya to delay her return to Berlin because of cholera in the city; she should not believe newspapers but wait for certain news about the end of the epidemic from him. Weierstrass wrote this letter on the island of Rügen where he was resting. Describing the beauty of the sea and the island with its meadows, forests and rye fields, Weierstrass expressed his regret that Sofya was not there with him,

> "How fine it would be were we both here. You with your soul that is full of fantasy, and I, excited and refreshed by your enthusiasm. We could dream and think here of the many problems that we have to solve: of finite and infinite spaces, of the stability of world systems, and about all the other great problems of the mathematics and physics of the future. But I have long since learned to resign myself to the fact that not every wonderful dream can be realized." [125, p. 167]

Sofya gave up her intention to stay in Zürich. She spoke about that with Litvinova who recalled their conversation:

> "'Well then, it is not my lot to stay here, I have to go to Berlin, and Yuliya is already there', Kovalevskaya said. 'Do you esteem Schwarz higher than Weierstrass?' I asked.
>
> "She said, 'Ah, not at all, but I am already at home with the ideas of Weierstrass, and here is, you know, the charm of novelty that fascinates me. But of course I can always manage and I live where I have to'.
>
> "And when I asked her the reasons for that "have to" she answered,
>
> "'My destiny, or, if you wish, the main goal of my life, but I like more the word destiny, because the goal of my life is in myself, while destiny is of divine origin. I feel that my destiny is to serve the truth, that is, science, and to blaze the trail for women, because that means to serve justice'.

"'I am very glad that I was born a woman, because this gives me a chance to serve both truth and justice at the same time. But it is not always easy to follow your destiny ...'

"The day before her departure from Zürich, Sofya wishing to please Schwarz went to his lecture with me. All during his lecture Schwarz's eyes were lit up with an inner fire, and after the lecture he could not restrain himself any more and told his students with whom they had the honour to attend the lecture." [137, pp. 45, 58]

In May Sofya Kovalevskaya returned for a while to Berlin. Her parents asked her many times to return to Russia for a rest. According to Malevich, Sofya went to Palibino together with Lermontova during the holidays of 1873 for several weeks. Early in the autumn she was again in Switzerland.

Good-bye, Berlin

In October 1873 in Berlin Sofya renewed her studies in mathematics. Weierstrass really worked very hard, which can be seen from his letters [125, p. 170], for instance:

"Berlin, Friday November 14, 1873 (morning)

"Dear friend, the day before yesterday I made a mistake when I said to you that I should be able to come to you yesterday or today. But today (Friday) I am definitely at home from 5 o'clock. Won't you come and bring me some mathematical material? I shall wait for you, if you don't refuse. On Saturday I am busy again, and I cannot yet determine whether I shall be free on Sunday or Monday.

Yours, K. W."

A few days later, on November 19, he writes that he is sending her another series of the *Songs without Words* that all are related to the minimal surfaces. This was when Schwarz was studying the minimal surfaces in compliance with the ideas of his teacher Weierstrass. It is possible that what Kovalevskaya recounted about her talks with Schwarz prompted Weierstrass to return to the theory of minimal surfaces. He collected the separate pages of his

notes and asked Sofya to keep them in the order he put them.

> "I wanted immediately to entrust them to your hands so that they will not become disarrayed again." [125, p. 171]

Weierstrass informed Sofya of his plan to discuss with her the theory of linear differential equations. Following many days of "most cheerless" efforts, Weierstrass asked Sofya on November 29 to come to him with all the notes and bring what she had done recently [125, p. 173].

In his letter of December 6 Weierstrass invited Sofya to come to him for the studies earlier, around half past two, because he had to leave at half past six [125, p. 174].

The mathematical problems that Weierstrass considered in his letters of 1873-74 are related to the theory of integer complex numbers, the reduction of Abelian integrals, and the equation of heat conduction. Weierstrass was very interested in the first problem, and although it was not one Sofya was dealing with, he wrote to her about these special complex numbers. The two other problems were connected with Sofya's dissertation. We shall consider the last problem.

Below I shall relate in detail how Kovalevskaya found the existence of series that can formally satisfy a partial differential equation but is nowhere convergent. This induced Weierstrass to study further the structure of solutions for the simplest parabolic equations. In a letter of May 6, 1874, he presented Sofya with his considerations and ended the letter with the words:

> "About all this and certain related problems we shall talk in more detail when we meet. You see, dear Sonya, your remark (that seems to you so simple) on the distinctive property of partial differential equations namely that an infinite series can *formally* satisfy such an equation but nevertheless not converge for any system of values of the variables, became a starting point for me for interesting and elucidating research.
>
> "I'd like my student to express her gratitude to her teacher and friend in the same manner again." [125, p. 178]

Weierstrass returned to the equation of heat conduction in his letter of May 9 [125, p. 178]:

"Here is a small problem. We have a partial differential equation

$$\frac{\partial \varphi}{\partial t} = \frac{\partial^2 \varphi}{\partial x^2} \qquad (*)$$

which has the partial integral

$$\varphi = (\mu t)^{-\nu} F(u), \qquad u = \frac{1}{\sqrt{\mu t}}(x - \lambda), \qquad (**)$$

where λ, μ and ν are arbitrary constants and $F(u)$ should satisfy the differential equation

$$F''(u) + \frac{1}{2}\mu u F'(u) + \mu\nu F(u) = 0. \qquad (***)$$

"What is the general solution of this equation?

"For $\mu = 1$, $\nu = \frac{1}{2}$ we can take

$$F(u) = f(\lambda)\, e^{-\frac{u^2}{4}}.$$

"Then the partial integral

$$\varphi = \frac{f(\lambda)}{\sqrt{t}} e^{-\frac{1}{4}[(x-\lambda)^2/t]}$$

yields the general integral

$$\varphi = \int_{-\infty}^{\infty} \frac{f(\lambda)}{\sqrt{t}} e^{-\frac{1}{4}[(x-\lambda)^2/t]}\, d\lambda.$$

"However, if $f(\lambda)$ becomes infinite for infinitely large λ to a greater extent than the function $e^{-c\lambda^2}$, no matter how small the constant c is, then the preceding equation does not make sense. Is it possible in this case to obtain a more suitable expression by using the general function $F(u)$, which satisfies the resultant differential equation for other values of the constants μ and ν? Or is the arbitrary function necessarily related to the constraint that

at $\lambda = \pm\infty$

$$\frac{\log|f(\lambda)|}{\lambda^2} = 0$$

in all cases?"

The last letter Weierstrass wrote to Sonya when she was in Berlin was dated August 18, 1874. She had decided to go to Russia, but Yuliya Lermontova intended to stay for a while in Berlin. Weierstrass offered his assistance,

"If I can be of use today in any way for you or Yuliya, ... tell your friend that from October 4 I shall come what may ..." [125, p. 187], and that she could ask him help if need be.

The Ph.D. Degree

Weierstrass admired the mathematical abilities of his student:

"As far as Kovalevskaya's mathematical education is concerned, I can testify that I have had very few students that could be compared to her in diligence, ability, commitment and absorption in science", he wrote to Fuchs [13, p. 346].

In 1874 Weierstrass petitioned Göttingen University to grant Sofya Kovalevskaya a Ph.D. degree in *absentia* and without examination. Weierstrass sent several letters to the professors of Göttingen University describing the three works presented by Kovalevskaya, each of which he believed, was sufficient for the degree [13, p. 344].

The first was "Towards a Theory of Partial Differential Equations" [1] and contains a proof for the theorem of the existence of a holomorphic solution for a system of partial differential equations in the normal form. In 1842 Cauchy produced a theorem for the existence of a linear system of partial differential equations and indicated a technique for reducing a non-linear system to this case [138, 139]. However, neither Kovalevskaya, nor Weierstrass knew of Cauchy's work.

Note that a holomorphic, or analytic function of the variables $x_1, x_2, \ldots, x_n$ in the neighbourhood of the points $x^0{}_1, x^0{}_2, \ldots, x^0{}_n$ is defined as a function that can be expanded into series

$$F(x_1, x_2, \ldots, x_n) = \sum_{S_1S_2\ldots S_n} A_{S_1S_2\ldots S_n} (x_1 - x_1^0)^{S_1} (x_2 - x_2^0)^{S_2} \ldots (x_n - x_n^0)^{S_n}$$

that is convergent for sufficiently small values of $|x_i - x^0{}_i|$, $i = 1, 2, \ldots, n$.

The Cauchy-Kovalevskaya theorem is formulated at present as follows [140].

Given a system of equations

$$\frac{\partial^{n_i} u_i}{\partial t^{n_i}} = F_i\left(t, x_1, \ldots, x_n, u_1, \ldots, u_N, \ldots \frac{\partial^k u_j}{\partial t^{k_0} x_1^{k_1} \ldots \partial x_n^{k_n}}, \ldots\right) \quad (1)$$

$$(i, j = 1, 2, \ldots, N, \; k_0 + k_1 + \ldots + k_n = k \leqslant n_i, \; k_0 < n_i)$$

in the *normal* form. The derivative $\partial^{n_i} u_i / \partial t^{n_i}$ must be among the derivatives with respect to t of the highest order n_i of each function u_i contained in the system, the system being solved for these derivatives.

At some point $t = t^0$ we are given the initial values of the unknown functions u_i and their first $n_i - 1$ derivatives with respect to t:

$$\left(\frac{\partial^k u_i}{\partial t^k}\right)_{t=t^0} = \varphi_i^{(k)}(x_1, x_2, \ldots, x_n) \quad (2)$$

$$(k = 0, 1, \ldots, n_i - 1)$$

(when $k = 0$, we have the function u_i itself).

All the functions $\varphi_i^{(k)}$ are defined in the same domain $G(x_1, \ldots, x_n)$.

Cauchy's problem is to find a solution of system (1) with the initial conditions (2). If all the functions F_i are analytic in a neighbourhood of the point $(t^0, x^0{}_1, \ldots, x^0{}_n, \overset{0}{\varphi}_{j,k_0k_1\ldots k_n})$ and all the functions $\varphi_j^{(k)}$ are analytic in a neighbourhood of the point $(t^0, x_1^0, \ldots, x^0{}_n)$ then Cauchy's problem has an analytic solution in the neighbourhood of the point $(t^0, x_1^0, \ldots, x_n^0)$ and it is a unique solution in the class of analytic functions. Here

$$\overset{0}{\varphi}_{j,k_0,k_1\ldots k_n} = \left.\frac{\partial^k u_j}{\partial t^{k_0} \partial x_1^{k_1} \ldots \partial x_n^{k_n}}\right|_{t=t^0}.$$

To prove the theorem, Kovalevskaya used Weierstrass's majorant functions

$$M\left(1-\frac{x_1+x_2+\ldots+x_n}{a}\right)^{-1}$$

and not Cauchy's

$$M\left[\left(1-\frac{x_1}{x_1^0}\right)\left(1-\frac{x_2}{x_2^0}\right)\ldots\left(1-\frac{x_n}{x_n^0}\right)\right]^{-1}.$$

The proof provided by Kovalevskaya is simpler than that given by Cauchy, and Poincaré said that she had given the theorem its definitive form. The theorem is now given in all basic courses of analysis [141, p. 380]. It is essential to note that Kovalevskaya established the importance of reducing the system of equations to the normal form. This is clear from Kovalevskaya's example of the simplest equation (the equation of heat conduction) for which Cauchy's problem does not have a holomorphic solution if the equation is not in the normal form. This was a significant discovery of the period. (Weierstrass wrote that originally Kovalevskaya demonstrated it for a composite equation.)

Kovalevskaya's example. Find the solution of the equation

$$\frac{\partial\varphi}{\partial t}=\frac{\partial^2\varphi}{\partial x^2},$$

satisfying the condition $\varphi(x, t) = 1/(1-x)$ for $t = 0$. It is easy to see that if there is an analytic solution, then it can be presented as a power series in t:

$$\sum_{n=0}^{\infty}\frac{(2n)!}{n!}\,\frac{t^n}{(1-x)^{2n+1}},$$

which, however, diverges at all $t\neq 0$. Therefore, an analytic solution of the problem does not exist.

In a talk "S. V. Kovalevskaya's theorem and its role in the modern theory of partial differential equations" presented at the Institute of Problems of Mechanics (the USSR Academy of Sciences) in 1975 to mark the 125th anniversary of Kovalevskaya's birth, Professor Olga A. Oleinik said that Kovalevskaya's theorem had and continued to have important and substantial applications in the theory of partial differential equations, and much of the most sophisticated of contemporary research bore witness to its profound and consummate character.

There were many people who questioned how independently Kovalevskaya had worked when she was studying a problem formulated by Weierstrass. Weierstrass wrote in this connection to Du Bois-Reymond on September 25, 1874,

> "Except for correcting her numerous grammatical mistakes I did not do anything other than formulate the problem for the author of the dissertation in question. And in this connection I also have to remark that as a matter of fact, I did not expect any result different to what is known from the theory of ordinary differential equations. To stay with the simplest case, I had an opinion that a power series in many variables that formally satisfies a partial differential equation must always be convergent within a certain domain and must, therefore, represent a function that really satisfies the equation. This is not true, as you can see from the example of the equation $\partial\varphi/\partial t=\partial^2\varphi/\partial x^2$ considered in the dissertation. This was discovered, to my great surprise, by my student completely independently, first for much more involved differential equations than the one cited, so that she even doubted that it would be possible to obtain a general result; the seemingly simple means she found to overcome the obstacle I value highly as proof of her mathematical flair". [142, p. 204]

The second work that Kovalevskaya presented to obtain her Ph.D. concerned the problem of the form of Saturn's ring. This is "Supplements and remarks to Laplace's investigation of the form of Saturn's ring" [5]. The problem is as follows.

A ring is filled by a homogeneous mass and is generated by the rotation of an ellipse about a straight line that does not intersect the ellipse, is in the same plane as the ellipse, and is parallel to one of its axes. The ring rotates with a constant angular velocity about the straight line. The surface of the ring is covered by an infinitely thin layer of a homogeneous liquid that is attracted by the gravitation of both the ring and the central body. The centre of gravity of the central body coincides with the centre of the ring. The problem is to determine the elements of the ring (the semiaxes of the ellipse and the distance from the ellipse's

centre to the axis of rotation) and the angular velocity of the ring while the liquid retains its equilibrium position with respect to the surface of the ring. In order to achieve this it is necessary and sufficient for the following equation to be satisfied:

$$V+\frac{M}{\sqrt{\rho_1^2+z_1^2}}+\frac{1}{2}n^2\rho_1^2=C, \tag{1}$$

where n is the angular velocity, V is the potential of the ring at a point of its surface, ρ_1 is the distance of this point from the rotation axis, z_1 is the distance from the point to the equator plane, M is the mass of the central body, which is assumed to be concentrated at its centre of gravity and C is a constant.

Laplace investigated this problem assuming that the distance from the centre of the generating ellipse to the rotation axis is very great compared with the semiaxes of the ellipse [143] and this made it possible for him to substitute the ring by an elliptical cylinder.

Kovalevskaya assumed that the line generating the ring differed very little from an ellipse and it had a symmetry axis whose continuation intersected the axis of the ring at right angles, and that every straight line parallel to the axis of symmetry intersected the ring curve at no more than two points. Kovalevskaya represented the equations of ring cross-section in the form

$$\begin{gathered}\sqrt{x^2+y^2}=1-a\cos t,\\ z=a\,(\beta\sin t+\beta_1\sin 2t+\beta_2\sin 3t+\dots),\end{gathered} \tag{2}$$

where t runs from zero to 2π; and a, β, β_1, β_2, ... are constants. Therefore the mean of the greatest and the least distance of curve (2) from the rotation axis is assumed to be unity. At the same time a is considered to be small with respect to unity, and β_1, β_2, ... and the sum of their absolute values are taken to be small with respect to β. Kovalevskaya provided a method for determining the coefficients β_1, β_2, ... such that the left-hand side of equation (1) everywhere differed from a constant by a quantity of any order of smallness with respect to a, and was only calculated as an approximation of the second order. She obtained a correction to Laplace's solution that produced egg-shaped

cross-sections of the ring, and found a relationship between the angular velocity n, the mass M, and the parameters α and β. Sofya Kovalevskaya pointed out that apart from the difficulty of the calculation, she restrained from determining more accurately the cross-section because Maxwell's investigations had rendered Laplace's surmise concerning the structure of Saturn's ring doubtful.

It should be mentioned that our modern concepts of Saturn's ring (or rather separate rings) are quite different; they are thought to be formed of mainly ice particles several millimeters in diameter. The latest research was performed with the use of the USA space probe "Voyager-1" (see *Umschau*, 1984, B. 84, No. 14/15, S. 432 (FRG)).

Kovalevskaya wrote that she had dealt with the problem of the stability of a liquid ring (when the mass of the central body is zero), but did not obtain any definite results, although the problem was of great theoretical value. Later it was established that such a ring is unstable [144]. In his *Course of Celestial Mechanics* Tisserand [145] presented Kovalevskaya's work on Saturn's ring in great detail and supplied it with graphs and explanations.

The third of Kovalevskaya's doctoral thesis was "On the reduction of a class of Abelian integrals of the third rank to elliptic integrals" [2]. This problem did not require great creative effort, but it was necessary to understand the theory of Abelian functions, one of the most difficult theories in mathematical analysis.

The rank ρ of a curve of the nth order is defined as the number by which the actual number of double points of the curve is distinct from the maximum possible number of double points a curve of that order may have. Thus if $\rho=0$, the algebraic curve possesses the maximum number of double points, which for an algebraic curve is

$$\frac{1}{2}(n-1)(n-2).$$

At the same time the rank of an algebraic function $f(x, y)$ equals the number of holes in the canonical Riemann surface for this function [146, p. 161].

Kovalevskaya wrote that she attempted

> "for the case when there is a *third* rank equation relating x and y to obtain the algebraic relationships between the coefficients of the equation, when the integrals $\int F(x, y)\,dx$ include integrals that can be reduced to elliptic ones by a transformation of the second order" [13].

It is assumed that y is a solution of the equation $f(x, y) = 0$, where $f(x, y)$ is an entire polynomial.

A transformation of the second order is defined as an algebraic transformation that is a generalization of the linear transformation (or transformation of the first order). This problem for an equation of the second rank and a transformation of the second order was considered by Koenigsberger [148] before Kovalevskaya did.

Kovalevskaya completely investigated the problem proceeding from Weierstrass's general results, presented in transcendent form. In particular, Kovalevskaya studied a hyperelliptic integral with the square root of a polynomial of the eighth order.

Kovalevskaya received her Doctor's degree *summa cum laude* (with the greatest praise).

Weierstrass rejoiced in the victory of his student more than anyone, at least more than Sofya herself. She had already travelled to Russia, and Weierstrass wrote to her (on September 21, 1874) about his visit to Heidelberg. When he was with Koenigsberger, Bunsen came and

> "brought a newspaper that carried the news that a learned lady from Moscow, Mrs. S. von Kovalevskaya had been granted a Doctor's degree by Göttingen's Philosophy Department... This was a sensation for everybody for it had been my intention not to say anything in advance. Naturally, later much was said about you and your both fellow students [Yu. V. Lermontova and A. M. Evreinova, the lawyer]". [125, p. 189]

This was the end of five years of intensive labour. Kovalevskaya travelled several times: she went to London, Paris, and Zürich. Her life in Berlin had been uncomfortable because of her inexperience and her inability to make arrangements.

Yuliya Lermontova recalled:

"On the whole, we lived in Berlin in an uncomfortable

apartment, with poor meals and unhealthy air. We worked hard continuously and there was no fun whatsoever. Life was so dismal that I recalled the time when we had just arrived at Heidelberg as paradise lost. After Sofa had received the Doctor's degree in Autumn 1874, she felt such a decline in her physical and spiritual faculties that when she returned to Russia she was unable to start any work for a long time." [64, p. 387]

The Memoirs of Yuliya Lermontova

Yuliya Lermontova left interesting recollections of her life in Heidelberg and Berlin with Sofya Kovalevskaya [147].

In this section we shall follow her life with Sofya during that period. Let me first cite the lines picturing the young Sofya:

"Her outstanding abilities, passion for mathematics, unusually attractive appearance, and great modesty won the sympathies of everybody she met. There was something really fascinating in her. All the professors she studied with were delighted by her ability; she was very industrious and could sit at a table doing her mathematical calculations for hours on end.

"Her moral appearance was complemented by a profound and complicated spiritual life, such as I have never seen in anyone else...

"Among all those women and girls, devoted to politics and more or less fatigued by life, she made quite a peculiar impression with her childish complexion, for which she was tenderly nicknamed the 'little sparrow'. She was already 18, but she seemed much younger. Short, thin, but with a rather plump face, clipped curly dark-chestnut hair, uncommonly expressive and live features, eyes that were constantly changing their expression, now bright and sparkling, now deeply dreamy, she was a unique mixture of childish naivety and profound intellect.

"She won everyone's heart by her ingenuous charm, which distinguished her at this period of her life. Everyone, old and young, men and women, were captivated by her. She was completely natural in her manner, without a

trace of coquetry, and did not seem to notice the adoration she induced." [147, pp. 377, 381]

As it has already been mentioned, in Autumn 1869 the Kovalevskys visited London and returned to Heidelberg, where they found Yuliya Lermontova waiting for them. Lermontova recollected the start of their life in Heidelberg with great pleasure. All three lived in the same apartment, and worked and attended lectures together and Yuliya had her studies at the laboratory. In the evenings and on holidays they would take long walks in the picturesque vicinities of Heidelberg, along the banks of the wonderful river Neckar. Lermontova remembered that during one walk they went rather far and happened across a smooth road; they started racing one another, like small children.

However, Lermontova later wrote that this was the only year she remembered Sofya happy. Lermontova always sympathized with Vladimir Kovalevsky and thought that during this initial period of their student life the three of them were so happy because of Vladimir, who was lively interested in problems of every kind, even those that bore no relation to science. The idyll was broken by the arrival of Anyuta and Zhanna Evreinova in early winter. Both girls were older than Sofya, and Zhanna was especially strict and made everything a matter of principle. The apartment where Sofya, Yuliya and Vladimir lived was too small for the five of them, and Vladimir had to move elsewhere, giving up his room to the newcomers.

The budding intimacy between Sofya and Vladimir was broken:

> "Sonya often visited him and passed whole days with him; sometimes they went on long walks together. Of course, the presence of so many women could not always have been pleasant for them, the more so since Anna and her friend were often unkind to Vladimir. They had reasoned that since the marriage was fictitious, he should not impart too much intimacy to his relations with Sofya. This interference of others into the life of the young couple resulted in many petty quarrels and soon spoiled the good relations that had been established between the members of our small community [147, p. 384].

Kovalevsky made up his mind to leave Heidelberg. He went to Vienna, then to Münich, and worked there intensely in his science.

As to life in Berlin, Lermontova writes that it was even more monotonous and secluded than in Heidelberg. She lived with Sofya, who passed days on end at her desk deep in her mathematical calculations, Yuliya worked in her laboratory until night. After a quick lunch, they went back to their studies again. Other than Professor Weierstrass, no one ever visited them. Sofya was always melancholy, she regarded with equanimity everything apart from her studies. Kovalevsky would come sometimes, and Sofya always slightly livened up on such occasions. However, from time to time their meetings became clouded with mutual reproach and misunderstanding. And when Sofya was with Yuliya, she would not go out for a walk or to the theatre or shopping.

Lermontova tells how Sonya studied. She could give herself up to the most intense mental work for hours on end without leaving the desk. At night, after a day of strenuous work, she would rise from her chair, deep in thought, and start walking around the room in quick steps; she seemed then completely to cut herself off from reality, apparently, her imagination had carried her far beyond the present.

She slept little and restlessly. She would often be woken up by nightmares and ask Yuliya to sit with her. She would readily recount her dreams, which were unfailingly original and interesting.

Sofya set herself the most complex aims, and then passionately desired to attain them.

> "However, despite all that, I never saw her so dismal and depressed as when she reached her goal" [147, p. 386].

Reality did not comply with Sofya's dreams. And

> "when she was seen to be sad and depressed amidst complete success, she invoked a deep compassion without intending it" [147, p. 387].

Nobody doubted that Vladimir Kovalevsky was deeply in love with Sofya. Beyond all question, Sofya came to love him too. But it was not only the fictitious marriage that was the obstacle. There were other circumstances that were ob-

vious to both the Kovalevskys themselves and to those who were close to them.

Analysing them, Lermontova explains:

> "She [Sofya] always felt an irresistible need for tenderness and heart-to-heart talks, she constantly wanted to have someone at her side who would share everything with her, and at the same time she made impossible the life of anyone who became too close. She was too restless, too much at odds in her nature to find any lasting satisfaction in the quiet life, full of love and tenderness about which she apparently dreamed so passionately. But she was by nature too selfish to pay enough attention to the aspirations and desires of the person who lived with her.
>
> "Kovalevsky was extremely restless himself; he was always obsessed with new plans and ideas. God only knows whether these two human beings, both immensely talented, could have lived a truly happy life together in any circumstances." [147, p. 385]

Late 1872 and early 1873 were a distressing period for both Sofya and Vladimir. Sofya was already sincerely attached to Vladimir, but he could not perceive it, and his pride suffered. He started to think of getting a divorce, believing that the situation was burdensome for Sofya, and began to send her letters that appeared very nervous. He was also tormented by jealousy because some rumours about Sofya reached him. She was alarmed as well and once asked Yuliya to mediate and write to Vladimir. At last, after a year of separation, a reconciliation occurred. In a letter of June 7, 1873, Vladimir writes to his brother that he has almost made up his mind to go to Petersburg to defend his dissertation.

> "Sofya goes with me too, and therefore things are going very well and we shall have fun". [116, p. 143]

During the summer the Kovalevskys stayed in Palibino, but went abroad again in the autumn to continue with their studies.

Chapter 4
RETURN TO RUSSIA

The Kovalevskys in St. Petersburg

By returning to Russia, the Kovalevskys intended to set to rights their family life. It is clear from Vladimir Kovalevsky's correspondence that they anticipated a joyful life in Petersburg, where Vladimir and "Sofya have so many anchoring hooks" [87, p. 328].

In August 1874 Sofya Kovalevskaya set out for her native Palibino, and the family warmly celebrated Sofya's name-day on September 17. General Korvin-Krukovsky himself was in charge of laying the table. At the end of the dinner Malevich proposed a somewhat flowery and touching toast:

> "A modest labourer who once slightly raised the veil separating his dear pupil from the temple of science proposes we consider this day the triumph of a woman's work, a work that attained impressive results and was crowned with the highest scientific degree". [95, p. 650]

At the beginning of the toast Sofya was a little embarrassed and she pressed herself to mother's bosom; she calmed down, and was glad when Malevich ended with "The health of the first Russian woman scientist, Sofya Vasilievna Kovalevskaya" [95, p. 652].

Everybody cheered with loud hurray and Malevich presented a bunch of flowers to his pupil, and she toasted her first teacher, which moved and somewhat embarrassed him.

In October 1874 Dmitry Mendeleev arranged a celebration in Petersburg for the first woman chemist Yuliya Lermontova.

Sofya Kovalevskaya wrote about it in the Autumn of 1874 when in a letter to Lermontova's mother Elizaveta

Andreevna she congratulated her on the success of Yuliya in passing her Doctor's examination and defending her dissertation in chemistry at Göttingen. Elizaveta Andreevna lived in Moscow and, naturally, was looking forward to her daughter returning home, but who was being delayed in Petersburg. Sofya Kovalevskaya asked Elizaveta Andreevna's permission for Yuliya to stay longer in Petersburg. Both girls were at Professor Mendeleev's who "sincerely sympathizes with the studies and success of Yuliya" [64, p. 241]. On one occasion he arranged a party to which he invited chemists to meet Lermontova. At other times the young women went sight-seeing in Petersburg, or visited the theatre, etc.

There is an autobiographical introduction in Kovalevskaya's novel *A Nihilist Girl* where Sofya describes herself, as she was just after she had returned to Russia. She was warmed by the success that she had achieved as a woman scientist, and she found pleasure in talking to people [67, p. 90].

The Kovalevskys moved in a circle of scientists and writers that included Mendeleev, Sechenov and Butlerov, Chebyshev and Gadolin, Turgenev and Dostoevsky.

> "Sofya immediately became the focus of one of the select cultured circles that were devoted to intellectual interests. They were a feature of the Russian capital and could rarely be found anywhere else in Europe," Anne-Charlotte Leffler noted in her recollections of Kovalevskaya [96, p. 132].

Through Vladimir Kovalevsky Sofya was introduced to other sort of people. For instance, Longin F. Panteleev wrote [123, p. 88] that when he returned in late 1874 from political exile, Vladimir and Sofya, whom he had just met, received him with uncommon warmth. Vladimir Kovalevsky had become involved with Panteleev before his exile. In 1865 Panteleev had been taken to Vilno to be sent to Siberia, Vladimir brought him money from friends and even offered to organize an escape. This is evidence that Vladimir Kovalevsky was very close to revolutionary circles. Natalya Armfeldt, whom I mentioned above, was related to Sofya and was also involved in revolutionary circles, and was close to the Kovalevskys.

L. F. Panteleev wrote of his first impressions upon his return [123, p. 409]:

"I haven't been in Petersburg a week before I was debating what was then a subject of constant talk in the liberal opposition circles, I mean the "go to the people" movement. A great many people had been arrested in this connection. All the names that were mentioned were, naturally, new to me, but in a way close to many of my acquaintances, especially to S. V. Kovalevskaya".

Later Sofya Kovalevskaya described the "going to the people" in her novella *A Nihilist Girl*.

Weierstrass and Kovalevskaya in 1874-75: Letters

In his first letter written on September 21, 1874, after he had returned with his sisters from a three-week trip along the Rhine, Weierstrass asked Sofya to rest:

"Whatever you miss in your study, you will easily catch on with a fresh effort. I know by my own experience how excruciating it is for a person to have so many problems at once and to be unable to cope with them because of lack of physical force." [125, p. 187]

First of all Weierstrass informed his student that her dissertation (the work on differential equations) had been published and published well; he wanted to keep 35 copies to distribute himself, 65 copies had been packed for sending to Petersburg, and 250 copies for Göttingen had been handed to the mathematician Lotze. Dissertations at the time were published at the expense of the authors, and Weierstrass was reporting to Sofya in one of his letters on the allocation of the money she had left for that purpose. Then Weierstrass described his trip to Heidelberg, Baden-Baden, Cologne, and Strasbourg. Reminiscences crowded into his mind, he recalled the path over which he had walked many years before with the friend who had convinced him to become a mathematician.

"I stayed faithful to my goals", added Weierstrass, "and I am happy with the result, even though not every flower has given forth fruit." [125, p. 188]

It is clear from the next letter, dated December 16, 1874, that Sofya had not written to him for a long time, and when she wrote, she asked him to scold her firmly for not being engaged in mathematics. Weierstrass responded [125, p. 190]:

> "... I took it for granted from the very beginning that after a period when you had long been deprived of the chance to move in society, you would not start constant and serious work during the first period of your stay in Petersburg.
>
> "And I am even not very much disconcerted when you write that this is how the matters stand: partly because I am convinced that a certain distraction will not harm you after such long work, partly because I am firmly sure that your serious mind and your attraction to ideal aspirations will not allow you to restrain from research for too long.
>
> "Now abandon yourself serenely to the unexplored delights of life in the big city insofar that you like it yourself, but do not yield to external influences. I know that you will not betray science, and the striving for creative effort, sometimes giving way to an understandable exhaustion, will be revited in you still more intensely."

Weierstrass conveyed his new results on the theory of the functions of complex variables; the results are related to representation of entire transcendental functions as infinite products, he advised Sofya to look through this section in her "elliptic" copybook and promised to send her a reprint of his article that would soon appear.

In his New Year greeting of January 1, 1875, he rejoiced in Sofya's scientific enthusiasm, which she relayed to him in her Christmas letter. He approved of her decision to use the winter to fill the gaps in her knowledge of "the more elementary parts of mathematics, namely analytical mechanics and mathematical physics". In particular, Weierstrass advised her to look through the *Comptes Rendus* of the Paris Academy of Sciences and get acquainted with the memoirs of Saint-Venant. He foresaw that Sofya

> "would be unpleasantly surprised by the lack of strict presentation" [125, p. 197]. "But do not give up because of that, since you primarily have to gain a general im-

pression of what has been done in the field of mathematical physics and its still unresolved problems. And you may deal with some uncomplicated problems to exercise yourself in the calculations, the point being, as I have often told you, a thorough mastering of detail." [125, p. 197]

In the same letter Weierstrass relayed to Sofya his thoughts in connection with what troubled him: he had long felt the need to finish and publish his earlier work. He wrote [125, p. 196]:

"I should not tarry for other reasons as well. At the present moment, since young mathematicians have found that writing large books (by the way, without references) is the most reliable means to win the esteem of the crowd and gain a good place in the field of analysis, to whose thorough investigation I devoted the best part of my life, they have become too outrageous, and it is high time to put an end to it...

"I do not believe that I am a scientific pedant, and I do not even admit there should be a common church in mathematics. But what I do require of a scientific work is a unity of method, a consistent implementation of a plan, the corresponding elucidation of detail, and an impression of independent investigation.

"It is just too bad that in this country, as in other countries, textbooks are often written by incompetent people, the French have at least the merit of a clear and eloquent presentation that to a certain extent atones for their lack of insight. But the highest and most difficult realms of the science, where something can be only attained by those who contribute their every effort, should not be handed over to those who write lightweight books."

In conclusion Weierstrass added. [125, p. 197]

"Forgive me, my sweet friend, this digression, in which you should see the proof of how deep I have the habit of making you a confidante in my thoughts, even the most joyless ones".

Sofya's good intentions to resume her mathematics went unfulfilled for the most part. She answered Weierstrass even more seldom. He expressed his anxiety in this connection

in letters dated February 18 and April 21. We can learn from his letter of May 7 that Sofya gave him some hope they might meet soon. But in the summer she contracted measles, and Weierstrass in a letter of June 17 wrote that he was very sorry. Sofya's next letter was sent only on September 17. In his answer, written on September 23, Weierstrass informed her, by the way, that he had sent her the recently reprinted correspondence between Jacobi and Legendre [152] now free from numerous misprints that had distorted it in Bertrand's edition.

Now let us deal in more detail with the content of the letters of the period. In the letter of June 17 Weierstrass wrote that he was distressed with her change in plan to come to Berlin, and explained this in the following manner:

> "For four years I have made you the confidante of my thoughts and aspirations, with whom I could talk as a friend who has stood close to me all my life. I have never found anybody who could understand so well the highest goals of science and investigate so thoroughly and joyfully all my views and principles as you did! [125, p. 211]
>
> "Last winter and this summer I have had several rather good auditors, among whom a Swede* has distinguished himself. I also started to give them private lectures at my home, but that did not succeed very much. The fact that these young men diligently put down everything I said but that I could not see by their faces whether they had understood me irritated me extremely. It was quite otherwise with you". [125, p. 212]

Essential information pertaining to Kovalevskaya's work on partial differential equations is contained in Weierstrass's letter of April 21, 1875. He wrote that the *Comptes Rendus* of the Paris Academy of Sciences had printed articles by Darboux [153, 154] dealing with the same problems Kovalevskaya had studied and he added [125, p. 206]:

> "I am very glad that my student has managed to anticipate her competitors with respect to both time and the subject itself, at least she has not fallen behind them. Darboux says that certain exceptions are of great interest;

* Mittag-Leffler.

I tend to think that he also met the same obstacles (as in $\partial\varphi/\partial t=\partial^2\varphi/\partial x^2$) you had so much trouble with at the beginning and have overcome so successfully; I do not deny that I couldn't restrain myself from a certain malicious delight that he was unable to cope with the exceptions.

"But still it seemed to me necessary to establish the pertinent facts in time and to notify both Darboux and the Academy that you have presented your article to the Göttingen Department of Philosophy in late July and that it appeared in September. In this connection I have sent a copy of your dissertation, complete with the diploma, to Darboux and another to Hermite (I did not have any other copies left), but without any additional notes. Now let us wait."

Although Weierstrass was told that Darboux praised Kovalevskaya's work he was still anxious and asked [125, p. 207]:

"What do you think? Will you write an official notice to the Paris Academy or shall we wait for what the commission says? The commission, like Darboux himself, will probably remain silent about this affair. However, I would like to ask Borchardt, in the conclusion of the 80th volume of the journal, where he is going to list the misprints that occurred in your work, to note categorically that it had already been published in August."

But Sofya was much less excited with her dissertation than Weierstrass. She answers him rarely and superficially. The exception was the letter informing Weierstrass of the death of her father, General Korvin-Krukovsky. In his letter of October 23, 1875, Weierstrass thanked Sofya for the detailed description of the circumstances of her father's unexpected end—in contrast with the scarcity of expression so characteristic of her when she spoke of anything purely personal. He was glad that there had been a reconciliation between father and children and remarked [125, p. 216]:

"I am convinced that all your life you will recollect with mixed joy and sadness this last year of your life

together and will think yourself lucky that you were able to make the last days of his life happy".

Then there was a long interruption in their correspondence. Sofya Kovalevskaya did not write to her teacher for almost three years.

Scientific Reviews

Sofya Kovalevskaya was widely educated, followed the literature, and liked the theatre. That is why she eagerly contributed to the newspaper *Novoe Vremya* (New Times), where she published theatre and scientific reviews. In 1876 Vladimir was invited to participate in the publication of this newspaper which was then liberally oriented and this stimulated her activity. In 1877, when the newspaper became reactionary, the Kovalevskys and the other progressive activists left.

I shall now cover Kovalevskaya's scientific reviews leaving a consideration of her theatre notices for the chapter on her literary and social activity.

Four of the scientific essays Kovalevskaya wrote are extant [23-26]. The first [23] deals with the direct utilization of solar energy and the problems of optics. Giving a rather detailed account of the history of attempts to concentrate solar light to obtain a thermal effect (Archimedes, Buffon, Saussure, and Ericksson), Kovalevskaya dwells on an invention of Mouchot from France, his solar steam machine. It consisted of a vertical copper cylinder painted black on the outside, and three concentric glass caps, or one cap and a truncated conical reflector silver-plated on the inside which was pointed at the sun. The cylinder was filled with water, which when boiling produced steam that could drive a machine.

Pointing out that coal reserves would diminish, Kovalevskaya emphasized the importance of the direct accumulation of solar energy: when humanity achieves this and

> "as it were, catches the light of the sun in flight, then we shall indeed have the right to be called 'sons of the sun'".

Kovaleskaya proceeds from the sun-powered machine to Werner Siemens's "artificial eye". It simulated the eye and

consisted of a glass ball (the eye ball), a lens (the crystalline lens), movable shutters (the eyelids), and a plate made of selenium (the retina). The electric conduction of selenium is related to the effect of heat and incident light. When selenium is exposed to light for a long time, its electric conduction drops (selenium becomes "tired"). This makes it possible to investigate the mechanism of light exciting the retina and the optic nerve. Siemens also succeeded in finding another application of the unusual properties of selenium: he invented a photometer for comparing light intensities.

Kovalevskaya dealt with aeronautics in the second scientific review [24]. She recounted the attempts by different people to rise into the air and related in detail how air balloons were utilized for reconnaissance. During the period of the first French republic, aeronautics was used for military purposes. The balloons were produced at the National Aerostatic Workshop established in 1794 in Médoc. According to Kovalevskaya, the balloons produced there were better than those than currently available being stronger and more air-tight. This was because the seams were well made and a special lacquer was used to impregnate the outer shell of the balloon.

Then Sofya Kovalevskaya discussed the investigation of bird flight, in particular, by French scientists. As for the German commission, which included the famous Helmholtz, to study possibilities for aeronautics Kovalevskaya informed her readers that its papers were "top secret".

The third review [25] dealt with the latest invention: the telephone, or speaking telegraph. Kovalevskaya started with a simplified account of the basic principles of telephone and wrote about its inventor Graham Bell, a Scottish-born American. He moved to the USA with his father, who was the inventor of a new method to teach the deaf and dumb, and at first Graham Bell followed his father's footsteps and taught the deaf and dumb. He then went on to invent the telephone. Kovalevskaya says that one of the most famous physicists and mathematicians, William Thomson, was so exhilarated with the telephone that he called it the most remarkable invention of the century. The sound of an orchestra in Philadelphia was sent to New York and that made an enormous impression. The same review gives a detailed description of the Remington typewriter.

The last extant review [26] was the most extensive and dealt with the process of fermentation and the nature of enzymes. Kovalevskaya defined an enzyme as a substance which when added in very small quantities to another substance caused it to transform chemically. The essence of fermentation is characterized as "life without air". Kovalevskaya discussed the incorrect theories of fermentation that had existed before Pasteur and presented his views. She touched on the problem of the independent origination of life, indicating that not one experiment supported the possibility. Finally, she discussed the problems of wine and beer making, pointing out the differences in their fermentation, and cited the practical advice of Pasteur based on his theory.

In order to have written these extensive essays, Sofya Kovalevskaya had to have read much and to have followed foreign publications especially; her diaries (in later years) testify that she read *Scientific American.* Her knowledge of physics allowed her to write the reviews with complete confidence.

Financial Troubles

The Kovalevskys had to think of the years to come, their work in their specialities, and the money they needed to live. Vladimir Kovalevsky's high hopes changed into low spirits. He "bitterly repented" his return to Petersburg where "no jobs can be had and there are swarms of scientists not on the permanent staff" [87, p. 337]. Vladimir was a stranger to the University and some of the scientists regarded him a dangerous contender.

Sofya Kovalevskaya, with her advanced mathematical education, could not apply her abilities at home. A man with a Doctor's degree, which roughly corresponds to a modern Soviet Candidate of Sciences, could teach at a university; he could occupy a chair after defending his Magister's and Doctor's dissertations. But a woman could only teach mathematics in the elementary classes of high schools for girls.

Vladimir Kovalevsky wrote to his brother on December 20, 1874 [87, p. 340]:

"They said at first that Sofa could give lectures at the Women's Courses, but of course it turned out right away that there were no girls well enough prepared, and possibly that would be settled next winter, but the matter still seems very doubtful".

Vladimir Kovalevsky had in mind the newly opened Alarchin Courses for Women. One can only wonder that Kovalevskaya was not admitted to teach at the Bestuzhev Higher Courses for Women, which had been established in Petersburg in 1878 (she had contributed to their foundation as a member of the Fundraising society). Later, during the "dark, leaden" years of the 1880s, she could not count on that at all. N. Mirovich recalled that Sofya Kovalevskaya offered to give lectures at the Courses free of charge, but still she was not invited and that brought about general disappointment and indignation; the injustice upset Kovalevskaya deeply [149, p. 312].

Earlier, in 1873, Vladimir Kovalevsky attempted to pass his Magister's examination (in Odessa), but did not succeed: a professor who had been criticized by Kovalevsky managed to fail him.

In late 1874 Vladimir passed his magister's examination in Petersburg*. In 1875 Sofya Kovalevskaya applied to the Rector of Petersburg University to be admitted to magister's examination. A session of the faculty of the Physico-mathematical Department provided the permission [150]**, but Kovalevskaya is known not to take the examinations. Obviously, the Minister of education did not approve the permission. Therefore, Kovalevskaya could not work at a level to which she had every right.

Before Vladimir Kovalevsky went abroad, he had handed over his whole publishing business to the book-sellers A. A. Cherkesov and V. Ya. Evdokimov with a provision that the money they gained from the book sales would be used to cover his debts. But when he returned to Russia, he found out that the business was still hard. Some of his books had been sold, but this had not covered all his debts. In particular, Kovalevsky still owed 20 thousand roubles to his father-in-law. Now Kovalevsky placed all

* *LOA AN*, f. 14, op. 3, ed. khr. 14816.

** See Appendix 5.

his hopes on the forthcoming edition of Brehm's *Life of Animals* that was being published by his company.

General Vasily Vasilievich Korvin-Krukovsky died in the Autumn of 1875, on September 30. He left his daughters fifty thousand roubles each, but took into account the debt of Sofya's husband, so Sofya inherited only thirty thousand. She would get interest of 900 roubles a year for this sum. Vladimir Kovalevsky was getting 600 roubles a year from his estate at Shustyanka, and he could count on the same sum of money if he could give lectures as a privat-docent. This was approximately the amount of money they spent when they lived abroad as students (Sofya received one thousand roubles a year from her father). But in Russia part of the money went into the publishing business.

Vladimir wrote to his brother in late 1875 [87, p. 355]:

> "This year we pay only 300 roubles for the apartment and give 25 roubles a month for the housekeeping; all the money we get will go for Brehm which will naturally repay us".

No doubt the couple had to discuss their finances, and Vladimir wrote to his brother on July 5, 1875 [87, p. 343]:

> "Sofa also insists that we should be engaged for a year or two not only in science but in other business as well, and then, when we have at least two thousand apart from a salary, we shall not be in such straightened circumstances and depend on the university and their parties; we shall be able then to think only of our research and devote ourselves to it entirely".

Vladimir presented a way of obtaining an independent position in an earlier letter [87, p. 340]:

> "We should set up a business here and start running it and this, I think, would be better and hurt us less than having to bow before such boors as our university people".

In the late summer of 1875 Aleksandr Kovalevsky wrote to his brother Vladimir that he wanted to buy a house for his family (he had three children). It was then possible to do it in Odessa rather profitably. He wanted to live in one apartment and let the others [87, p. 345].

Vladimir zealously took up the idea. He developed it and wrote in a letter of August 25 that a man with energy should not just buy houses but construct them.

> "This idea came to me this very minute and I am presenting it to you. Of course, it means a lot of trouble, but Yazykov constructed and did not stop being a lawyer!" [87, p. 348]

This was a period when Sofya Kovalevskaya had a deep confidence in the ability of her husband to manage business and defended him before Aleksandr Kovalevsky, who doubted it. Naively she thought that she herself as a mathematician could offer substantial help to her husband. She ascribed his former failures to various contingencies. Vladimir Kovalevsky began to apply all his energy to the construction of the houses with Sofya being his active assistant. Besides houses, they constructed public baths in the belief that they were needed on Vasilievsky Island.

The Kovalevskys entered an era of material well-being. They occupied a large flat, managed greenhouses, and even had a cow to feed their daughter with fresh milk.

However, already by 1879 it transpired that their well-being was only apparent. The houses they were building were mortgaged and remortgaged, and the interest for the mortgages was more than the profit they could realize. Vladimir lost heart completely, but Sofya showed "miracles of energy, wisdom and persistence", as her husband said of her. In spite of this, the Kovalevskys went bankrupt. When one of their creditors distrained their property, he "seemed amazed to find it so meagre," as Sofya informed Aleksandr [71, p. 267].

I have already mentioned that Vladimir Kovalevsky worked at the newspaper *Novoe Vremya*. He became its *de facto* editor which paid well but was rather troublesome and took a lot of time. When he left the newspaper for reasons of principle, he lost this income.

In addition to all his construction failures, in the Spring of 1879 Vladimir Kovalevsky received a cruel and unexpected blow. He was sent an article, published in a Geneva newspaper, alleging he had been a spy while he lived abroad. None of his friends believed the libel, but the attack tormented Vladimir.

After the failure of the construction business, the Kovalevskys wanted to leave Petersburg, and they started to think of moving to Moscow. The chair of paleontology in Moscow University was headed by an 80-year-old Professor G. E. Shchurovsky, and there was a hope that he might make Vladimir Kovalevsky his deputy. But there was no vacancy.

Kovalevsky was then offered a position in "The society of Russian factories of mineral oils of Ragozin and C°". The head of the company V. I. Ragozin helped Vladimir to become appointed a director of the society. In this connection Vladimir wrote [71, p. 269] to his wife: "my occupation is extremely indefinite and therefore dangerous". It was difficult to think over the business at once, but it seemed strange to him that there were a great number of outstanding promisory notes if the business were sound. But he decided that assuming the worst, it was still good (*Ibid.*).

Ragozin proposed that Kovalevsky take shares in the company on credit. All told, Vladimir took sixty shares, a half of which he mortgaged for a thousand roubles each. Aleksandr Kovalevsky and Yuliya Lermontova also bought shares.

Sofya Kovalevskaya busied herself too. Together with Yuliya Lermontova, she began to cooperate with the inventor of the electric "candle" Pavel N. Yablochkov, to organize electrifying the street lighting. Their life in Moscow became easy again: they rented a large apartment, and bought heavy furniture.

Vladimir Kovalevsky was mainly attracted to his new job by the business trips he would have to make abroad, where he hoped to meet his scientific colleagues, and maybe, this would help him return to science. His first mission abroad took place in October 1880.

The invitation to lecture at Moscow University quite probably was issued then, and he had to return to give lectures in January 1881. In early December 1880 the physico-mathematical Department unanimously elected Kovalevsky a staff docent for the chair of geology and paleontology. The University Council approved the election, and it was sanctioned on January 3, 1881.

Back to Science

Sofya Kovalevskaya began to return, little by little, to mathematics. The change in her mood can be dated to the Summer of 1878 when she was expecting a baby and so had started to lead a more serene life. In August of that year she wrote to Weierstrass for mathematical advice.

Weierstrass did not know Kovalevskaya's real circumstances. He thought that Sofya was rich and was not engaged in science because she was attracted by society life. In his letters of 1874-75 he had tried to sustain Sofya's interest in mathematics.

On August 15, 1878, Weierstrass wrote to Sofya from the island of Rügen, where he was resting, that he had received her almost unexpected letter and expressed his amazement at her three-year silence. During that time, the old teacher heard of her only twice: once his student, the Swedish mathematician Gösta Mittag-Leffler, who visited Kovalevskaya in Petersburg in 1876, and once from Pafnuty Chebyshev in Berlin. Weierstrass did not happen to talk with the latter himself, Chebyshev only told Borchardt that Kovalevskaya had dropped mathematics altogether.

> "I would be glad to hear from you protestations of the opposite", wrote Weierstrass. Then he related what he thought of Mittag-Leffler:
>
> "Mittag-Leffler was a very nice student of mine. Along with his formidable knowledge he possesses the wonderful ability to assimilate the subject and an intellect aiming at an ideal. I am sure that if you continue your friendship with him, it would be a stimulus for you.
>
> "The position he holds in Helsingfors is unfavourable. They go further there than anywhere else in the creation of a national Finnish mathematics, and when Leffler was there, each semester local newspapers published editorials against Weierstrass's mathematics. Leffler is careless enough to mention my name in his lectures and articles more often than necessary". [125, p. 218]

Weierstrass wrote of himself that although he hadn't had any serious illnesses for the past three years, he had began to feel that a two-hour lecture without an interval

exhausted him. He had been working for the past years on periodic functions of several variables and on the theory of differential equations.

Weierstrass did not receive any answer to this letter. The birth of daughter (on October 17, 1878) and a subsequent prolonged disease distracted Sofya from the mathematical interests that started to wake in her. She wrote to Litvinova:

> "Thank heavens I had not lost my strength in the study of mathematics; now, at least, my little girl will inherit fresh intellectual capabilities." (Quoted from [284] p. 138.)

In early 1880 the VIth Congress of Natural Scientists and Physicians was held in Petersburg, and there was a section of mathematics. P. L. Chebyshev suggested that Sofya Kovalevskaya should present a paper at the section. Her daughter was only one year old and there was the threat of bankruptcy looming over the Kovalevskys. But Sofya accepted the offer willingly. Overnight, "with a feeling of joy and pride" she prepared her work on Abelian integrals for presentation.

> "And in the morning I delivered my abstract at the congress, made a great impression, merited praise from Chebyshev and joined the ranks of scientists again." [91, p. 49]

Kovalevskaya demonstrated once again that she "was a born mathematician".

In Spring 1880, after they moved to Moscow, Sofya Kovalevskaya, counting on the positive attitude of the new and liberal Minister of Education A. A. Saburov, applied again for permission to be admitted for the Magister's examination and started to prepare for them. However, in spite of the support of Professors A. Yu. Davidov and N. S. Tikhonravov, she was refused. In a talk with a Professor, Saburov expressed an opinion that Sofya Kovalevskaya and her daughter "will both get old before women will be admitted to university" [87].

It has already been mentioned that in October 1880 Vladimir Kovalevsky went abroad on business. Sofya made up her mind to go for a while to Weierstrass to get "recharged"

for her work. She began to return to her former outlook on the destiny of her life: to blaize the trail for women striving for science. She was frustrated at not being admitted to the examination for which she had got ready during the summer. She wrote to Aleksandr Kovalevsky after her husband has gone abroad:

"Well, what shall I do? Since it is essential for me to prepare as many mathematical papers as possible to support our women's reputation at least by that, I resolve to risk rather heavily: I am going to go to Berlin for a month or a month and a half and leave my daughter here in care of Yuliya Lermontova and Mariya Dmitrievna.*" [73, p. 167]

In the Autumn of 1880 Sofya Kovalevskaya sent Weierstrass a letter informing him of her decision to come to Berlin. Weierstrass answered on October 28 [125, p. 221]:

"Above all, my sweet friend, you can be sure that I shall be sincerely glad to see you again after such a long separation. However, before you resolve to make such a long trip here, I have to acquaint you with my particular cirumstances in order to avoid that you come here full of hope for treatment that I shall not be able to meet in the nearest future. No doubt that you will be amazed on hearing that over the past years my additional obligations have not decreased but have increased. I have such a mass of work this winter that I do not know how to cope with it.

"No doubt, you know that my friend Borchardt passed away last summer after a grave disease. Since April 1 I have had to accept the editorial work in the journal** and now, together with Kronecker, I have to go on with it.

"Now we decided in the Academy to start publishing the complete works of Jacobi, Dirichlet and Steiner. Borchardt had taken Jacobi, and I had taken Steiner. When Borchardt fell ill, I had to continue with the publication of Jacobi's work because we could not find another competent person for the task. I did not suppose

* The child's nanny.
** Crelle's Journal.

before that this enterprise would take so much time and effort.

"Besides, Borchardt made me the guardian of his six children. I was closely related to Borchardt and his family for 25 years so I cannot refuse help to Mrs. Borchardt or refrain from advising her in the management of her fortune because she is quite inexperienced in this respect.

"Besides, I can tell you without shame that I am partly compelled to take up jobs like the above to raise my income. The salary I get as professor is insufficient to cover my expenses, which rise from year to year.

"I am mentioning all this, my dear friend, only to explain to you why I shall have little free time for you this winter and why I'd prefer you come some time later, I mean in the spring, if life allows you. If this is impossible, I say again that you will be a welcome guest at any time and that all I can do for you will be accomplished. But if you come later, we have to start mathematical correspondence. It will work well, because I learned how to write letters during those faculty meetings, doctorate examinations, etc. For instance, two years ago I exchanged letters with Borchardt on the mean arithmetic-geometrical of four elements, and almost all my letters were written in the Senate hall."

Weierstrass wrote that he had had pneumonia and got a liver disease, and that lectures he had to give for a hundred and fifty people exhausted him.

We can see that Weierstrass is a little cross with his student for the long intervals in their correspondence and wrote about their meeting with reserve. But this letter did not reach Kovalevskaya in Russia for she had left for Berlin and arrived there on October 31, 1880. In early 1881 she was in Petersburg again because her husband was to return by then.

Now Kovalevskaya was deep in mathematics and full of interest for a new and exciting problem of light refraction in a crystalline medium. However, first in Petersburg and then in Moscow (she arrived in Moscow on January 8), she was abruptly faced by the fact that their financial affairs "had begun to take a bad turn". She was stunned by

the news that Vladimir owed the Ragozin company a considerable sum [87, p. 375].

Vladimir, who was abroad, had been distracted by his scientific interests and came back to Russia only in late February instead of early January, when he had been scheduled to start lecturing.

In the Spring of 1881 (the date is unknown) Sofya rushed again to Berlin and took her daughter (nicknamed Fufa in the family) and nanny with her, while Vladimir, as soon as he saw them off, went straight to his brother in Odessa.

Militsa V. Nechkina [100, p. 91] surmises that they departed so suddenly because they wanted to avoid the possible persecution that anyone suspected of nihilism would face after the assassination of Alexander II by revolutionaries on March 1, 1881. To characterize the current situation in Russia, suffice it to cite a few excepts from the diary of M. V. Bogdanovich [151]. She wrote on March 25, 1880 that 400 thousand people were "under surveillance" in Russia; a year later, on March 14, 1881, she wrote "They say nearly 70 people were arrested in the city, most of them from intelligentsia"; March 29, "Speransky was there, he said that he saw the names of those implicated in socialism, and that there are 617 known people"; April 18, "More people arrested"; May 5, "Many new arrests". Anyway, feelings in Russia were far from favourable for research.

Sofya Kovalevskaya was then focussing on her scientific interests. She was awaked by the self-esteem of a person whose talent had been buried and whose life's aim had been forgotten for a time. She said in a letter to her husband [73, p. 171]:

> "You write, and it is quite true, that no woman has ever accomplished anything, but this is exactly why it is necessary for me, while there is still energy left and we have financial resources, to put myself in such settings where I can show whether I can accomplish anything or whether my wit is not enough."

Sofya's life was by no means serene. She was troubled by the financial position of her husband and therefore tried to cut down her expenses. She was anxious because of the separation from her husband. She wrote that she missed

him, wanted to meet him, and if he missed her and missed his daughter she would return to Russia for good.

When Weierstrass and his sisters invited Sofya to spend the summer with them in Marienbad, she agreed only "not to hurt the good old people" and stayed with them for two or three weeks together with Fufa. She also told her husband that her trip there had been inexpensive.

She wrote to him from Marienbad:

"My sweet darling,

"We have been four days in Marienbad; we walk all day long; the neighbourhood is very pretty; there is a lovely fir and pine forest, and the hills are to my taste, i.e., not very steep. I am tending to become a tourist with years. How delighted I would be if we could travel with you again. I was unable to appreciate anything before, but now it would be different. What if it is really true that you won't be able to leave Russia in August! The idea is so bitter that I have all but decided to go back home for the winter. But will you allow me to go to Paris for a month or two so that I can, if possible, bring my work to an end there? The Weierstrasses have invited me to go to Switzerland, but I won't go there for the world, since as soon as I see the verdure and the forest, I can't work any more; but my conscience only allows me to spend your money abroad if I am smothered under a heap of papers. I am going to wait here only until your letter arrives. Please, answer me soon and thoroughly: may I go to Paris, where should I stay there, in whose care will you send me money for the next month if your answer is positive?

"Fufa and Marya Dmitrievna are in a state of bliss here. There are many Russians here, and we talked to some of them, but did not really meet anybody. Fufa was riding a donkey today (sitting astride in a man's style, because there are no lady's saddles here) and she was not afraid in the least. She will make another Ida Pfeiffer*, this is already clear. Good-bye, my dear sweet darling. I hold you tight. Many kisses; I am waiting for a detailed letter from you.

* A famous horse rider.

"Oh, how I'd like to see you soon and for us both to be abroad! I want to see you awfully. It seems we haven't seen each other for a whole year. Write to me what to do; we shall do as you decide.

All yours, Sofa".*

In another letter she related her joy that he was coming soon. "I'm just jumping with pleasure," she wrote. However, Vladimir did not hurry to Sofya and did not recall her home. We can explain this now: his finances had become ever more entangled, and he wanted to stay away from his wife in order to carry the burden of financial responsibility alone. But Sofya did not know that at the time and ascribed Vladimir's alienation to his becoming cool to her. Yuliya Lermontova had hinted as much to her.

In early 1882 Sofya met Vladimir in Paris; he had arrived there on business for the Ragozins. The pair wanted to go together to Cannes in the south of France, and to stay there with the family of Aleksandr Kovalevsky, but only Vladimir went; Sofya stayed in Paris for economy's sake and waited for her husband to travel back via Paris. Vladimir sent her a postcard (on January 6) from Darois, South France, very nervous in its tone [RM 2]:**

"Sofa, I had to go, and it's good you stay in Paris, you are going to be a station and point de ralliement. I'll probably telegraph you to go to André and take his word he will not tell anything to the banker Gros, who went to Moscow, but leave a note to me; you may read him a copy of my letter to Thornton and ask his opinion. Say that I went to Cannes to see Loris-Melikov and the Deputy Minister for railways to get their support for the future ... Good-bye; good girl you stay in Paris. Buy the financial journal *Semaine Financier*".

But after that "he wrote to me from Cannes in quite another tone and now, evidently, something strange is happening to him," wrote Sofya to Aleksandr after she had not

* *LOA AN*, f. 300, op. 2, No. 76.

** Here and hereafter the letters in square brackets identify a file [see Appendix 6] and the number is that of a letter.

heard from her husband for about two weeks. She was afraid he could get into trouble because of money.

"If V.O. could make up his mind to calm down and limit himself to the university, then naturally I would have to return to Russia, and that would not be so terrible, if only V.O. could calm down truly and did not torment me and himself by his endless inventions," Sofya wrote then. [71, p. 283]

She was clearly determined to work, and she could no longer understand the state Vladimir was in. But she carried out his assignments and tried to help him as much as she could. There are notes in her Paris diary of 1882:

"January 30. All day long I wrote papers for the lawyer. Had dinner at 7, slept 1 hour until 9, in the evening write letters ... and going to pack.

"January 31. Arrived in Berlin." [64, p. 177]

It appears that Kovalevskaya did not stay long in Berlin. Wise Weierstrass wrote to her Paris address on April 11, 1882 [125, p. 230]:

"My dear friend,

"More than a quarter of a year has passed since your departure from Berlin, but I haven't written to you even once. You have the right to complain, if you did not know from experience that one can clearly realize one's duty but nevertheless delay its performance from day to day and by no means because of carelessness or laziness.

"Your first letter from Paris also made me wait for long, and I can frankly confess to you that it could have been very difficult to answer it immediately.

"From each line of your letter and moreover from what could be read between the lines it was clear that for the reasons that you did not want, and could not, relate in detail you were seized by emotions and troubles threatening to interfere for long with your arduous wish to devote yourself serenely to your works.

"You are not accustomed being frank with your friends in such cases and believe that each has to try and manage on one's own what one has to bear.

"I sympathize with you completely in this respect and therefore I could not ask for explanation or more detailed information from you. But I am still your sincere friend and confessor, and could hardly pass in silence what you communicated by hints and what I have managed to adduce.

"This is the true reason why it was so difficult for me to resolve to write to you."

In this long letter Weierstrass wanted to orient the thoughts of his student to mathematics: he informed her of his latest investigations and drew her attention to the work of other mathematicians such as Kronecker, Hermite, Poincaré, Fuchs, Schwarz, and Klein.

When Sofya Kovalevskaya wrote Weierstrass at last about her personal affairs, she received the following answer [125, p. 230]:

Berlin, 14.6.82

"My dear friend,

"I was very distressed but not surprised by all that you relate in the first part of your long-awaited letter.

"In fact I have already long since guessed the true reason for your protracted stay in Paris and your absolute silence. The few hours during which I was able to meet Mr. Kovalevsky sufficed to convince me that there is an internal split in your relations which threaten to rupture them completely.

"He does not comprehend and is not interested in your ideas and aspirations, and you cannot get used to the tumultuous course of his life. Your characters are too different for you to hope to find in him what is necessary for a happy marriage, namely his backing and support, and for him to find in you a complement to his own nature. In any other case even a certain delusion on his part could not interfere with a candid reconciliation.

"When I considered it my duty to object to your plan to hold the position of privat-docent in Stockholm while he was going to hold his job in Moscow, I proceeded from my conviction that such relations between spouses are unnatural. No one can make me change my opinion that such a plan could never have occurred to you if you

had felt yourself intrinsically bound to your husband and loved him as any husband wants to be loved.

"I cannot rebuke him for having declined your plan and maybe that is why he is still more armed against your mathematical aspirations.

"It appears that the current state of affairs makes your former relations impossible. I would only like all to be settled so that you can find youself free of emotion and trouble, which is necessary for your existence. As soon as possible you must get out of your loneliness and have little Sonya at your side.*

"Your care of her and her development will take your time and will make you happy.

"I have offered my standpoint frankly and without extra words. Thank you for your confiding in me. I know you too well to thrust advice up on you and I am convinced that you are strong enough to manage your destiny on your own. When you think that my advice and my support can be in any way beneficial for you, you know that you can turn to me without hindrance."

Anne-Charlotte Leffler has said, apparently repeating Sofya Kovalevskaya herself, that the Ragozins understood her penetration with respect to their financial speculations and wanted to part her from her husband; with this in mind they succeeded in making her jealous of her husband [96, p. 139]. Sofya Kovalevskaya received from Yuliya Lermontova a letter with hints of this kind: Vladimir lodged the wife of an engineer in his apartment.

Let me note that Sofya had also to suffer in another way when she had to leave the girl in the care of Yuliya; "I must confess that I'm jealous for Yuliya who substituted for me seemingly with success in every aspect," she wrote to Vladimir.**

This comes out in the following letter written by Sofya in Spring 1882:

"Dear Vladimir Onufrievich,

"The fact that you do not write to me or your brother allows me to conclude that in all probability things are

* In July the girl was already in Paris with her mother.
** *AAN*, f. 300. op. 2, No. 76.

going very poorly. This troubles me awfully, and when I consider our situation more seriously and with coolness, my fear that your "profit hunting" would bring deplorable results increases. I am ready myself to do absolutely anything to alleviate your care of our living and not to be a pretext for you to plunge into enterprise. I am racking my brains in order to cut down our expenses, I even write down every day each centime I spend..., and nevertheless coal, coke, and various things to feed us four (including the cook) devour a terrible amount of money, so it is clear that together with Fufa we cannot spend less than Fr 800 a month (the rent and the cook's salary amount to almost Fr 400). Until I get from you some accurate idea on the state of our finances, I'll stay in Paris, because it is more sensible still to act on the basis *j'y reste où je suis** than to rush about. If you write that you can, without overstrain and without compromising your position at the university, send me Fr 800 a month (from what source?), I'll stay in Paris; if at the present moment you do not have reasonable and non-compromising resource besides your university salary, then naturally we shall have to think up of another combination for us. Maybe your brother will agree to take Fufa and M. Dm. for the summer for Fr 200-300 a month, and I can live in Berlin for about the same money. If this is still too expensive, then naturally there is nothing left than for me to come back to Moscow. Please, write and tell me frankly about your situation and your hopes...

"As to our mutual relations, there is nothing for you to worry about. Our characters are so different that you are capable sometimes of driving me crazy for a while, but when I am left to my own devices, I come back to reason, think it over with coolness and find that you are absolutely right and the best thing for us is to live for a while separately. But I do not feel any malice against you or a desire to interfere in your life at any price. Believe me, if only our finances or their lack do not clip our wings, I shall not be a nuisance to you in any way. But I repeat again, do not try to get rich *à tout prix***, expe-

* I stay where I am.
** At any price.

rience has already given you many lessons. Yours, Sofa".*

But Sofya's advice was already too late.

Once in a period when Sofya Kovalevskaya was hard pressed for money, her daughter had fallen ill. It was in a noisy Paris hotel with draughts and banging doors.

Sofya sent the girl to Aleksandr Kovalevsky while she stayed in Paris, worked and cut her expenses down to a minimum. When she heard that Aleksandr and his wife were not happy with Fufa's nanny Marya Dmitrievna and wanted to dismiss her, she asked them not to do that because she wanted Marya Dmitrievna to bring the girl, whom she missed very much, back to her [64, p. 265). She wrote that she had sent a hundred roubles to pay the nanny. Yuliya Lermontova had obtained the money by selling Sofya's wardrobe.

All this points to the fact that the circumstances were very unfavourable for Sofya's research. However, the same circumstances motivated her to complete her work as soon as possible because it gave her a hope for financial security.

Vladimir Kovalevsky was living through a terrible drama of gradual ruin. At the beginning he did not want to recognize that before his brother. Thus, in this letter to Aleksandr from Paris in early 1882 [129, p. 379] he takes offence at his brother's distrust and says that he is ready to return him his shares any time. In another letter to his brother, dated May 7, 1882 [87, p. 448], he wrote of his wife's family jewels, which he had kept after her departure, and her capital from which he pledged himself to pay three thousand roubles a year, i.e. 6% from fifty thousand. Obviously, he could not keep to his initial intention not to touch his wife's inheritance as he had decided before Sofya's mother died. Now he suffered because he was unable to keep his pledge. He expressed the hope that "in a couple of years all the capital will be returned or firmly secured in a house or some other means" [116, p. 453]; but these were just illusions. Several months later he was unable to send his wife anything and could only hope for the help of Sofya's sister who lived in Paris. He wrote to his brother in March 1883:

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* *AAN*, f. 300, op. 2, No. 33.

"I wrote to Anyuta and asked her to keep S. V. in Paris at all costs... It is terrible that I am unable to help, although this is my prime responsibility, but I cannot find any means to earn a living." [116, p. 456]

Vladimir could not bear the torment he was having to endure in connection with the increasing entanglement of his affairs in the Ragozin company and the threat of an impending fraud trial.

Considering his position hopeless, Vladimir Onufrievich Kovalevsky decided to commit suicide and wrote last tragic letter to his brother. On April 27, 1883, Vladimir took his life by covering his face with a chloroform mask.

Mittag-Leffler and Kovalevskaya

Let us return to the VIth Congress of Russian Natural Scientists and Physicians that was held in Petersburg from January 1 to 11, 1880. Scientists from all corners of the vast Russian state came to the Congress. Professor Gösta Mittag-Leffler from Helsingfors University, Weierstrass's student, came to the Congress from Finland. Mittag-Leffler and Sofya Kovalevskaya met at the Congress for the second time.

They met for the first time before the Congress, on February 10, 1876, when Mittag-Leffler came to Petersburg and visited Sofya on the instructions of Weierstrass. Sofya made a great impression on him, which he related in his letter to old Professor Malmsten [136]:

"She is charming as a woman. She is beautiful, and when she speaks, her face lights up with an expression of feminine kindness and great intellect that is amazing. Her manner is simple and natural, without any pedantry or affected erudition... As a scientist, she shows a rare clarity and exactness of expression and is exceptionally quick-witted. It is easy to see that she has reached deep in her studies, and I quite understand why Weierstrass believes her to be his best student."

After the Congress Mittag-Leffler and Sofya Kovalevskaya started a lively correspondence that lasted until the end of Kovalevskaya's life.

Gösta (Gustaf) Mittag-Leffler* was a mathematician of the first rank. His biographer Nörlund praised his scientific and organizational activity on an international scale, his being a great and "commanding" figure in the mathematical world, and an outstanding and favourite teacher who made his auditors enthusiastic [155, p. 1]. Mittag-Leffler made a considerable contribution to the theory of functions. Mittag-Leffler's theorem on analytic functions with essentially singular points comes into modern courses of mathematical analysis.

Mittag-Leffler's father Johann Olof Leffler (1813-1884) was master and then headmaster of several Stockholm schools and was a member of the Swedish parliament in 1867-1870. Mittag-Leffler's mother Gustava Wilhelmina Mittag was a respected woman. Sofya Kovalevskaya grew strongly attached to her when she lived in Stockholm. Gösta Leffler joined the names of his mother and father and became Mittag-Leffler.

Mittag-Leffler had a sister and two brothers. His sister Anne-Charlotte (Anna Carlotta) Leffler, Edgren by her first husband, was a well-known writer. Some of her stories were translated into Russian and published in Russia through the good offices of Sofya Kovalevskaya.

The older of the two brothers, the poet Fritz Läffler, was a philologist by education and in 1881-1883 he became a professor of Nordic languages in Uppsala University. Sofya was a great friend of his and when she died he wrote a fine elegy (see pp. 284-285). Arthur Leffler, the younger brother, was an engineer.

Gösta Mittag-Leffler started his education at the school where his father taught and revealed his mathematical ability early. He could easily comprehend the mathematics of the senior classes. When he went on to a Stockholm high school, he was exempt from the mathematics lessons, and he studied advanced mathematics independently. He studied at Uppsala University from 1865 to 1872 and received his doctorate there for a treatise "On the separation of the

* The basic biographical data on Mittag-Leffler are drawn from his obituary by Professor T. Carlemen [156]. A xerox of this obituary was kindly sent to me from the *Institut Mittag-Leffler* by Mrs. Silvia Ljungqvist-Carleson.

roots of a synectic function of one variable" [155]. In the same year of 1872 he became a docent at Uppsala, where Professors Malmsten and Daug were his principal teachers.

For 1873-1876 Mittag-Leffler received a Byzantine travelling scholarship for studies abroad; he went to Paris, Göttingen, and Berlin. In 1876 he was offered a professorship in Berlin University, but declined the invitation and occupied (after Lindelöf) the chair of mathematics at Helsingfors (now Helsinki).

In 1881 Mittag-Leffler returned to Sweden and became an ordinary professor at the University (Higher School) of Stockholm; he held this post for about 30 years. In 1886 and 1891-1892 he was Rector of Stockholm University.

Russian mathematicians regarded Mittag-Leffler with respect, and in 1896 he was elected Corresponding Member of the Petersburg Academy of Sciences, and in 1925 he was elected Honorary Member of the USSR Academy of Sciences.

Mittag-Leffler made an impressive presentation at the Mathematical Congress in 1925 in Copenhagen. He described the mathematical environment in which he moved in his younger years. He spoke of the mathematicians of the past, such as Abel and Galois whom he admired, just as he admired his teacher Liouville. He warmly recalled Malmsten, his teacher at Uppsala, a fine lecturer who developed a course of analysis from the works of Abel and Cauchy.

In Paris Mittag-Leffler attended lectures given by Hermite on the theory of elliptic functions and came into close contact with the French mathematicians Liouville, Briot, and Bouquet, as well as the Norwegian mathematician Broch, who lived in Paris.

Mittag-Leffler attended the lectures of Weierstrass in Berlin in 1874-1875. He reminded the participants of the Congress that Weierstrass once said, "The highest point within our science is accessible only to one who is to a certain extent a poet and has prophetic vision and a sense of beauty" [156, p. 4].

Mittag-Leffler also named the prominent European mathematicians he met later during the 1880s, among them Chebyshev, the most famous Russian mathematician.

Mittag-Leffler was a tall handsome man. Sometimes he grew his hair long, and when he skated, they said he sported a "lion's mane".

Once, after a visit to a theatre in Stockholm with Anne-Charlotte, Mittag-Leffler's sister, Sofya wrote a letter to Mittag-Leffler (undated) that is interesting both as it is and in the joking description of his appearance [SK 205]:

> "Dear Gösta, yesterday night, instead of being diligent, I was light-headed and allowed nasty Anne-Charlotte to entice me to go to the theatre with her. However, I had the unexpected pleasure of seeing you there—yes, do not dare deny it. And strange as it may seem, instead of modestly taking a seat together with all of us, you went to the stage in the mask of the Professor of Mathematics in Paul Heyse's comedy "Speaking Between Us, Brothers". True, it was said in the poster that this role was performed by Hillberg, but the carriage that was straight as a rod, the squint in the eyes, and even the somewhat long frock-coat, were so undoubtedly yours that even Knut Wicksell, who was also incidentally at the theatre, came down in the interval to ask whether we thought that Professor Leffler was very much like himself.
>
> "Today I really intend to work; if I succeed in finding this fine solution, I shall come to you after dinner; otherwise we shall meet tomorrow.
>
> Sincerely Yours, Sonya".

In 1916 Mittag-Leffler made a will according to which he and his wife Signe donated money for the organization in Djursholm of a mathematical institute for investigations in the field of pure mathematics in the four Nordic countries and especially in Sweden. Mittag-Leffler gave the Swedish Academy of Sciences his house and his fine library, which was praised by Hardy [165].

We are obliged to Gösta Mittag-Leffler in that all the correspondence Sofya Kovalevskaya had with other mathematicians, in particular with Mittag-Leffler himself, is preserved and kept in his archive in the *Institut Mittag-Leffler*. It is available for investigation and publication.

The first letter from Sofya Kovalevskaya to Mittag-Leffler, dated October 14, 1880 [SK 1], begins with the "woman's" question: is it true that Helsingfors University admits women to its lectures?

"Dear Sir,

"I would be very grateful if you were so kind as to inform me clearly with respect to the attitude your University holds to us: does it open the doors for us without restraints, or does it admit us only in exceptional cases and as a special favour, or does it refuse us completely?"

This was interesting for Sofya Kovalevskaya since a girl she knew, one Pokrovskaya, wanted to study mathematics.

On October 19, 1880, Mittag-Leffler answered that women can attend any lecture at the University without hindrance, but the problem of their being admitted for the examinations had not yet been solved: the University Council had requested permission to award diplomas to women, but the emperor* had not yet answered. Anyway, Pokrovskaya was free to study mathematics with Mittag-Leffler. He taught a course in the theory of algebraic functions and Abelian integrals (according to Kronecker, four hours a week).

Mittag-Leffler wrote about himself: he had not been to Petersburg since the Congress in January, had fallen ill in Paris, and had to ask for a leave until autumn. He passed the spring and summer in Italy and Switzerland, returned to Stockholm via Göttingen, Berlin, and Christiania**, and stayed for a day in Rüdersdorf where Weierstrass was resting at the time [ML 1].

In a letter of March 23, 1881, Mittag-Leffler discussed the idea that Sofya Kovalevskaya teach at Helsingfors University. He had heard from Pokrovskaya's mother that she was quite willing to have a job at Helsingfors University and started to explore the ground for that but found the university circles reluctant:

"All my university friends know about your outstanding talent, so do not doubt that you would be invited here if you were Finnish or of any other nation except Russian... But if you are here, it is quite probable that you will be followed by some Russian women students, and one can never guarantee that among these there will be none belonging to a revolutionary party." [ML 2].

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* Alexander II, because Finland was a part of the Russian Empire from 1809 to 1917.

** Now Oslo.

Therefore, the same misfortune had befallen Sofya Kovalevskaya in Helsingfors as had befallen her in Petersburg and Moscow. Wherever she went, government circles feared that she would be accompanied by a penetration of "nihilism" into the institution she was going to teach in.

Mittag-Leffler expressed in the same letter his hope that he would succeed in inviting Kovalevskaya to the chair of mathematics he was going to head in the newly opened University of Stockholm.

He related that Stockholm was one of the most beautiful cities in Europe, and many scientists of renown lived there. There is an Academy of Sciences that is structured like the Petersburg Academy, and there is a large and good Polytechnical School. The astronomer Gyldén was a good mathematician, as was Holmgren, the professor of mechanics at the Polytechnical School. In Uppsala, two hours by train from Stockholm, there was an old university with 1500 students; Professor Malmsten, a mathematician, retired there on pension but still enjoyed great influence in the scientific circles. Mittag-Leffler believed that the Swedish language would pose no trouble for Kovalevskaya. In connection with these prospects she wrote from Berlin on June 7, 1881:

> "As far as your excellent plans with respect to Helsingfors are concerned, I have to admit, dear Sir, that I have never taken them seriously, though I wish they could be implemented very much. I do not intend to place too much hope on Stockholm; however, I admit that I would be delighted if I had the chance to apply my mathematical knowledge to teaching at a university: the functions of a professor are somehow noble in themselves and they have always been attractive for me. To say nothing of the great significance the duties of a docent would have for me, I would be delighted to open a new career for women... But I repeat that I do not want to give myself up too much to these fine projects that probably will have the same end as most fine projects in the world." [SK 3]

Ann Koblitz writes [284, p. 144] that it was earlier that

> "Mittag-Leffler... determined, if at all possible, to find a position for her, even at the risk of jeopardizing

his own future. Gösta realized that his attempts to obtain a university job for a woman would make him unpopular in certain quarters, but he decided to proceed anyway."

In the summer (on June 19, 1881) Mittag-Leffler informed Sofya Kovalevskaya that the Department of Mathematics at the new University of Stockholm opened from the first of September and held out hope that she would agree to take the post of docent or professor; the job, however, would not be salaried at the start. He had already talked it over with his friends, Hugo Gyldén and the physiologist Retzius. In the autumn he would see Malmsten, who would do anything possible to help him in this problem [ML 3]. Sofya Kovalevskaya assured Mittag-Leffler (in a letter from Berlin dated July 8, 1881) that if she was offered the post of docent, she would accept it whole-heartedly.

"I have never counted on any other position. I can even confess that at the beginning I shall feel much less timid and embarrassed if I am just offered a chance to apply my knowledge in the field of higher education and thus to open up for women the access to universities that has not so far been granted them, with the exception of specific cases that are specific favours. Besides, the favour can be revoked just as easily and arbitrarily as it was granted, as has happened at most German universities." [SK 4]

Sofya Kovalevskaya at the time felt herself financially secure yet. She wrote:

"I am not very rich, but I still have some means to be absolutely independent. Therefore the problem of salary does not influence my decision in any way. My major goal is to serve the cause that is closest to my heart and to provide myself with the possibility of being devoted to work among the people engaged in the same task." [SK 5]

She was afraid that the efforts on her behalf would undermine Mittag-Leffler's own position at the University of Stockholm, and asked him not to hurry and undertake any decisive steps until he knew for sure the attitude of

his colleague professors concerning the admission of a woman to the faculty. She wrote to Mittag-Leffler:

> "So far I ask you not to attempt and not to talk too much about it until you are assured that you can count on those around you and on whom it depends to a considerable degree whether your stay in Stockholm is pleasant or not." [SK 5]

In a letter of July 15, 1881, Mittag-Leffler developed his idea of the work for Sofya Kovalevskaya. He wrote that the new university had proclaimed the most liberal principles, and the position of women in Sweden was quite different from that in Germany. There were about 20 girl students at Uppsala, and there was no worry that the rights of women would be taken away [ML 4]:

> "Sweden is a free country and it is not governed, like Germany, by a despotic minister*. True, it is not clear whether a woman can become a professor at the old Universities of Uppsala and Lund, but so far no woman capable enough has appeared.
>
> "Naturally, there will be great obstacles at Stockholm University as well." But he enthused:
>
> "I never fear any trouble when I work, like in this case, for a scientific cause of the greatest significance." He hoped to overcome all the obstacles. "You are going to be accepted with great sympathy, and you will have eager students. The theories of Weierstrass have the ability to interest and captivate."
>
> In Finland Mittag-Leffler had been first accepted with distrust, but he left with more than ten followers "inspired by the grandeur of our science and imbued with the desire to devote their lives to mathematical study."

He wrote that to begin with he was going to give an introduction to the function theory according to Weierstrass, and then lecture on the function theory itself.

However, later (in a letter dated November 21, 1881) Kovalevskaya referred to the opinion of Weierstrass and wrote:

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* I.e., Bismark, the "Iron Chancellor".

"... the appearance of a woman in the position of docent at a university chair is so serious a step (which may have such serious consequences for the cause that I basically want to serve) that I do not have the right to resolve to take it until I prove my abilities by my purely scientific work." [SK 5]

It took Mittag-Leffler months to answer this letter. On February 25, 1882, he accounted for his long silence by saying that he had been very busy arranging an apartment and purchasing furniture, as he had married and moved to Stockholm. His bride was Signe Lindfors, the daugher of General Lindfors from Helsingfors.

Mittag-Leffler gave lectures on the theory of elliptic functions three times a week and had 20 to 25 students, almost all of them holders of a Magister or Doctor of Sciences degrees from the Universities of Uppsala and Helsingfors. They would be the students Sofya Kovalevskaya would teach.

Initially, when the subject of her being a member of faculty had been first raised, Sofya had been anxious about her family life. Telling her husband about the project at Helsingfors University, she asked,

"Shall I accept the invitation or not? It is boring to constantly live alone, but the honour is great."*

But later, in early 1882, she had begun to consider the work in Stockholm as a chance of material security for herself and her daughter. This is clear from a letter she wrote to Aleksandr Kovalevsky from Paris in 1882 about Fufa, who had fallen ill while living with her uncle. Sofya understood that this was a lot of trouble for the family of Aleksandr Kovalevsky, but nevertheless she asked him to keep looking after the girl for a while more and added,

"I work very hard; possibly, I shall be able to arrange something for myself by the spring or summer". [71, p. 282]

In May 1883, while Sofya was staying in Paris with the Polish revolutionary Maria Jankowska, she received the news of Vladimir Kovalevsky's suicide. The blow was very

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* *AAN*, f. 300, op. 2, No. 76.

severe for her and she fell ill. She retired to her room and did not take any food. She lost consciousness on the fifth day. The doctor, whom Sofya had rejected before could then start treating her. She regained consciousness on the sixth day, reached for a pencil and paper, and began to perform mathematical calculations. During this period Maria Jankowska cared for her.

Sofya was extremely emaciated when she got better. She went to Berlin, where beside her teacher, she could become somewhat stronger both physically and spiritually. Then she returned to Russia.

Later, whenever she recalled Vladimir when she met Yuliya, she always wept and reproached herself for having left her husband alone at a crucial moment in his life.

Sofya arrived in Russia in late August. She spent much effort and succeeded in rehabilitation of Vladimir when she "proved to the investigator that Vladimir had acted in the Ragozins' speculations in conscientious error and without any material advantage either for himself or for his family" [71, p. 176].

When Sofya Kovalevskaya wrote to Mittag-Leffler in August 1883 (from Russia), she was finishing a paper that she had been engaged in for two years: the refraction of light in a crystalline medium. She was then ready to go to Stockholm to start giving a special *privatissimum* course, i.e. an optional course of lectures. She was very nervous and thanked Mittag-Leffler and Stockholm University. She wrote [SK 8]:

> "I am ready in advance to become as attached to Stockholm and to Sweden as to my native country, and I hope that having arrived there I'll stay for many years and find my second motherland."

She also wrote that she would probably spend two or three months with Weierstrass, if he had returned to Berlin by the end of October, and so she would arrive in Stockholm by the New Year. She intended to start lecturing on the theory of linear differential equations, which were well-known to her from literature by Fuchs, Tannery, and Poincaré.

Sofya Kovalevskaya was invited to the VIIth Congress

of Russian Natural Scientists and Physicians that was held in Odessa from August 30 to September 9, 1883.

This time only a few scientists participated in the mathematics sessions. The mathematicians included Professor V. P. Ermakov from Kiev and those working on mechanics included N. E. Zhukovsky, who later wrote about Sofya,

> "I have the most gratifying recollection of her. Lively and cheerful, she could speak with equal interest of mathematics and sea trips." [159]

Sofya Kovalevskaya presented at the Congress a paper "On the refraction of light in a crystalline medium". This had been her main topic in 1881-1883. She had focussed on this subject after investigating work by Lamé.

Vito Volterra later dealt with the same problem and found that Kovalevskaya's solution, like that of Lamé, was incorrect since it did not satisfy all the assumptions of the problem because of the multi-valued nature of the functions entering the solution [13, p. 279]. However, Kovalevskaya's work is of interest because it presents Weierstrass's theory of integrating linear partial differential equations with constant coefficients which Weierstrass himself did not publish elsewhere. This part of Kovalevskaya's work was later reprinted in a collection of Weierstrass's works [162].

Weierstrass did not return to Berlin in the Autumn of 1883, and Mittag-Leffler advised Sofya Kovalevskaya to go to Stockholm as soon as possible. He had already announced a course of linear differential equations for 1884, and had recommended that she give the lectures on the theory of partial differential equations, namely her own investigations, and Sofya had agreed.

Mittag-Leffler worried about her life in Stockholm: whether she would take her daughter with her, whether it would be better to rent an apartment at the beginning or to live *en pension*, which would cost only 100 to 150 kronor* a month. Sofya wrote that she wanted to leave the girl with her god-mother Yuliya Lermontova. She telegrammed Mittag-Leffler on November 16 that she was leaving Petersburg [SK 11], and on November 17 that she was leaving Hango, "Arriving steamer *Express*" [SK 12].

* 1 krona was about 50 kopecks.

Chapter 5

YEARS OF RESEARCH

Stockholm

Sofya Kovalevskaya, a fragile shy woman, arrived from Petersburg in Stockholm on November 18, 1883. The weather was murky, and the beautiful city of Stockholm with its square royal palace, large Lake Mälar (or Mälaren), and its picturesque shore was not at its best.

Mittag-Leffler met Sofya with joy. He took her to his apartment, and his wife, the fair pretty Signe, welcomed her and invited to live for some time with them. On the following morning Anne-Charlotte, Mittag-Leffler's sister, took Sofya to the dentist because she had a toothache.

Several Swedish women willingly helped Sofya in her daily life. Teresa Gyldén, the astronomer Hugo Gyldén's wife, Julia Kjellberg, Amélie Wickström, and other women helped Sofya at different times choose an apartment and furnish it. The rest of November and the whole of December Kovalevskaya stayed with the Mittag-Lefflers. She went to look for an apartment on January 5, 1884 with Amélie Wickström.

Sofya Kovalevskaya soon made friends in Sweden. The writer Ellen Key became a great friend of hers and later wrote some fine recollections about her, while Swedish society as a whole soon began to call her just Sonya (Sonja Kovalevsky).

In a letter to Maria Jankowska dated December 26, 1883 Sofya wrote that two universities were vying with each other in Sweden: the newly founded Stockholm University (the Swedes called it the Higher School), to which all the youth and the free-thinking people were striving, and the old University of Uppsala, a city two hours by train from Stockholm. Uppsala University had been in

existence for several centuries and was "a conservative centre of orthodox science and old traditions" [64, p. 274].

There was yet another old university, at Lund. Public opinion in Stockholm at the time could not agree with the inveterate tradition and medieval order of the older universities. Besides, Stockholm was chagrined at not having its own university. Hence when the idea of founding a higher education establishment in Stockholm appeared in the late 1870s, it was supported by many wealthy people. Half of the money needed to found a university was collected by subscription among the rich of the Swedish capital, the other half was given by the City Council. The founders of the university were broadminded, and the government of Sweden granted it autonomy. From the start there were many private special courses that attracted auditors not only from Stockholm, but also from Uppsala and Lund [64, p. 145].

Sofya Kovalevskaya immediately found many friends, but also many enemies, the latter were mainly at Uppsala University. She wrote [64, p. 274]

> "As soon as my course in Stockholm was officially announced, the mathematics students at Uppsala put up a notice about it in their *Verein**, and this brought about a burst of indignation from the Uppsala professors. A meeting which lasted a whole evening was devoted to my denigration; they decried my scientific merit, and hinted at monstrous and also ludicrous reasons for my arrival in Stockholm."

The arrival of Sofya Kovalevskaya aroused great interest in Swedish society, and the newspapers wrote much about the event. One democratic newspaper reported:

> "Today we must inform you *not* of the arrival of some commonplace prince or similarly highly placed but totally ignorant individual. No, rather it is a princess of science, Mrs. Kovalevskaya, who has honoured our city with her visit and will be the first woman *privat-docent* in all of Sweden." [64, p. 276, and 284, p. 179]

As soon as Gösta Mittag-Leffler and Sofya Kovalevskaya had agreed that she would lecture on the theory of partial

* Social association quarters.

differential equations, to which she had contributed considerably in her dissertation, she started to prepare meticulously for the course and wrote to Aleksandr Kovalevsky (in October 1883) [64, p. 271].

> "I think it will be very chic if a woman at the start of her lectures ... talks of her own research on the subject."

Sofya kept in touch with Aleksandr until the end of her life, because she saw him as a friend of hers and her daughter.

During January 1884 Sofya received letters from mathematicians Weierstrass, Kronecker, and Hermite, and from the physicist Lippman. They all wished Kovalevskaya the fortunate New Year and happy birthday*.

Lippman's mother wrote separately. Gabriel Lippman himself greeted the appearance of Kovalevskaya at the faculty of Stockholm University, but added [75, p. 115]:

> "France in this respect is not so advanced: the invitation of a woman to a university chair would stupefy us all."

Hermite in his letter suggested Kovalevskaya send a short memoir about the refraction of light in a crystalline medium to the Paris *Comptes Rendus*.

Sofya sent letters to Berlin for Weierstrass and Vollmar (a social democrat she met in Paris) and to Petersburg for her aunts and Yuliya Lermontova. She asked Weierstrass to consent to the publication of her memoir in *Comptes Rendus*; he replied positively on January 19 [125, p. 248].

Kovalevskaya's diaries from January to April, 1884, [64, p. 171] indicate how she was getting the feel of new surroundings.

At the end of 1883 and the beginning of 1884, i.e. the Christmas holidays, she was in Stockholm mainly busy making acquintances among Swedish society. She passed the evening of January 3 with Amélie Wickström, the secretary of the literary society "Nya Idun" in which Sofya later also took part. The Bendixsons were among the guests and Ivar Bendixson soon became Sofya's auditor. She used to go sledging with Amélie and sometimes passed a whole

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* January 3(15).

day with her. Sofya visited the Gyldéns and Lindhagen who was the Rector of Stockholm University; went sledging with the Lefflers, the Gyldéns, and the Lind av Hagebys. She dined with the Lefflers, the parents of Mittag-Leffler, and their daughter Anne-Charlotte read her a play which Sofya found very striking [64, p. 178].

Sofya's new Swedish friends assisted her greatly. However, at the bottom of her heart she could not rid herself of her feelings of loneliness. One can only imagine how a Russian felt coming to a country where the only person she knew, Gösta Mittag-Leffler, had met her only twice before. She had to speak German, a language she never mastered completely, at least she spoke it less fluently than French. Once Weierstrass hurt her a little by remarking that Mittag-Leffler spoke German better than she did. She had to lecture without having ever had any experience in it, except for a few reports on the theory of Abelian functions she had presented to a group of young mathematicians in Berlin in 1883. But now she had to give lectures to strangers, and to give them in German. Mittag-Leffler wrote to Weierstrass on February 18, 1884:

> "On January 30, 1884 Kovalevskaya gave her first lecture, in German, on partial differential equations. The auditorium was full; people were aware of the historic nature of the occasion. Not only the twelve enrolled students, professors, university officials, and interested citizens came to see the "princess of science" begin her teaching career. Sofya was nervous, and stumbled at first, but finished her talk to applause. It was clear even from the first class that she would be a good lecturer" (cited from [264, p. 182]).

There is a reminiscence of one of Kovalevskaya's first lectures from the astronomer V. V. Vitkovsky [163, p. 122] who attended it with his friend A. M. Zhdanov, Gyldén's student. Sofya Kovalevskaya was dressed in a black velvet frock and wore no decorations. She armed herself with some chalk and started the lecture before 15 auditors, "very simply and whole-heartedly, about the Dirichlet principle" [163, p. 124]. But she seemed to feel constrained because she did not once turn from the blackboard and left directly when she finished the lecture.

Her diary shows that her feelings of loneliness were especially keen during the first few days after she started her lectures. On January 30, 1884, when she gave the first lecture, she wrote [64, p. 179]:

"Gave the first lecture today. Don't know whether it was good or bad, but I know that it was very sad to go home and feel so lonely in this world. I feel especially lonely at such times. *Encore une étape de la vie derrière moi.**

February 1. Prepared all day for the lecture.

February 2. Gave the lecture. Came home feeling awfully melancholy and sat contemplating my loneliness.

February 5. Prepared for a lecture.

February 6. Gave 3rd lecture.

February 9. Lecture 4.

February 13. Gave 5th lecture. Received a letter from D. F. Selivanov this morning. Strong spell of melancholy... Leffler gave my article to the Stockholm Academy."

Then she mentioned that there was a lecture on February 16, and the last time a lecture was mentioned was on April 16.

Sofya had acquired a great friend in Gösta Mittag-Leffler, who was as devoted to her as a brother. Later, in a difficult moment, Sofya Kovalevskaya exclaimed, "I think your friendship is the best I have had in my life!" [SK 321]

Mittag-Leffler cared for Sofya in many ways. From her arrival in Stockholm he introduced her to a wide circle of his acquaintances. On January 3, 1884 he wrote a message that he had stayed at home because he still had catarrh and had suffered from insomnia, but he said that Mrs. Palme, a banker's wife, would come to visit her the next day to invite her to a grand reception at the banker's house. He advised her to accept the invitation because there would be many interesting people. Gösta himself was going to attend with his wife, sister, and father-in-law. They did not gather earlier than half past eight, so she wouldn't lose much time [ML 6].

It seems that at first Kovalevskaya did not feel quite at ease with her auditors. But later she became an outstand-

* I am over one more stage in my life.

ing lecturer who took into account the individual tendencies of her students and awoke their abilities. As one of her female students put it, she always felt that Mrs. Kovalevskaya saw through her "as if I was made of glass", but at the same time she felt at rest under that tender and sure gaze [64, p. 413].

When Sofya Kovalevskaya gave her last lecture in the spring semester of 1884, the students presented her with a photograph of them all in a gorgeous frame and made an enthusiastic speech.

After Kovalevskaya had successfully delivered her first special course in mathematics (on partial differential equations), her position became stronger and she was appointed professor at Stockholm University for five years. Ann Koblitz [284, p. 187] writes:

"...Mittag-Leffler, the geographer-explorer Eric Nordenskjöld, and the astronomer Hugo Gyldén consented to make a trade. They would withdraw their opposition to the promotion of two protégés of the Uppsala school from privat-docent to ordinary professor, which was the equivalent of an American full professor. In return, Kovalevskaya would be appointed to a five-year 'extraordinary' professorship, which was the approximate analogue of an assistant professorship.

"'You see how expensive I am,' Sofya remarked humorously..., 'I went for two ordinaries!'" [64, p. 281]

On June 2, 1884 Mittag-Leffler wrote [ML 10]:

"God knows I have not accomplished much in life, but one really big thing will always be written in my list of merits. God grant us only strength and health enough to work together long! Maybe we shall achieve much in due course!"

In the autumn of 1884 Sofya wrote to Aleksandr Kovalevsky [64, p. 508]:

"Of course, my lectures are a great trouble for me. I try hard to give them properly and clearly; sometimes I succeed and then I'm happy, but sometimes things are not so smooth; I notice that I do not manage to make my auditors interested and to present my material clearly enough, and this is a moment of distress."

Sofya spent the winter holidays in Berlin. She sent a letter from Berlin to Mittag-Leffler and wrote, among other things, about an article of August Strindberg, an opponent of women's emancipation. She joked [SK 30]:

"As a Christmas present, I received an article by Strindberg from your sister, in which he proved as clearly as $2\times2=4$ that a monstrous phenomenon as a woman professor of mathematics is harmful, useless, and inconvenient. I find that he is essentially right. The only thing I do not agree with is that in Sweden there are so many mathematicians that are far better than I am and that I was invited out of courtesy only."

However, as Georg Brandes remarked, Strindberg's article did not have any effect on Swedish society.

In fact, when Mittag-Leffler invited Sofya Kovalevskaya to Stockholm, he was taking care not only of her, but also of the interests of the chair. He believed that the admission of Kovalevskaya to the faculty of the university would give him with her assistance the opportunity to establish a very advanced chair of mathematics. He wrote in a letter of June 19, 1881 [ML 3]:

"As far as I am concerned, I shall be extremely happy if I have a chance to invite you to Stockholm as a colleague, and I do not doubt that with you in Stockholm our faculty will be one of the most advanced in the mathematical world."

In fact, he succeeded, together with Sofya Kovalevskaya, to set very high standards for teaching mathematics at Stockholm University.

Sofya Kovalevskaya gave twelve courses over seven years. The 1884 course, included the theory of partial differential equations; then followed a course on algebraic, Abelian, and elliptic functions according to Weierstrass; one on the theory of motion of solids (in 1886-1887); one on the curves defined by differential equations, following Poincaré (in 1887-1888); and one on the application of mathematical analysis to number theory. Sofya Kovalevskaya's lectures were always a success. Usually she gave them twice a week, two hours each time [186, p. 391].

Mittag-Leffler took care that Kovalevskaya's lectures would be successful. He did not want to compromise the admission of women to professorships. At the beginning, Kovalevskaya had to write each lecture and show it to him. He invariably visited the first lectures of each course.

For instance, in early February 1885 she sent him a message [SK 79]:

> "Please, have a look at my lecture for tomorrow and return it no later than noon so that I can look through it once more."

She added that this was the beginning of the lecture and that the history of the problem she had borrowed from Mittag-Leffler but had shortened.

Acta Mathematica

Gösta Mittag-Leffler was a prominent public figure in science, and he contributed much to the rise of mathematics in Sweden and Scandinavia on the whole.

One of the first ideas Mittag-Leffler brought to life was the foundation of a journal, which he named in Latin *Acta Mathematica*. Sofya Kovalevskaya was made an editor. The Scandinavian countries were represented as follows: Bäcklund (Lund), Daug (Uppsala), Gyldén and Lindstedt (Stockholm) from Sweden; Bjerknes and Broch (Christiania), Sophus Lie (Leipzig), and Sylow (Fredrikshald) from Norway; Lorenz, Petersen, and Zeuthen (Copenhagen) from Denmark; and Lindelöf (Helsingfors) from Finland. The secretary was G. Eneström from Stockholm. Kovalevskaya came into contact with all of these mathematicians. Apart from Mittag-Leffler, she met Eneström especially frequently.

Mittag-Leffler wanted to make the journal an internationally renowned one. He strove to interest the best European mathematicians in sending their articles to the journal. Each time Kovalevskaya went to Germany, France or Russia he instructed her to urge mathematicians to contribute.

As an editor, Kovalevskaya had to look through the papers submitted by the authors, and some of them were sent to her directly.

One or two issues of the journal appeared annually. In the tenth issue, which appeared in 1887, Eneström included a name index of the contributors with short biogra-

phical data and a detailed list of the articles they had published in *Acta*; the authors' other papers on the same subject were also listed, as well as notices and reviews of the articles in *Acta* written by mathematicians elsewhere. The greatest number of items in *Acta* (eleven articles) had been published by Poincaré; his participation had mainly been solicited by Mittag-Leffler and, possibly, by Kovalevskaya.

Eneström then summarised the publications in the journal's ten issues, the first of them having been printed in December 1882 and the tenth in November 1887. They contained 162 articles, of which 95 were in French, 66 in German, and one in English. There were 28 German authors, 16 French, 11 Swedish, four Danish, four Italian, three Russian, and three Finnish, other countries were represented by one or two authors. All in all, there had been 77 contributors from 13 countries.

Eneström also produced a list of all the mathematicians whose names had been mentioned in *Acta*, that is those cited as well as the names of the authors. Two portraits had been printed, one of Abel at the beginning of the first issue and one of Weierstrass in the seventh issue (1885).

The Russian contributors were Chebyshev (two articles), Sonin, and Markov. Kovalevskaya and her two papers had been listed among the Swedish authors. Later, Russian mathematicians published 16 articles in issues 11 to 20.

Mittag-Leffler wanted to make the journal international also in the sense that it should be sponsored by donations from many countries, and while mainly from the Scandinavian ones, not exclusively. Also, he expected the journal to be supported by subscription in the belief that all university libraries and even the libraries of many schools should have it. Mittag-Leffler compiled a special memorandum (he called it a *pro memoria*, or P. M.), where he listed the number of copies required by the subscribers in several countries, namely the Scandinavian ones and France. Kovalevskaya was to recruit subscribers in Germany and Russia.

The summer holidays in Stockholm would begin on the 15th of May, but Kovalevskaya had finished her lectures earlier and was in Russia on April 29, 1884, from where she sent a sad letter to Mittag-Leffler. The situation in Petersburg was gloomy [SK 14],

> "it seems as if everybody is being suffocated by a nightmare and acts in a sharp contrast with the common sense".

Ibsen's "Nora" was hissed off stage, whereas five years before

> "this fine drama would have been received in Russia with understanding".

She approached Chebyshev twice, but he was not in the mood and there was no hope that he would appeal to the Minister of Education for a wide subscription to *Acta*. Chebyshev said that the mathematics in the journal was too abstruse, "foggy", and completely useless, so his conscience could not allow him to speak in favour of the journal.

Kovalevskaya added that two weeks later Chebyshev would be going to his estate (Okatovo) and possibly his mood would be more positive when he returned to Petersburg. Indeed, Chebyshev's feelings did change both with respect to West-European mathematicians and to *Acta Mathematica*, in which he started to publish his articles from 1886.

At the end of the letter Kovalevskaya wrote that she was going to Berlin and would try to fulfill Mittag-Leffler's instructions with respect to *Acta*.

The basic topics in later letters from Mittag-Leffler concerned two new projects: (1) a prize promoted by *Acta* and (2) fundraising for an investment from which the interest would be used to pay prominent foreign mathematicians for lecturing in Stockholm. The second project was implemented much later. We know that Stockholm University invited Paul Painlevé to give lectures in 1895 and Vito Volterra in 1896 [156, p. 6].

Mittag-Leffler managed to involve Oscar II, the king of Sweden and Norway, in the first project. There was an announcement in issue 7 of *Acta* to the effect that on January 21, 1889, the king's 60th anniversary, Oscar II had decided to endow a prize for an important discovery in the field of mathematical analysis.

The prize consisted of a gold medal, worth 1000 francs, with the image of Oscar II, and a sum of 2500 kronor in gold (a krona was 1.4 franc worth).

Four problems were offered for the competition:

(1) The problem of joint motion of n solids.

(2) The development and generalization of the functions whose theory Fuchs advanced in a number of memoirs but for which he could not find an explicit form.

(3) The study of functions defined by a differential equation of the first order in the following form: an entire rational function of an independent variable, the function, and its derivative being equal to zero.

(4) An investigation of functions that had been called Fuchsian* by Poincaré, who had applied them to the integration of linear differential equations. It was necessary to study the formation and properties of algebraic relationships between two Fuchsian functions with a common group.

Other memoirs were also admitted.

In 1889 the prize of Oscar II was awarded to Poincaré for his work "On the problem of three bodies" and to Appell for his paper on the integrals of special functions and their application to the expansion of Abelian functions in trigonometric series. Both extended memoirs were published in *Acta Mathematica*, issue 13 [166, 167].

Correspondence with Friends

During 1885 Kovalevskaya and Mittag-Leffler frequently corresponded with each other. Mittag-Leffler was ill and needed treatment outside Stockholm, so he spent much time abroad, in Switzerland or in the north of Sweden and Norway. In May Sofya cheered up Gösta, who was going to leave, with a note [SK 196]:

> "Happy journey! The main thing for you is to recuperate... Thank you for everything. Faithfully yours, Sonya."

On board the steamer Gösta posted Sofya a long letter [ML 28] cheering her in the end with

> "My heart-felt thanks for your faithful cooperative work in the past academic year. I wish we should meet healthy and gay. Very truly yours, Gösta."

The two friends agreed to number their letters, and that summer they sent fourteen letters each. Mittag-Leffler's

* These are called now automorphic.

letters are full of advice and instruction. Earlier, in a letter of January 22, when he was en route to Hamburg and Kovalevskaya was in Berlin, he implored her,

> "Bring me Weierstrass's problems for the prize, whether they are ready or not, I have to get them."

Weierstrass could send his corrections and additions later, but Mittag-Leffler had to hurry with the preparation for the announcement of the Oscar II prize competition.

"Moan and groan, but do not forgo the business," he added [ML 26]. The result was that Weierstrass did send his problems.

Then Mittag-Leffler conveyed several requests concerning the University: when Kovalevskaya was in Stockholm, he wanted her to visit Councellor Lindhagen and speak to him about further lectures; if she could not see him, she should then go the following day: "energy!!!" He also wanted her to see Professor Rubenson; if he was not at home, she was to go to the Meteorological Institute of the Swedish Academy of Sciences, and "take care" of the curriculum for the autumn; Gösta provided his lecture hours and advised her to make her schedule harmonise with his. Kovalevskaya had to wait for Eneström to coordinate the lecture schedule. Then Mittag-Leffler urged Kovalevskaya not to miss the Linné anniversary festivities, "You should be seen there" [ML 28].

In another letter, he replied to Kovalevskaya's report of the Linné anniversary, and Mittag-Leffler surmised why Warming, a Danish botanist who worked in Stockholm University and was its Rector in 1882 to 1885, had said nothing when the wreath was being laid to the statue of Linné: he had not wanted to compromise himself in Denmark, where Linné was not recognized because he was Swedish [ML 29]. Mittag-Leffler's guess was probably correct because Warming became Rector of Copenhagen University in the same year, 1885.

In the Summer of 1885, Holmgren, the professor of mechanics at the University (and the Polytechnical School) became ill, and Mittag-Leffler was troubled with the problem of filling the chair of mechanics. Kovalevskaya agreed to fill the position for a year at the University, but Holmgren died, and a full-time professor of the highest expertise

was needed. Mittag-Leffler talked to all the prominent mathematicians about possible candidates. He discussed Herz with Weierstrass and received a positive report from him: Herz was then Helmholtz's assistant and a privat-docent in Kiel. He was then invited to become professor of mathematical physics at the Polytechnicum of Karlsruhe.

However, it was difficult to discuss other possible contenders with Weierstrass as he was never in sufficient contact with his disciples. His weakness was that he was surrounded by mediocrities. "He is forced to this also by Kronecker" [ML 28].

One of the candidates was a young German mathematician Schering who possessed good personal qualities, knew a little Swedish and, besides, was a relative of Professor Malmsten. Sofya was to ask Eneström to bring her Schering's works so that she could look through them. Mittag-Leffler also considered Italian mathematicians. He received a good opinion of Maggi, Gasorati's son-in-law, and asked Eneström also to send Kovalevskaya Maggi's articles for review. Mittag-Leffler was to write to Betti and Beltrami. Mechanics in Italy was a prominent subject, but the positions were underpayed; on the other hand, there was a shortage of professors there, so there was no hope of getting someone from Italy. Mittag-Leffler was in Bern for only a few hours, but he had enough time to see the mathematicians Schläffli and Stern. His talk with Schläffli was particularly interesting, as he was an outstanding person.

Mittag-Leffler wanted Kovalevskaya's advice on possible candidates for the chair of mechanics, for instance, Carl Runge. Nor did he forget at the same time to give her "diplomatic" instructions as to how to keep to the customs in Stockholm when she went for her summer leave: she had to send visiting cards to her colleagues and their wives, as well as to all her other friends who had invited her. She should visit several persons, including Gustaf Ugglas, the politician responsible for Stockholm University.

When it transpired that the Council of the University did not object to Kovalevskaya taking over the mechanics lectures Mittag-Leffler wrote to her [ML 30]:

> "It is clear that you may retain the mechanics chair as long as you want it, and I feel that You should retain

it until you can bring up a really good candidate. This will take more than one, perhaps two or even three years... It would be important, pleasant, and prestigious for you to bring up an expert in mechanics who would be Your successor."

Gösta Mittag-Leffler was preoccupied with the mechanics course Sofya was going to give: Holmgren had been an exceptionally good lecturer. In Uppsala, Lundqvist, the professor of theoretical mechanics, also gave very good lectures and was aware of all the new results in the field. To ward off unfavourable comparisons, she would have to take other topics and use more attractive mathematics. Therefore it was necessary for Sofya to know the material given by Holmgren and Lundqvist. She had to send a servant to Lindhagen and ask him to loan her a synopsis of Holmgren's lectures for a day or two. And Bendixson had to get Lundqvist's lectures in Uppsala [ML 30].

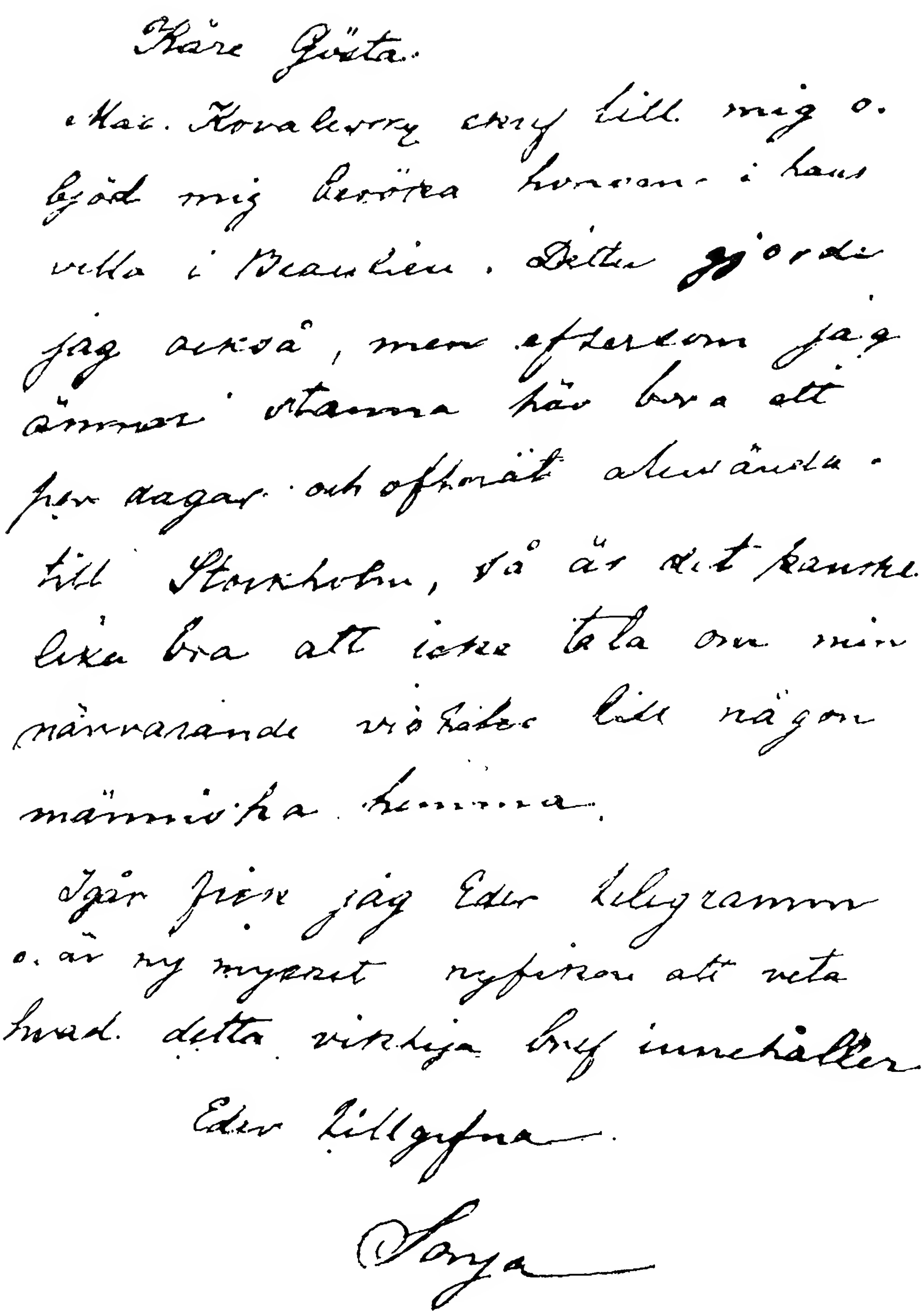
Kovalevskaya then informed Mittag-Leffler of what was going on in the University and the news she was getting elsewhere. Thus, she was visited by Eneström who had been in Berlin. He told her that Kronecker had severely criticized the theory of functions of a real variable and was very rigorous about Cantor. Kronecker seemed to have been annoyed that he had not been invited to be a judge for the Oscar II prize competition. In Berlin, Eneström heard how Cantor had been giving lectures on the philosophy of Leibniz. He had initially 25 auditors, then their number went down to three, two, and finally to one. But this last Mohican apologized, said he was very busy, and withdrew. To the joy of his wife, Cantor promised her never give any lectures on philosophy.

In a letter dated May 24, 1885 Mittag-Leffler shared with Kovalevskaya his plans for the future [ML 30]:

"Dear Mrs. Kovalevskaya,

"With time we shall make Stockholm the best place to learn the mathematical sciences, if we just manage to live and stay at least relatively healthy for several more years, and if Your spirits do not sink half-way."

When Sofya Kovalevskaya was appointed, starting from the autumn, to take over the lectures of the ailing Holmgren, it was agreed to keep this secret so as not to trouble

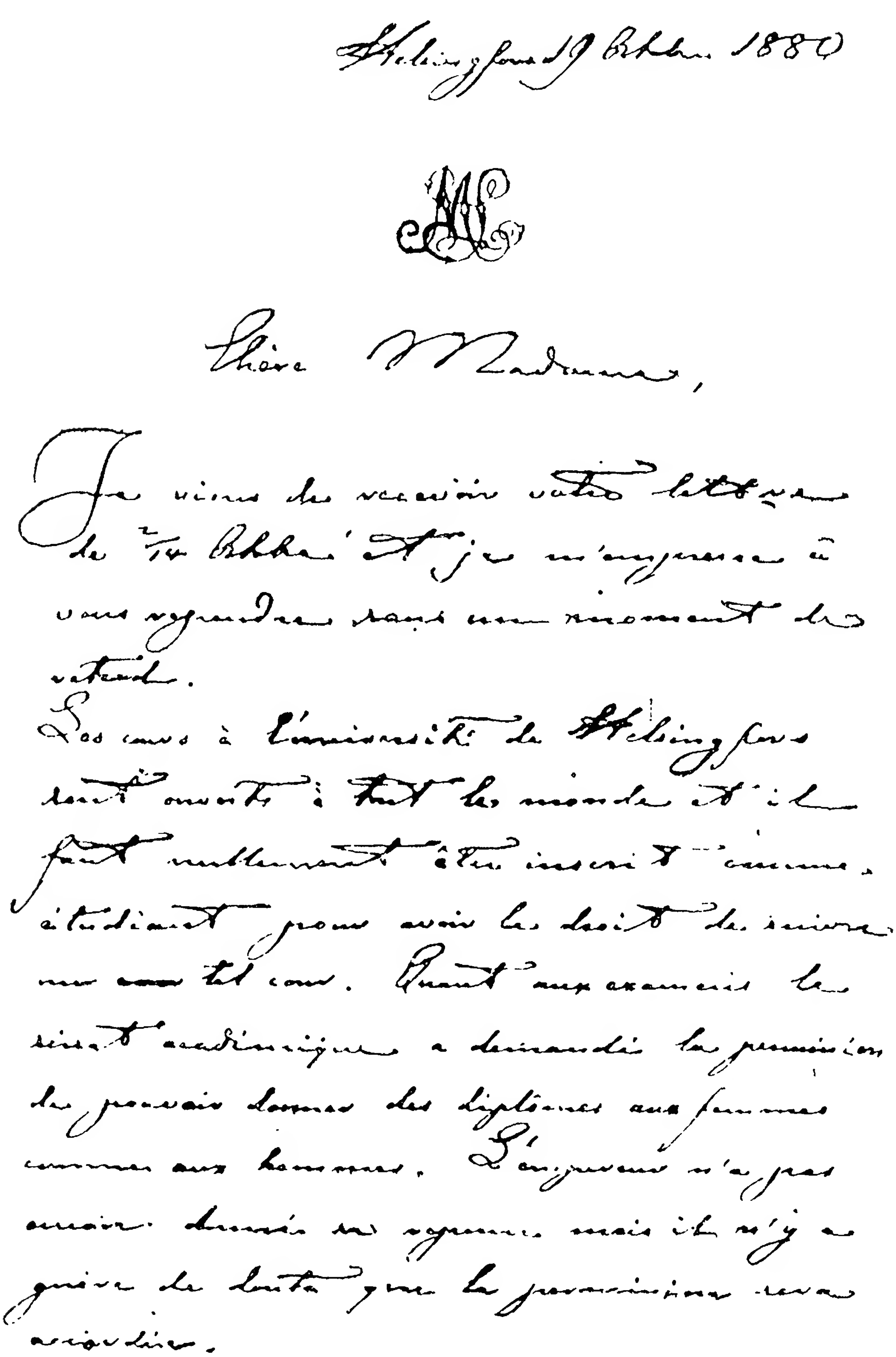

Käre Gösta.
Mr. Kovalevsky skref till mig o. bjöd mig besöka honom i hans villa i Beaulieu. Detta gjorde jag också, men eftersom jag ämnar stanna här bra att par dagar och oftnät alldeles till Stockholm, så är det kanske lika bra att icke tala om min närvarande vistelse till någon människa kunna.
Igår fick jag Eder telegram o. är nu mycket nyfiken att veta hvad detta viktiga bref innehåller.
Eder Lillgufna.
Sonya

A letter from Sofya Kovalevskaya to Gösta Mittag-Leffler

him. Of course, the decision had run into some opposition, and she wrote (in early June 1885 from Russia) [SK 43]:

"I am absolutely happy the matter has been settled, and I shall now strain every effort to prepare the course of lectures as best as I can."

Here is another excerpt from this letter revealing how witty Kovalevskaya could be. Mittag-Leffler had repoached



Helsingfors 19 Oktober 1880

Chère Madame,

Je viens de recevoir votre lettre du 14 Oktober et je m'empresse à vous répondre sans un moment de retard.

Les cours à l'université de Helsingfors sont ouverts à tout le monde et il faut nullement être inscrit comme étudiant pour avoir le droit de suivre un tel cours. Quant aux examens le sénat académique a demandé la permission de pouvoir donner des diplômes aux femmes comme aux hommes. L'empereur n'a pas encore donné sa réponse mais il n'y a guère de doute que la permission sera accordée.

A letter from G. Mittag-Leffler to Sofya Kovalevskaya

her that she was discouraged too easily and then would start to complain. Sofya answered [SK 43]:

"When a Swedish woman is tired or in a bad mood she pouts and does not talk. That is why her bad mood

enters her organism and becomes a chronic disease. On the contrary, a Russian woman moans and wails so intensely that it produces the same mental effect as limeleaf tea produces physically in influenza. On top of that, I have to tell You that I only moan and start wailing when I am slightly pained. When I am in great distress, I too am silent and no one can detect my anguish."

When Kovalevskaya agreed that she would give the lectures on mechanics, she wrote to Hansemann that she had become "a squared professor" [64, p. 292]. Holmgren died in the summer and then the problem was not to fill his position temporarily, but to find someone to give the course of mechanics full-time. The professors were in disarray because many did not want Kovalevskaya to be appointed permanently. On September 3, 1885 she wrote [SK 47]:

"Lindhagen asked the professors whether they felt that we needed a chair of mechanics. Rubenson and Pettersson claimed that, in their opinion, mechanics was not very important."

One professor told Kovalevskaya [SK 47] that Holmgren's position in the Polytechnical School was already

"occupied by someone whose name he did not know but who had a reputation of a completely paltry creature".

Kovalevskaya added:

"I am afraid some people would imagine I am very interested in this, so I think that if only we could find a candidate up to our standards it would be most reasonable to appoint him immediately."

Let me mark another episode in the correspondence between Kovalevskaya and Mittag-Leffler. In the Summer of 1885 he wrote there was a vacancy in the Swedish Academy of Sciences and he wanted to nominate Kovalevskaya. She answered on June 25, 1885, from Russia [SK 37]:

"The picture of a pretty academician's dress is constantly before my eyes, and you can have no doubts that I, in my turn, will do everything possible to help You to get it for me... I am joking, my dear friend, but you cannot imagine how touched I am by each new proof

of the interest and friendship I receive from you. You know that in effect I am rather indifferent to the honour and the outward signs of respect that are rendered to me. But I am ever sensitive to the proof of attention from my friends."

She considered this project of Mittag-Leffler's only as a sign of his attention and wrote to him on July 15, 1885 [SK 44]:

"The more I think about the vacancy in the Academy, the more I come to the conclusion that You should not exert every effort and even not exert any at all to get the position for me... Just think: it hasn't been a year since I was appointed professor; this winter I shall probably have to take the positions of two professors... If we add to this my election to the Academy, I am afraid it will upset too many people in Sweden and bring about too much envy and ill-will. Strindberg has already said that I was patronized because I was a woman. Reasonable people do not believe this so far, but it will be very unpleasant if other people reiterate it, and I fear that if we triumph now, it will cost us dear in the years to come."

Then she added:

"What is put off, is not lost. Even if You succeed in my election, it will be against the wish of many Members. Therefore, it will bring their ill-will against us. It is much easier now to pick an astronomer out of our friends... I am sure Gyldén will be of the same opinion. He does not object to my election only in fear of his wife, but I am confident that in his heart of hearts he does not want it."

Gyldén treated Kovalevskaya well but he did not want her to be his colleague in the Academy.

Social Life in Stockholm

In the second half of the last century many social clubs appeared in Stockholm for various groups. During the 1860s, a group of scientists, scholars, writers, and artists, headed by Professor Axel Key, founded a society whose aims were

"on the one hand, to meet together for relaxation and to help each other over difficulties, and on the other hand, to school the younger generation in the traditions and aspirations that should hold sway over the meetings of the society" [168, p. 102].

The society took the name of the ancient Scandinavian goddess of youth and spring, Idun, who possessed the apples of eternal youth (*idunsapplen*). The society only admitted men.

Twenty years later, some women decided to establish a similar society with the same goals and rules as "Idun", and they called it "New Idun" ("Nya Idun") [169]. Ellen Key was the chairwoman of the society and more members were attracted each year. The secretary of the society was Amélie Wickström. The first meeting was held on February 7 1885. Members of the society, including Sofya Kovalevskaya, gave talks on art, the rights of women in different countries, etc. The following topics were among the first reports: "German Katheder-socialism", "Hotel Ramboillet", "New Athens", "Business activities of Swedish women in the 17th century", "Madame de Montpensier and the Parisian salons", "Duchess Dashkova", "Dwellings in East London", "The ring of the Nibelungs", "Literary criticism", "Contribution of women to fine arts", "Investigation into the position of workers in this country", "Graphology", "A trip to America", "On some types of Ibsen's women", "Saint Bridget as a type of the time", and "Models to reform dress fashion". Anne-Charlotte Leffler presented "Aesthetic trends in England". The women sang at the meetings to the accompaniment of the piano or guitar [168, p. 103ff].

In the Winter of 1887, another society was formed and in contrast to "Idun" and "New Idun", it included both men and women. The meetings were to be of a family nature and were to encourage contact between experts in different lines. The first chairman was Professor Kurt Wallis, the first secretary was Mrs. H. Palme. The society was named "Heimdall" after the ancient Scandinavian hero, the son of nine mothers, because the founders of the society included several women, including Mrs. Palme, Mrs. Gyldén, Mrs. Curman, Miss Leffler, and Mrs. Kovalevskaya. The first meeting was held on February 23, 1887 with a talk by the

archeologist Hildebrand. Other speakers were Wallis, Geijerstam, Leche, Brögger, J. Leffler, and Tigerstedt, with topics such as "A visit to the Mormons", "Suggestion in common life", "New discovery of fossil people", and "Nansen's polar trip". Poets, artists and musicians were invited to speak. Any pair of members had the right to invite someone else. The talks or performances were followed by dinner with as many as 50 to 60 people at the beginning [168, p. 73].

In her unpublished recollections, Sofya Kovalevskaya (she mixed up "Idun" and "Heimdall") wrote that the members (evidently, of "Heimdall") arranged walks and trips.*

Sofya Kovalevskaya was respected in Swedish society. On January 8, 1950, the 100th anniversary of her birth the newspaper *Svenska Dagbladet* wrote:

> "The first woman professor in Sweden made a dazzling impression on the citizens of Stockholm during the 1880s."

An article in the same issue, signed by "Corinna", said:

> "The little foreign bird was met with great enthusiasm by the citizens of Stockholm of the 1880s, especially in the circles close to the University. They were won over by her charm, wit, and intelligence. She was so popular that once, when Professor and Mrs. Retzius invited people to a guest-night, they wrote on the back of the invitation card, 'Professor Kovalevskaya and Fridtjof Nansen have promised to come.'"

Sofya Kovalevskaya is known to be very interested in Fridtjof Nansen's project of a ski march to cross the ice plateau of Greenland. Many years later, when Nansen was in the Soviet Union, N. K. Verzhbitsky, who accompanied him on his trip to Armenia, asked Nansen about Kovalevskaya. Nansen said [170, p. 253]:

> "Kovalevskaya? She was a person of rare intellectual and physical refinement, the most clever and fascinating woman in Europe." He then paused for a long time before going on, "Yes, no doubt, I was attracted to her, and I

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* *AAN*, f. 603, op. 2, No. 2-6.

guessed the feeling was mutual. But I could not break my word, and I returned to the woman I had promised to marry. I am no longer sorry about it."

The same issue of *Svenska Dagbladet* cited recollections of the Gyldéns' younger daughter

"who remembered many of the small things from the daily life of the great Sonya Kovalevskaya".

She remembered the good jam Kovalevskaya used to make and the beautiful embroidery she presented to her friends. The paper also published an interesting story concerning friendship between Kovalevskaya and Anne-Charlotte Leffler titled "Michelangelo of Conversation":

"When the eminent mathematician Sonya Kovalevskaya settled in Stockholm in the Autumn of 1883, she was met by a hopeful and cordial academic community. Those who welcomed her included the Mittag-Leffler family. Professor Gösta Mittag-Leffler is a famous mathematician himself, and he has energetically arranged everything concerning the material welfare of the new woman docent. His sister, the famous Anne-Charlotte, whose married name was then Edgren, was ready to make friends with this clever woman...

"They can be called two outstanding women who must have matched each other well. They were very different in character and as personalities, and therefore they stimulated each other. Sonya Kovalevskaya seems to have enlivened the entire intellectual community of Stockholm during the years she stayed here.

"Ellen Key described Anne-Charlotte as the mistress of her house: peace and comfort triumphed during her parties, she had no desire to distinguish herself and instead revealed the best points in her guests and made them interesting by talking of things that were close to them. She never forced a person to seem to be a wit or a genius because the hostess was natural herself. She was undemanding. When she was silent, she looked somewhat sullen as is clear from her pictures. But when she talked or smiled her face lit up. She was very cautious in her judgement and never wanted to formulate her views categorically.

"This was Anne-Charlotte, the 'traditional Swede'...

"And what was Sonya the Russian? Ellen Key described her tenderly and with respect. A bright light in society, a 'Michelangelo of Conversation', she could touch upon a subject and develop it as a genius. She was delighted when she intuitively constructed a personality, or a destiny, from nothing, from a gesture or an intonation. She took pleasure in discussions for discussion's sake; "she built the windmills herself and then ended fighting them". Smiling Anne-Charlotte listened to her and never interrupted the stream of Sonya's broken Swedish: it was deep, and turbulent, and humorous, so apt and lyrical.

"The harmonious nature of Anne-Charlotte soothed the nervous Sonya, though sometimes it could cause irritation as well. And Sonya's spiritual wealth and liveliness stimulated and provoked Anne-Charlotte.

"When they met for the first time in 1883 at Professor Mittag-Leffler's, they went out for a short walk, and Anne-Charlotte incidentally talked of the play she was thinking over (it was "How to Do Well'). Before they had parted, the play had become clear in more detail than Anne-Charlotte had first thought, 'so great was Sonya's power over the internal world of another person,' as she said later in her autobiography when she recalled their first encounter" [171].

An interesting article appeared in the newspaper "Stockholms Tidnigen" on January 14, 1950. It began,

"'If the Academy starts electing women as Members, which of the beings in creation should stop at?' This was the response of Professor Lindhagen, the secretary of the Academy of Sciences, when in the 1880s he heard talk about electing Sonya Kovalevskaya, the professor of mathematics at Stockholm University and a famous European mathematician, to membership.

"She was not elected, and thirty years had to pass before the Swedish Academy found at which of the beings in creation it could stop without jeopardy. The person, as we know, was Selma Lagerlöf."*

* These were two different Academies: Mittag-Leffler wanted Kovalevskaya to be elected a Member of the Academy of Sciences of Sweden, while Selma Lagerlöf became Member of the Swedish Academy, i.e. the Academy for the study of the Swedish language and literature.

Weierstrass's Jubilee

Weierstrass was going to be seventy on October 31, 1885, and German mathematicians had started preparing for the jubilee in 1884. Sofya Kovalevskaya received a letter [FM 14] dated February 3, 1884, from a Berlin mathematician, Lazarus Fuchs, who was the chairman of the committee organising the celebration. Kovalevskaya was asked as a member of the committee to send (mainly to Russian mathematicians) 25 copies of the announcement concerning the celebration. A. V. Vasiliev from Kazan had not answered by then, therefore Fuchs decided that they would have to send the announcement to press without his signature.

Fuchs informed Sofya that they had started talks about an album, a bust and a medal with Weierstrass's image.

Sofya Kovalevskaya then received a letter from A. V. Vasiliev (1885, undated), whom she had once met in Berlin where he had been attending Kronecker's lectures [RM 20]:

"Dear Sofya Vasilievna,

"First of all allow me to thank You sincerely for remembering me. I have wanted several times since early winter to write to You in Stockholm and congratulate You upon the fulfilment of Your cherished wish to occupy a chair, but I was arfaid that this would be too much of a liberty on my part.

"It goes without saying that Your flattering proposal for me to subscribe to a gift for Weierstrass was very pleasant. Having thought it over, I decided to print an appeal, a copy of which I am enclosing. I have sent it to all the Russian mathematicians I know by the name, and I have asked someone in each of the cities with a university to take a more active part in the subscription: Sokhotsky in Petersburg, Bugaev in Moscow, Ermakov in Kiev, and Sleshinsky in Odessa.

"It would not be best to send money to me in Kazan, therefore I have asked that it be posted to Professor Fuchs in Berlin. Besides, I thought that it would be most gratifying for many of our compatriots to send their money for the gift to Weierstrass through his most renowned Russian disciple, and therefore I have taken the

liberty to forward Your address as well. I hope You will not grudge me for this.

"Much respected Sofya Vasilievna, this is what I have managed to do, but I do not know what will come of it. In a week's time I intend to send money donated by our small mathematical community..."

Kovalevskaya showed Fuchs's letter to Mittag-Leffler, and they started an animated discussion about the celebration, because both of them highly respected Weierstrass, his name was mentioned in almost every letter of theirs. In many letters Kovalevskaya wrote that she was distressed by the overloading of their teacher. Thus, in a letter of January 8, 1881 she wrote that Weierstrass, unfortunately, was snowed under by work that could be done by a younger mathematician whose time was not so valuable: Weierstrass was giving lectures to audiences of 250 people, editing the works of Jacobi and Steiner, and attending meetings of the Academy and the faculty Council, none of which gave him chance to finish his own investigations.

"I do not understand," she wrote, "why other Berlin mathematicians do not make the Minister understand how urgent it is to free Weierstrass from his extra load." [SK 1]

The death of Borchardt, who appeared to be the only influential friend Weierstrass had, was a great blow for him and entailed an increase in his load. In particular, Weierstrass had been editing the "Journal of Pure and Applied Mathematics" with Borchardt, but now he had to do it alone.

Kovalevskaya and Mittag-Leffler were members of the committee for Weierstrass's jubilee, and many people wrote to them in this connection. Disagreements between mathematicians began to appear. In late 1884, Georg Cantor wrote to Kovalevskaya saying that he could not put his signature under the announcement that Fuchs had sent him.

"Halle, December 30, 1884

... This announcement is so cool, colourless and wishy-washy. It does not say anythyng positive to such an extent and forshadows such failure that I don't quite know what they hope to attain.

"Consequently, in my opinion, it is necessary to compile another announcement where the great achievements of Mr. Weierstrass are reflected most fully and fittingly.

"Maybe You, madam, will be able to influence in this direction; then later I shall be willing to participate in the effort, if my friend Mittag-Leffler agrees...." [75, p. 121; 172]

The announcement Cantor mentioned had been sent to other members of the celebration committee including Mittag-Leffler. Immediately on receiving Cantor's letter, Kovalevskaya wrote to Mittag-Leffler that she was waiting impatiently for his response. She seconded Cantor's opinion in that the announcement was inadequate. It would not voice the admiration for the theories of Weierstrass his followers have, but would be just an expression of formal respect. Kovalevskaya insisted that she too would not sign anything until she knew how Mittag-Leffler thought.

Some of the mathematicians were against presenting Weierstrass with a bust because Kummer had not been given one during his jubilee. In this connection Kovalevskaya wrote:

"It would be sad if the presentation of a bust became a pretext for an exchange of caustic remarks between German mathematicians."

Mittag-Leffler took care to put in *Acta mathematica* a good picture of Weierstrass and an article about him in time for his jubilee.

Neither Kovalevskaya nor Mittag-Leffler were able to attend Weierstrass's jubilee. But the celebration was fine, and Weierstrass was pleased with it. Some time later he described the event in a letter to Kovalevskaya where he magnanimously forgave his disciple for her absence—though, naturally, her presence would have made the celebration still nicer. I would like to quote this letter (with some omissions) [125, p. 263]:

December 14, 1885

"My dear friend,

"You are a fervent sophist. You are my student of a specific kind, and You had no desire to mix up with the 'crowd' on 31 October, but preferred to make Yourself

heard a week later. I think You have the right to be called '*egregia*'*, but wouldn't it have been better to salute an old friend before everybody did?...

"First of all I have to confess sincerely that the celebration of my 70th anniversary that was organized by my old and young auditors was really a great joy for me. It had no official flavour other than the Minister of Culture sending me his semiofficial congratulations. And though it was not quite free of exageration, it was an unblemished demonstration of the feelings of the participants. Apart from my local colleagues, present were: Cantor, Schwarz, Lindemann, Killing, Tomé, and P. Du Bois, who gave me the gifts on behalf of the Committee.

"Fuchs delivered a good speech guarded by his wife's cautious glances, so a woman did adorn the celebration.

"You may have already got a gypsum copy of the bust. I will be interested to know what You think of it. My sisters don't like it very much. You and Mittag-Leffler will be sent a copy of the medal made of gilded bronze. The album with photographs (more than 500 pictures) is very good, everybody likes it.

"I have to thank especially the publishers of *Acta* for their generous gift. The photograph is very good and the passe-partout has been made with a delicate taste. But the dedication is too wordy, as is the inscription on the medal.

"In the evening, there was a proper banquet..."

After Fuchs and Kronecker had made their speeches, the floor was given to Weierstrass [125, p. 265].

"My response to both speeches was very short: I was too exhausted. Then a torrent of speeches followed.... My brother talked very humorously, trying to prove how poor his science of philology was compared with mathematics.

"In the end, all of us went to drink some beer, I stayed till midnight and my brother till dawn.

"(Two days later, we had a meeting of the Mathematical Society, and the young men behaved very well.) My brother liked the company so much that he stayed till five in

* Outstanding (Latin).

the morning, while all the older people went together with Schwarz....

"Now you, my dear friend, have an accurate account of the Weierstrass's jubilee that you probably wanted as a member of the Committee. I could send You some Berlin newspapers. You would laugh at the legends attributed to me. I have also to inform you that a poetical toast to those present was given not by my brother but by myself....

The poem by Weierstrass was enclosed in the letter. The first quatrian is a citation from a poem by August von Platen, the remaining stanzas belong to Weierstrass [125, p. 131]:

"*Schönheit* ist das Weltgeheimnis das uns lockt in Bild und Wort,
Wollt ihr sie dem Leben rauben, zieht mit ihr die Liebe fort.
Was auch lebet, zuckt vor Abscheu, alles sinkt in Nacht and Graus,
Und des Himmels Lampen löschen mit dem letzten Dichter aus".

Also der Poet, Der Forscher, dem ein güt'er Gott verlieh
Zu verstehn des Geiestes Welten und der Sphären Harmonie,
Sagt uns: *Wahrheit* ist die Sonne, deren Licht das All erhellt,
Und des Wissens Gut has Höschste, was an Schätzen beut die Welt.

Alles Schönste aber das des Menschen sehnend Herz beglückt.
Alles Höschste, das des Menschen Geist dem Erdenstaub entrückt,
Im Gemüthe edler Frauen ist's vereint zu schönem Bund,
Das uns allen kund es werde durch der Liebe Zaubermund'.

Here is an English rendering:

"*Beauty* is world's sacred secret,
lure in image, lure in word.
Drive it out of life forever,
love will never then be heard,
All the living things will start and
night will cover all in trance,
Stars of sky will die together
with the poet's heart at once."

So said the poet. Men of Science,
who, by God's will, feel the glee
With their inner understanding
of the world, its harmony,
Tell *the truth*: the sun is giving
light to lives, to all that is,
And the knowledge is the treasure
lifting us up from our knees.

All that's finest, striving always,
make our hearts beat happily,
All the noblest, making our spirit
shake off dust, is elegantly
Intertwined in gentle women's
lofty souls and tender hearts.
Magic words of love they tell us —
train of best ideas starts.

(*Translated from the German by M. Burov*)

Cheerless thoughts that some time before began to take possession of Weierstrass, came to his mind again after the joyful celebration. His ailments made him reflect upon them. When Weierstrass was only 35, he had fits of vertigo. Once during a lecture he had to sit down in a leather armchair near the podium, the students helped him out, and he stayed in bed for a long time recuperating very slowly. These fits returned in the course of 12 years and he lapsed into apathy and was unable to do anything. His doctors called it "brain exhaustion". Then varicose veins developed, his legs swelled and ached. In order to give a lecture, he would sit down, and a student would write the formulas on the blackboard. For a long time Weierstrass thought about retiring and he wanted to settle somewhere in Switzerland.

But Weierstrass was dispirited for other reasons, too. When Borchardt died he was left without his most intimate friend at the University, and he felt lonely. In a letter dated March 24, 1885 to Kovalevskaya he wrote [125, p. 256]:

> "What I need more and more is friendly cooperation with my colleagues founded upon agreement in principle and sincere mutual recognition."

But he ended the letter cheerfully, modestly expressing his satisfaction with his life-long labour:

"No one knows better than I how far I am from the goal I set myself in my youth, but no one can take away my awareness that my aspirations and achievements were not quite in vain, and the road I took to the truth was not a false one."

Weierstrass was seriously ill for his last eight or nine years. He could not walk for the three years before he passed away, and two attendants carried him from his bed to a wheelchair and outdoors, and sometimes they took him for a ride in his wheelchair to a park. But he retained almost to the last his clarity of thought and could talk to his students.

Weierstrass's 80th anniversary was celebrated quite differently from his previous jubilee. The students and friends who had gathered could not by doctor's orders talk with him for more than two hours at a time. His brother Peter had been taken ill and stayed in another town. He sent a telegram in verse, which was read aloud to those assembled.

A year later the great Weierstrass died of pneumonia.

Weierstrass had immensely influenced the development of mathematics. In the recollections of his students and in his letters he appears great both as a mathematician and a person.

His works were published in Germany in seven volumes over the course of 34 years, from 1894 to 1927. Volumes 1 (1895), 2 (1895) and 3 (1903) contain his articles, volume 4 (1902) contains his work on Abelian functions, volumes 5 (1915) and 6 (1915) that on elliptic functions and volume 7 (1927) that on the calculus of variations.

The last posthumous gathering in honour of Weierstrass was his 150th anniversary. It was held both in Münster, where he had been educated, and in Düsseldorf. A collection of works [173] was published (see also [174]), with part I devoted to Weierstrass's biography (by H. Behnke, Kurt-R. Biermann, C. Frostman, G. Hohmann, and R. König). Part II was devoted to his lectures, theoretical work, and their development, and part III to various problems related to the function theory.

In Moscow the Mathematical Society celebrated the anniversary at a meeting where I had the honour to present a paper "On the 150th anniversary of Karl Weierstrass" [130].

1886-1887

As in the years before, Sofya Kovalevskaya's life can be followed by her correspondence with Gösta Mittag-Leffler. We have already seen that they were great friends.

In the Summer of 1886 Kovalevskaya sent Mittag-Leffler a letter from Paris saying that her friend Maria Jankowska, a Polish revolutionary, had talked her into moving for ten days to her well-appointed apartment, because she was going to leave Paris for a time. She also wrote about "our famous friends". She had visited Hermite, Poincaré, and Lippman the day after she arrived in Paris; she had lunch with Poincaré and saw Tannery and Boutroux at his place; and she had been invited to dinner by the Lippmans. Her conversations with Poincaré were very interesting. Hermite told her of Poincaré's article in the newspaper *Temps* about *Acta* and herself [SK 115]. She saw Jonas Lie, a Norwegian novelist, and handed him a new book by Anne-Charlotte, which Lie praised.

The most important piece of the news Kovalevskaya sent from Paris was that she told the French mathematicians of the last results she had obtained concerning the rotation of a solid body around a fixed point, and they had considered the results significant. Bertrand told her that a meeting of the Paris Academy of Science was soon to be held, and problems for prizes were to be discussed. He decided he would propose the rotation problem for the great academic prize of 1888. The day before (the letter dated June 26), Hermite, Bertrand, Jordan, and Darboux (all the members of the commission for the prize to be announced) discussed the project with her and made her once again expound the results in detail, so she had a very good chance of winning the prize. The only inconvenience was that the results could not be presented in Christiania at the Congress of Natural Scientists to be opened on July 7, 1886, as she had planned. She even began to reconsider whether it was worthwhile going to Christiania at all (earlier she had agreed to go to Christiania with Mittag-Leffler [SK 116].

Comptes rendus of the Paris Academy of Sciences announced another suitable major prize competition at the end of 1886:

"The Prix Bordin [the problem proposed for 1888]. To improve in an important point the theory of motion of a solid body. The prize will consist of a gold medal worth three thousand francs.

"Manuscripts for memoirs entering the competition will be accepted by the Secretariat of the Institute of France until June 1, 1888. They should be accompanied by a sealed envelope containing the name and address of the author. The envelope will only be opened if the memoir to which it belongs is awarded the prize" [283].

Each manuscript was to have an attached maxim identifying the author.

In connection with the success of Kovalevskaya's research, Mittag-Lefler wrote that if he were envious, he

"would envy the luck of being able to make a new mathematical discovery and then presenting it to the most competent public in Europe" [ML 50].

He was displeased with his fruitlessness, he had missed a lot of things over the year, though nothing seemed to interest him more than his mathematical investigations.

However, Kovalevskaya did go to Christiania, though she was late, as it follows from a telegram she sent on July 8, 1886, from Havre to Mittag-Leffler in Christiania "Coming tomorrow steamer Kungdag" [SK 117]. Kovalevskaya was warmly welcomed at the Congress, though she did not make a presentation. She wrote Maria Jankowska [64, p. 509]:

"Yesterday I received a great ovation. I was elected to chair the mathematical section. At the banquet Professor Bjerknes gave a long speech in my honour, and all the participants, mostly students from Christiania, applauded so much that it seemed the walls would crash."

When Kovalevskaya went to Moscow in August, she was met with the cheerless news that her sister Anyuta was suffering from a serious disease, and the doctors said she would not live for more than a year. Sofya stayed for a while in Gatchina (a place near Petersburg) with her sister to care for her. She asked Mittag-Leffler if he could grant her leave to care for the patient, but he answered that if she did not return to Stockholm in the autumn, it would entail

some difficulties. The decision to continue her course of mechanics in the autumn semester had been just approved, and Mittag-Leffler had managed to collect money to pay for the lectures: 500 kronor had been donated by an anonimous sponsor, 300 kronor by another, and Mittag-Leffler put by 200 kronor from money earned from an insurance company for some calculation.

> "We have to keep the mechanics for ourselves. So far there is no one who could occupy the position except Lindstedt, and if he gets it, he will scarcely wish to part with it later" [ML 53].

Leave was not given for personal reasons, with the exception of treatment for oneself or scientific work. If Kovalevskaya were given leave to care for her sister, it would bring about a storm and would serve as a pretext for their opponents in the women's question and an argument against Mittag-Leffler and Kovalevskaya in their struggle for the University, in that a man would never ask for leave to care for a patient. Gösta hoped Sofya would arrive and he was ready to render any assistance, be it the removal of her furniture or dealing with Vladimir Kovalevsky's collection of fossils, which Sofya wanted to sell. In the end, Kovalevskaya returned to start her lectures.

In all his letters Mittag-Leffler voices his côncern about their journal: "Be sure to get good articles for *Acta* without hindrance." Especially original was his instruction' "Do not behave so as to be suspected of nihilism." However, he wanted Sofya's advice as to whether he should run for election to the Rigsdag (The Swedish Parliament).

Mittag-Leffler informed Sofya of himself. He was not feeling well and was getting tired too soon. He wrote from Düfed, a city in the north of Sweden, where his doctor had advised him to go, about a Lapp festivity the Swedish authhorities had organized during which

> "the Lapps held a public worship ceremony. 1000 spectators and 12 Lapps were present. The Lapps had no desire to be shown off" (a letter dated August 10, 1886).

Mittag-Leffler included an interesting description of University festivities in Uppsala in a letter dated May 22, 1887:

"There is no place like this for celebrations, and now the motherland has presented its oldest University with a room for ceremonies, the best one in the whole of the North, — an Assembly Hall. It is gorgeous and might almost compete with the 'Eden' or 'Alcazar', or, as they call them there, the grand Parisian cafes. The Assembly Hall has been ornamented after these cafes. The public at large is delighted.

"They gathered on the hill above which the '*Carolina rediviva*' (library) towers. A ceremonial procession of 1500 white caps* with flags and standards moved from there. They were followed by professors with wrinkled, as if from bitter almonds, or Petterssonian physiognomies. Then came the nobles or military in full-dress uniform with orders and ribbons, and then respectable commoners. Attendants in uniforms and with ceremonial rods were everywhere. The procession went down to the Grove of Oden and climbed another hill, where the University building is. The bright May sun, the chimes of the cathedral bells and high feelings. Everything was very imposing. Then we took our places in the Assembly Hall, which seats more than three thousand people. A fine choir of young fresh voices sang a short rhymed compendium of Boström's philosophy from K. D. Wirsén. Then the archbishop ascended the rostrum, majestically and gracefully. He walked exquisitely indeed. He thanked our Lord and warned us all against science and learning. Then he descended from the rostrum just as ceremoniously as he ascended it. Then a psalm was sung, and the archbishop sang with a watch in his hand to see that it wouldn't take too much time. Then the Rector, Sahlín, minced to the rostrum. A Dane's words at the jubilee in Uppsala came to my mind: 'It's a pity Bellmanian** types should have died out in Sweden, I have seen only one in Sweden, the magnificent Rector of Uppsala.' Then he thanked the King and Crown Prince, and then we had to listen for an hour to an extended compendium of Wirsén on Boström's philosophy, Socrates, Plato, Christ, Leibniz, and

* i.e., students.

** Bellman Carl Michael (1740-1795), a Swedish poet who wrote drinking songs and parodies on the Bible.

at last on the great one whose name was too holy to be mentioned on this occasion (Boström). Science had caught enough of it, and we were warned about publications. They think they are high and mighty, it was said. Erudition was praised. The youths were reminded that they could not possess the expertise of those of mature years, and therefore they had to study obediently what mature people had to teach them. The goal of a university was to give youth the fruit of science.

"I listened and pictured what I would say were I Sahlín. I thought I would tell youth that all the truly great and novel thoughts were born in young heads; however, they were not always put forward by young people, but when mature people advanced them to the world, they just expounded what they dreamed and thought of in their younger years. And I would try to warm up the enthusiasm of the young from this standpoint. I would tell them the goal of a university is not to shove ripe fruit into young throats, but to train young people to work so that their own labour would reward them with the best of fruit. At long last, Sahlín minced down from the rostrum, and the same fine choir sang some verses of Wirsén where he warned against the verdantists* and said that the new Assembly Hall (a concert hall in Uppsala) would become a stronghold to smash the depravity of our time. Then we proceeded in the company of His Majesty to participate in a magnificent and well-organized banquet. There were toasts to the glory of the King, the Queen, the Crown Prince and the other Princes; our ministry and the Rigsdag were thanked and both of them reminded of their duties and warned against transgressing them. Sonorously and in regal fashion, the King proposed a toast and commented on the ideas that had just been presented, the culmination being the grand words about the Assembly Hall: 'To think freely is great, to think correctly is greater!' I heard many people saying that he should have added a third phrase: 'To act correctly is still greater!' The words 'to think correctly' were interepreted by everybody as being 'to think correctly from the moral point of view'. The King was extremely

* A Students' society in Uppsala.

gracious to me and to my father-in-law. A dinner was then thrown by the governor Count Hamilton to which the King and yourself were invited. My father-in-law and I became guests at the last minute."

Mittag-Leffler added some self-critical remarks:

"I have made a few interesting acquaintances and I lavished praises on *Acta*, but I allowed some uncautious remarks about Uppsala and the University building to escape my lips. As always I've been mixing wise and favourable chess moves with thoughtless, precipitate, and quite unfavourable ones" [ML 56].

In the same letter Mittag-Leffler said a few words about his relations with General Lindfors, his father-in-law.

"The days I stayed there, in Uppsala, were... terribly tiring. I couldn't sleep for three nights. And when I came home, my father-in-law didn't give me a minute of rest: dinners, cafes, theatres, cognac, champaigne, cigars—anything I can't stand.... My trip to London will most probably be cancelled primarily for reasons of economy and secondly for health considerations. My father-in-law would surely go with us, and it will kill me if I have to roam with him to all the pleasure spots of London." [ML 56]

The year 1887 was overshadowed for Kovalevskaya by the disease and death of her sister. In the spring Victor Jaclard had been given four days to leave Petersburg as part of the repressions after the attempt on the life of Alexander III on March 1, 1887. Kovalevskaya requested Anna Grigorievna Dostoevskaya* to appeal to Pobedonostsev for an extention because Jaclard had to take his gravely ill wife to Paris. An extention of ten days was obtained, but this was not enough as Anna could go to Paris only in the autumn. In Paris Anna Jaclard died after a serious operation and consequent pneumonia.

Mittag-Leffler became interested in health resorts in the Pyrenees to treat his disease. He received information about the resorts through Kovalevskaya from Victor Jac-

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* F. M. Dostoevsky's widow.

lard. In addition, she sent him letters to two French doctors, to Letourneau who was the editor of the *Revue philosophique*, and to a young doctor who "would do anything he can". Sofya told Mittag-Leffler about her daughter Fufa, who found that Stockholm was more fun than Moscow.

In response to her letter, Mittag-Leffler wrote to Kovalevskaya on July 2, 1887 [ML 58]: he was glad she had started working. It was essential that her work was good and ready before the dead-line as it would strengthen her position. It was necessary that the work which promised to be very extensive, should be presented for the Oscar II prize because it was an answer to the first problem for the prize.

> "It would be an unheard-of triumph," Mittag-Leffler wrote, "if you could get both prizes. And besides, it would be a small fortune for you to start you working in France, if, as I think, you get there." (*Ibid.*)

Kovalevskaya thought now of working outside Sweden, if not in Russia, then in France. The magnanimous Gösta was ready to help her anyway, even if it was against his interests and plans and would be very bitter for him.

When Kovalevskaya returned to Stockholm in August, she spent three days looking for an apartment and found a suitable one in Sturegatan, opposite the Sture park. It consisted of five small rooms on the fifth floor, near where Anne-Charlotte lived, and cost 900 knonor a year.

Every day at 4 p.m. she visited Mittag-Leffler's brother, the poet Fritz Läffler, who was suffering from nervous disorder. He was a pleasant person, who had read widely and thought deeply; he showed her his poems on patriotism. In the evenings, she went walking with Lindstedt, the professor of mechanics in Uppsala. Gösta had asked Sofya to induce Lindstedt to start writing his scientific articles again. He had a large family to support and therefore had been taking on other work. After a while, Lindstedt published a number of articles on the three body problem.

Kovalevskaya then received two articles for *Acta*, one from a German and the other from a Russian author. Mittag-Leffler wanted her to see whether the articles could be reduced and then for her to pass them to Eneström.

Mittag-Leffler was having trouble: notes on Weierstrass's

theory of Abelian functions had gone missing. A courier was to take them to Phragmén but had taken them somewhere else and didn't know where. In the autumn Kovalevskaya was to continue with her course on Abelian functions. Mittag-Leffler advised her to alternate lectures on the Poincaré curves defined by differential equations and the theory of Abelian functions. He hoped that the notes would be found. Otherwise, he would have to order through Professor Knoblauch for some parts of Weierstrass's theory to be copied (it would be very expensive to copy the entire work, as a student could charge 40 pfennings, i.e. about 20 kopecks, per page). Gösta hoped that Sofya would deliver lectures on her own work in the course of mechanics. He was happy that her apartment was not far from the Lefflers' and thanked Sofya for her attention to his brother.

In late summer, Mittag-Leffler was en route in Paris and Berlin. Weierstrass was hurt because she had not written to him. Gösta explained to him that Sofya was writing a play "The Struggle for Hapiness" together with Anne-Charlotte, and she was ashamed to write to him. Let me note that they had given the play to Mittag-Leffler to read. Mittag-Leffler was opposed to Sofya's literary work because it distracted her from her main job, and he also criticized the drama. Kovalevskaya answered his criticism by a letter (quoted in Chapter 7).

The Prix Bordin

For Kovalevskaya the year 1888 was full of important events. She had at all costs to make her results in the rotation problem presentable for the competition. In the summer, Kovalevskaya strove to finish the work, and submitted it in late summer. By December she was informed by the Paris Academy of Sciences that she had been awarded the prestigious prize. It was announced at a ceremonial session of the Academy on December 24, 1888. All this was preceded by other events, of which we know from Kovalevskaya's correspondence.

In April, Gösta, Signe, and Anne-Charlotte travelled to Algeria. The trip had been recommended to Mittag-Leffler for health reasons. During his stay in Algeria, a scientific conference was held in Oran, and the English mathema-

tician Sylvester was present. Mittag-Leffler wrote to Sofya [ML 64]:

"...we have been met with great hospitality. Both he and I, as well as my two ladies, are considered guests of the town, and we were the joint honorary chairmen of the conference. They talk rubbish, however, at the mathematical section, but there were some very interesting excursions. The other day, an attack by 1000 Arabian horsemen was staged. It was grandiose, but we were almost knocked down by the horses that went wild. I lost track of Signe and Anne-Charlotte completely. We all thought of the 'bravery' You would have shown."

Mittag-Leffler was interested in what was going on at the University: how was Phragmén getting on (he had to sit for his doctoral examination), what happened at the meeting of the society of thirteen* (a free-thinking society they were members of) and how Sofya was getting on with her work. A Miss Lagerborg had to take her exams in mathematics and mechanics and Mittag-Leffler wrote that she was not to graduate until she was completely prepared, "otherwise it will be a great scandal for us." [ML 64]

In a letter of May 18 Mittag-Leffler conveyed his summer plans. He had decided to go to Finland to visit his father-in-law General Lindfors, and in mid-July he was going to Lake Siljun, because he had on doctor's orders to stay in the mountains and rest. But he wanted to go to the Harz, when Weierstrass would be there, and other young mathematicians will gather around him. At the end of the letter he wrote about Anne-Charlotte: she was talking with one Señor del Pezzo about boat trips and Dante. (Señor del Pezzo was an Italian mathematician. After his father's death he became Duke di Cajanello. Anne-Charlotte married him in 1890).

In a letter of July 5, 1888 from Paris, Sofya wrote that

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* The "Society of Thirteen" ("Tretton-sällskapet") originally included the following 13: professors, scientists, and social activists: Brögger, Gyldén, Hildebrand, A. Key, Kovalevskaya, Leche, Lindstedt, Lovén, Mittag-Leffler, Montelius, Retzius, Smitt, and Tigerstedt. The members met in turn at each other's homes and presented reports [286, p. 260].

she became acquainted with French physicians who treated by means of hypnosis [SK 242]:

> "I saw much interesting in the field of hypnosis, and was present at a séance by Charcot, Luys (from the Medical Academy), and Dr. Berillon (editor-in-chief of the journal *Hypnose*); the latter met you in Oran. Tomorrow I am going to a lecture given by Voisin, and Berillon will present me to him. I have to confess that all I saw here strongly undermined my faith in hypnotism. I was very attentive to all I saw and made notes. They are in Russian, but I shall translate them for you when we see each other again."

Mittag-Leffler was very interested in hypnotism because of his brother's nervous disorder. Sofya Kovalevskaya published two articles about her visit to two French hospitals where the patients were treated by hypnosis [41, 42].

In the Summer of 1888 Mittag-Leffler went to Switzerland and Kovalevskaya to Paris. She wrote on July 5 [SK 242] that all the mathematicians in Paris seemed interested in the results of her work, and so it was very important for her to send at least a poor copy and substitute it before October by another one that is better edited. They thought that everything would be all right and Kovalevskaya would get the prize. Then she wrote that Bertrand had given a dinner and invited Sofya Kovalevskaya, Hermite, Picard, Halphen, and Darboux; and three toasts were proposed in her honour. She was to leave Paris on Luly 17 and move to the Harz where Weierstrass was resting. She intended to work hard there although Weierstrass wanted her to stay with him for several weeks during the summer.

> "He says," she added, "that he is growing weaker, and he has much to tell me and is afraid that he will not have a chance to do it later." [SK 243]

In late July, Kovalevskaya was in Wernigerode (in the Harz), where she was going to stay until August 15 at the latest. She continued her work with great ardour. A group of younger mathematicians came to visit the veteran Weierstrass, such as Mittag-Leffler, Cantor, Schwarz, Hurwitz, Hettner, and Volterra. Naturally, they talked among them-

selves a great deal and Sofya was vexed that she had to work and could rarely participate.

In the autumn, on September 11, Kovalevskaya wrote that she was in the Harz, in the small town of Thale, and for the last five days had stayed alone with Weierstrass. His sister, who had lived with him in the summer, had gone to Berlin to tidy up the apartment. Weierstrass had not felt very well for a time.

> "Poincaré's article makes him work hard, but he cannot understand it properly." [SK 244]

Weierstrass suspected there was a mistake or incomplete proof in one of Poincaré's works, and he was annoyed by it.

Kovalevskaya wrote that she was leaving in a few days for Stockholm, after passing several days in Berlin en route. She hadn't quite finished her work, but there wasn't much left.

At last, the work over the rotation problem was completed and sent for the competition. In the second half of December, Kovalevskaya received the long-awaited-for announcement from Paris. It was written on a sheet of paper adorned with the image of Minerva, the goddess of wisdom.

Institute of France
Academy of Sciences

Paris, December 18, 1888

Permanent secretaries of the Academy
to Madam Sofya Kovalevskaya in Stockholm

Madam,

We have the honour to notify You that the Academy of Sciences awarded You the Prix Bordin (improvement in an important point of the theory of motion of a solid body).

We invite You, Madam, to be present at a public session that will be held on Monday, December 24 of this year at one o'clock p.m. exactly and at which time the results of the competition will be announced in public. We take this opportunity to convey our personal congratulations and testify to our confidence of the benefit the Academy anticipates Your work and Your advances will bring.

Madam, receive our assurances in our highest respect.

L. Pasteur J. Bertrand

[SK 241]

There were two signatures because the Paris Academy of Sciences had two sections: one for the natural sciences and one for physics and mathematics.

The award ceremony was held on December 24, 1888 in the great hall of the Institute of France. Sofya Kovalevskaya and her guests from Russia, Maksim M. Kovalevsky and E.V. de Roberty*, were given a side box.

The chairman of the session, Academician Janssen, an astronomer, handed the diploma to Kovalevskaya and said [176, pp. 1035-1036]:

"Gentlemen, among the laurels we are giving today, one of the finest and most difficult to attain we lay on the forehead of a woman. This year the great prize of the Mathematical sciences** is conferred upon Mme. Kovalevskaya. Our colleagues from the geometry section having studied the memoir submitted for the competition, discovered in it not only evidence of wide and deep expertise, but also an indication of a mind of great inventiveness.

"Mme. Kovalevskaya is a Professor at the University of Stockholm, where she teaches young scientists. She is a descendant of a King of Hungary, Matthias Corvinus, who was both a great warrior and an enlightened patron of Science, Literature, and Fine Art. Obviously, these latter qualities have been inherited by Mme. Kovalevskaya from her famous ancestor, and we congratulate her upon it."

The announcement that the prize had been conferred on S. V. Kovalevskaya was published in *Comptes Rendus* in December 1888 [176]:

The Prix Bordin

(Judges: Messrs Moris Lévy, Phillips, Résal, and Sarrau, and Mr. Darboux, rapporteur.)

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* A philosopher and sociologist.

** Janssen made a mistake, it was the Prix Bordin.

> The Academy proposed that the topic for the Prix Bordin to be awarded in 1888 was 'To improve in an important point the theory of motion of a solid body.'
>
> The prize Commission has unanimously awarded the prize to the memoir registered under No. 2 and having the maxim "Say what you know, do what you must, and whatever will be, will be". This remarkable work contains the discovery of a new case in which it is possible to integrate the differential equations describing the motion of a heavy [solid] body fixed at one of its points. Therefore, the author has not only added a result of the greatest interest to the ones established by Euler and Lagrange, the author has also made a discovery, for which we are obliged by the author's profound study which involved all the resources of modern function theory. The properties of theta functions with two independent variables make it possible to give a complete solution in an exact and elegant form. Consequently, we have a new and remarkable example of a problem in Mechanics using these transcendental functions, whose application has until now been limited to pure Analysis and Geometry.
>
> The commission has requested that the prize-winning memoir be published in *Mémoirs des Savants étrangers* [Memoirs of Foreign Scientists].
>
> Having concluded this Report according to the Regulation, Mr. President opened the sealed envelope attached to the Memoir and announced that the author's name is Mrs. Sofya Kovalevskaya."

Because of the merits of Kovalevskaya's work, the Paris Academy of Sciences had raised prize money from 3000 to 5000 francs. Kovalevskaya has written that about fifteen memoirs were submitted for the prize, but only her work was rewarded. The rotation problem had been proposed for the Prix Bordin three times before, but the prize had not been given [64, p. 149].

In 1889 Kovalevskaya was awarded a prize by the Swedish Academy of Sciences for additional research in the rotation of a solid body. In all she published three articles on the subject [6-8].

Hermite perceived the significance of Kovalevskaya's investigations and wrote to her that he waited their publication with excitement.

In a letter to Mittag-Leffler in late December 1888, Kovalevskaya wrote [SK 247] that she had been five days in Paris and was staying in a furnished apartment consisting of a large sitting-room and bedroom, costing 200 francs a month.

The day after her arrival, Kovalevskaya saw Hermite, who was very obliging and invited her to dinner on Tuesday 25th, when the Hermite and Bertrand families would gather. She also visited Bertrand, Poincaré, and Darboux. Bertrand wanted to throw a party in her honour and invite the Swedish ambassador. Kovalevskaya was embarrased because she did not know his name, and she asked Mittag-Leffler to forward it to her: she would visit him [SK 247]. Sofya Kovalevskaya received many invitations, where she was congratulated and praised.

Chapter 6

KOVALEVSKAYA AND THE MATHEMATICAL COMMUNITIES

Russia

In the second half of the 19th century, mathematicians in Russia were headed by P. L. Chebyshev, in Germany by Karl Weierstrass, and in France by Charles Hermite. Sofya Kovalevskaya established at various times scientific contacts and had friendly relations with each of them.

Chebyshev and Hermite were almost the same age. Weierstrass was older, but we have seen that he entered the "official" scientific arena late, remaining a school teacher in a provincial city until he was forty.

Chebyshev had a shrewd and penetrating mind, his work was full of novel ideas. He investigated the integration of irrational differentials, the theory of best approximation, probability theory, algebra, number theory, and other major problems in mathematics and mechanics. The integration of irrational differentials and the theory of best approximation are of special interest for us in connection with Sofya Kovalevskaya.

In the late 1850s, Hermite, Chebyshev, and Kovalevskaya established close scientific relations, mainly owing to Chebyshev's research on the integration of irrational differentials, in particular differentials in the form

$$[(x + A)\, dx] / \sqrt{P(x)},$$

where $P(x)$ is an integral polynomial of the fourth degree.

Chebyshev developed some elegant transformations making it possible to find out whether the integral could be expressed in elementary functions, such as the logarithms of algebraic functions. He was especially interested in the case when the coefficients of $P(x)$ were rational numbers

[220]. E. I. Zolotarev [221], Chebyshev's disciple, generalized the problem and strictly proved it applying the theory of elliptic functions [224].

One of Chebyshev's papers appeared in 1857 in the Liouville's journal [223], and Weierstrass responded by publishing also in 1857 a paper on his own technique of solving the problem with a full reference to Chebyshev [226]. A French mathematician E. Rouché was also engaged in the problem [225]. Hermite was astutely interested in the work of Chebyshev and later works on the same subject, Rouché's included. On June 27, 1858, he wrote to Chebyshev noting that Rouché had underestimated Chebyshev's work [220, vol. V, p. 427]

> "...this theory belongs to you in a degree that is much greater than is admitted by M. Rouché".

Having commenced a correspondence with Chebyshev, Hermite continued it until Chebyshev died. Ten letters of Hermite to Chebyshev are known [220]. In one of his first letters, Hermite informed Chebyshev of the investigations undertaken by Weierstrass [226] and advised Chebyshev to consider more general methods based on elliptic functions. But Chebyshev did not take this advice, and he clung to his "algebraic" method. This allowed others, first of all his disciple Zolotarev, to expand the problem and the techniques needed to solve it.

When Weierstrass began to work with Kovalevskaya, one of the three problems he gave her was to reduce certain Abelian integrals of the third rank to elliptic integrals [2], [15].

An Abelian integral has the form

$$\int R(x, y)\, dx,$$

where R is a rational function of its variables and y satisfies the algebraic equation

$$f_0(x)y^n + f_1(x)y^{n-1} + \ \ldots\ + f_n(x) = 0,$$

where $f_0(x)$, ..., $f_n(x)$ are integral polynomials.

A particular case of an algebraic equation is

$$y^2 = x^n + a_1x^{n-1} + \ldots + a_n.$$

The corresponding integral is termed an integral of the third rank (or genre) if $n=7$ or $n=8$.

A similar problem for an Abelian integral of the second rank was proposed by Weierstrass and considered by Koenigsberger (for $n=5$ or $n=6$) [148]. Chebyshev's problem involves an Abelian integral of the first rank.

On returning to Russia in the Autumn of 1874, Kovalevskaya attended a party at D. I. Mendeleev's home. P. L. Chebyshev and A. V. Gadolin (who was a crystallographer and artillerist) had also been invited, and Kovalevskaya argued with them until 1 o'clock in the morning. Most probably, the gist of the debate with Chebyshev was related to the approach to integration of Abelian integrals: the "algebraic" approach (according to Chebyshev), which leads to a result after a finite number of steps, versus the "transcendental" approach (according to Weierstrass), which reveals the general properties of the integrals of a given rank.

Weierstrass is known to have burned all the letters Kovalevskaya sent him; in one of her letters she most probably told him of the discussion with Chebyshev, and he answered her on January 12, 1875 [125, p. 204]:

> "You wrote to me recently that Chebyshev liked to pose problems for you concerning the integration of elliptic differentials through logarithms. This encourages me to take up once more my old work in the subject."

Then Weierstrass presented his treatment of the problem, so it was "almost a small paper".

Therefore, Kovalevskaya though not a disciple of Chebyshev, was intimately acquainted with his research. And when in 1879 he suggested she make a presentation at the VIth Congress of Russian Natural Scientists and Physicians, she took up the reduction of Abelian integrals, although she could have chosen one of the two other problems from her doctorate dissertation.

These debates over the reduction of Abelian integrals probably induced Kovalevskaya to include the following lines in the paper that was published in 1884 [2]:

> "In conclusion I should say that my aim was not to prove the results obtained, because they can be obtained much more quickly using purely algebraic methods, but to

give an example of the application of the theories of my respected teacher, which I presented in the introduction. He induced me to carry out and *completely* investigate the case for $\rho=3$, $k=2$."

(Here ρ is the rank of the integral and k is the degree of the substitution.)

Another field of Chebyshev's investigation that Kovalevskaya was interested in was problems of the best approximation of functions by "simpler" functions [220]. Chebyshev's remarkable results in the theory of functions that deviate least from a given function had drawn the attention of scientists at once. That polynomials which deviate little from zero can be presented by trigonometric funtions, such as $\cos(n \arccos x)$, aroused such admiration that Bertrand included it in his course of analysis [227], and high school teachers tried to treat Chebyshev polynomials in an elementary way (at least those of the second, third, and fourth degrees) [122]. Zolotarev sought a polynomial $x^n+p_1x^{n-1}+\ldots+p_n$ given $p_1=\delta$ and found it could be expressed through elliptic functions [228-230].

On January 8, 1881, in one of her first letters to Mittag-Leffler, Sofya Kovalevskaya expressed regret that Russian mathematicians exhibited complete indifference to Abelian functions and ascribed it to the fact that the books by Neumann [231] and Briot and Bouquet [232] on the subject, which were the ones known to Russians, were not well-written. She added:

"...recently I had a very vigorous debate with several professors at Moscow University who claimed that Abelian functions were not yet ready for serious application and the whole theory was so dry and entangled that it could not be a subject of a university course." [SK 2]

Then Kovalevskaya asked whether Mittag-Leffler was familiar with the work of Chebyshev and Zolotarev on integral polynomials of x of the nth degree that for all real values of x between given limits deviate least from zero. The discussion Kovalevskaya had on the application of Abelian functions made her undertake an investigation in connection with Chebyshev's problem. The point was that at the end Zolotarev's paper gave an expression of

the polynomial for a given condition through elliptic functions. Then Kovalevskaya believed he made

> "the bold generalization that a polynomial wherein there are two or more relationships given between the coefficients could not be considered from the viewpoint of modern mathematics".

But Kovalevskaya believed that

> "this is quite possible for anyone who is familiar with Abelian functions".

We do not know whether Kovalevskaya obtained any results in the field. Great hopes had been placed on Zolotarev, a talented mathematician, but he had died in an accident at the age of thirty-one by the time the letter was written. However, a number of other scientists developed the work of Chebyshev and Zolotarev, and covered polynomials with two and more conditions for their coefficients. Chebyshev thought much about the problem of the greatest and least values. He wrote [220, vol. III, p. 264]:

> "Despite the current development of mathematics with regard to the theory of the greatest and least values, practice goes further and requires us to solve newer varieties of greatest and least value problems, which are essentially distinct from the two that are solved in differential calculus and calculus of variations. There is a problem that is common to any operation, that is 'how to distribute allocations in order to maximize the gain?'"

Chebyshev appears frequently in the correspondence between Kovalevskaya and Mittag-Leffler. When Kovalevskaya started work in Stockholm, she often went to Russia on holiday and invariably visited Chebyshev, conveying Mittag-Leffler's requests for support of *Acta Mathematica* and discussing mathematics and mechanics with him.

One topic of conversation was the problem of the equilibrium shapes of a rotating liquid. According to Lyapunov, Chebyshev suggested to Zolotarev and Kovalevskaya a problem of transition at certain angular velocities from certain ellipsoidal and rotational shapes to other equilibrium shapes [233, p. 328]. Lyapunov himself later tackled this problem.

When the first issue of *Acta Mathematica* was printed, Mittag-Leffler sent a copy to Chebyshev with a letter dated December 2, 1882, requesting him on behalf of Oscar II, the King of Sweden and Norway and the patron of the journal, to present it to the Petersburg Academy of Sciences and to publish in its proceedings (Izvestiya Akademii) a few words in connection with the presentation. Mittag-Leffler also requested Hermite and Weierstrass with respect to the Paris and Berlin Academies of Sciences. He proposed to send his journal in exchange for the *Proceedings* [Izvestiya] *of the Petersburg Academy of Sciences* and "possibly for still other printed material of the Academy in the field of mathematics" [220, vol. V, p. 449].

Chebyshev fulfilled Mittag-Leffler's wish. At the session of Physico-mathematical section of the Academy of Sciences on January 18, 1883, he praised the journal and its editor. He said in particular:

> "Gösta Mittag-Leffler is a famous mathematician to whom transcendental analysis is obliged for his fundamental contributions. He is editor-in-chief of a new mathematical journal published in Stockholm under the title of *Acta Mathematica*".

He pointed out that its editorial board included A. Bäcklund, H. Daug, H. Gyldén, H. Holmgren, K. Malmsten, K. Bjerknes, O. Broch, S. Lie, L. Sylow, L. Lorenz, J. Petersen, G. Zeuthen, and L. Lindelöf and added that the new journal

> "will not only promote the development of the mathematical sciences in Sweden and Norway, but will also spur their progress all over Europe" [220, vol. V, p. 384].

In a letter of April 8, 1884, Mittag-Leffler expressed his gratitude for Chebyshev's kind words. He called Chebyshev "one of the greatest ever masters of analysis" and asked him to send an article to *Acta Mathematica*. He wrote that he had studied Chebyshev's excellent investigations on maxima and minima. Sending him three volumes of the journal, he asked Chebyshev to accept them

> "as a modest gift to a genius who has lavishly enriched our science with his immortal discoveries" [220, vol. V, p. 450].

But Chebyshev was slow in sending his material. Mittag-Leffler therefore decided to include a translation of Chebyshev's paper "On the presentation of the boundary values of integrals by means of integral residues" [234] which had been published in 1885 in the *Proceedings of the Petersburg Academy of Sciences*. This is an article where Chebyshev raises the problem of the greatest and least boundaries of the integral

$$\int_u^v f(x)\,dx,$$

if there are n values

$$A_{k-1} = \int_a^b x^{k-1} f(x)\,dx, \qquad k=1,\ 2,\ \ldots,\ n,$$

and if $a<u<v<b$, provided $f(x)$ is non-negative within the interval $(a,\ b)$. Chebyshev solved the problem by using a Cauchy integral expanded into a continued fraction

$$\int_a^b \frac{f(x)\,dx}{x-z} = \cfrac{1}{\alpha_1 z + \beta_1 + \cfrac{1}{\alpha_2 z + \beta_2 + \ldots,}}$$

Sofya Kovalevskaya translated this paper into French, and it was published in the ninth volume of *Acta Mathematica* in 1886 [69].

This interest in his work made Chebyshev give the matter further consideration, and he presented his result in a letter to Kovalevskaya on September 20, 1886. The letter was published in the same volume of *Acta Mathematica* under the title "On the Sum Composed of the Coefficients of a Series with Positive Terms. A Letter Addressed to Mrs. Sofya Kovalevskaya" [70]. The letter begins [70, p. 182].

"I am very happy with the honour you rendered me by translating my paper on the boundary values of integrals. The interest you have exhibited in my investigations of the subject encourages me to inform you of a result I have just drawn with respect to determination of the

boundaries within which the sum of a number of the first coefficients of a series

$$A_0+A_1x+A_2x^2+A_3x^3+\dots$$

or

$$\frac{B_1}{1^x}+\frac{B_2}{2^x}+\frac{B_3}{3^x}+\dots$$

remains the same provided all terms of the series are positive."

Then Chebyshev gives inequalities that are "most remarkable in their simplicity" [70, p. 183]. He obtained these and other inequalities for an integral of a non-negative function. He skilfully applied these inequalities to sums of the series mentioned above.

A letter Chebyshev sent to Kovalevskaya on October 8, 1886, is related to the publication in *Acta Mathematica.* Chebyshev wrote [RM 17]:

"I am very glad you found it possible to print my letter in your journal. Currently, I am engaged in some work for which the first of the formulas I gave you is very essential." And then: "At the yesterday's session of the Academy of Sciences, presentations were made concerning its three new members, Markov, Beilshtein, and Beketov*. I shall be looking forward to the holidays with the hope that you will do me the honour of visiting me and then we can talk about mathematics and mechanics."

The secretary of *Acta Mathematica* earlier wrote to Chebyshev that it was desirable to change the title of his paper (the initial title is unknown). Chebyshev wrote to Kovalevskaya in response [RM 22]:

"October 14, 1886

"Dear Sofya Vasilievna. At the request of Mr. Eneström, I changed in the proofs the title of the paper compiled from my letter to you for the following one: "Sur les sommes composée des coéfficients des séries à termes positifs". If you find this title better defines the nature of the problem, whose solution, in fact, was implied in the consideration of the integral

* A. A. Markov was a mathematician, F. F. Beilshtein and N. N. Beketov were chemists.

$$\int_0^\infty e^{-tz}F(z)\,dz$$

"and is of special interest, let it be. However, if you prefer to attach another title, I give you my consent in advance and think it unnecessary for you to send me the proofs again with the new title. There are no other changes or corrections I can send, save for the few misprints which I indicated in the proofs. Once again I express my tender thanks for your translation..."

Two years later Chebyshev made up his mind to publish another paper in the journal of Mittag-Leffler, and he wrote to Kovalevskaya on October 20, 1888 [RM 23]:

"The flattering attention you paid to my first work on the boundary values of integrals has given me hope that you will assist me in the publication in French of a second paper on the same subject, and continuing on from the first paper. I am enclosing a translation made by a young mathematician [J. Lyon, Darboux's student] who received his higher mathematical education in Paris. He has made an excellent translation. Of all the foreign journals that could publish this paper I prefer *Acta Mathematica*, and *not* just because it printed my first paper and my letter to you concerning the same subject. Please convey my deep respect to Professor Mittag-Leffler together with my anticipation of seeing my paper in his journal. There is no mathematical news here; I'm toiling over a memoir on simple hinged systems and hope to end it before long and present it to the Academy of Sciences."

Chebyshev's paper was published under the title "On the integral residues giving approximate values of integrals" in the 12th volume of *Acta Mathematica* in 1889 [235].

Yet another paper by Chebyshev "On two theorems related to probabilities" [236]—was published in Lyon's translation in 1890-1891 in the 14th volume of the journal. Apparently, Kovalevskaya looked through it as an editor of the journal. But the last of Chebyshev's papers was published without Kovalevskaya. It was titled "The approximate presentation of the square root of a variable by means of

simple fractions" [237] and was published in 1894 (the year Chebyshev died) in a German translation by O. Backlund.

Chebyshev's last letter to Kovalevskaya was sent in October 1889, soon after Kovalevskaya had been refused a position in Russia [RM 19]:

> "Dear Sofya Vasilievna,
>
> "Nobody doubts that you are devoted to your homeland with all your heart and that you would gladly move from a Swedish university to a Russian one. There can be no doubt in this; one can only doubt whether you would agree to exchange your university chair in Sweden for the position of a teacher at the Higher Courses for Women here. I believe that such a change would be a great sacrifice for you, and a sacrifice to the detriment of the development of mathematics. Because the current statutes in our educational institutions for men do not admit women to any chair in any case, the only thing left for us is to be proud and rejoice that our woman compatriot occupies a chair so successfully in a foreign university, where the national feelings are far from being favourable for her. I have heard that an answer was sent to the letter of Mr. Kosich, who raised the question of offering you a position in Russia in exchange for the one you have in Stockholm. I had the chance to read his letter, and I admit I was very surprised how little your relative is familiar with what is universally known about your learned career."

Kovalevskaya had little communication with other Russian mathematicians of the older generation. E. F. Litvinova claims that all of them, except Chebyshev, remained cool to Kovalevskaya, considering her a "Westerner", and an advocate of Western trends in mathematics [91]. But Kovalevskaya was not a "Westerner" in the sense that she preferred all Western things to all that was Russian. She was familiar with the ideas of Weierstrass, but that was natural. Chebyshev was superior to other mathematicians and he accepted Kovalevskaya's arguments with him reasonably.

Kovalevskaya became acquainted with the young Russian mathematicians D. F. Selivanov and A. V. Vasiliev only when they studied abroad.

Dmitry F. Selivanov travelled abroad in 1882-1884 and visited the universities of Zürich, Heidelberg, and Berlin. In 1885 and 1886 Selivanov wrote to Kovalevskaya from Russia. He became the Professor of mathematics at Petersburg University and the Higher Courses for Women.

Selivanov gave his lectures in a strict and methodical manner, began and ended his lectures punctually and was dressed neatly. His lectures in algebra were lithographed.

Selivanov wrote and defended his magisterial dissertation in 1885 under the title "The Theory of Algebraic Solution of Equations" [238]. In the introduction he wrote:

> "We have advanced here the considerations drawn from the lectures given by Mr. Kronecker and included his 'Arithmetischen Theorie der algebraischen Grössen'." [239], [240].

In 1889 he wrote "About the Equations of the Fifth Degree with Integer Coefficients" [241] for his doctorate in mathematics.

There are eleven letters from Selivanov to Kovalevskaya; they are written in a legible and elegant script [RM 2-9, 13-15]. In his first letter dated July 17, 1882, he recounted that he had attended lectures given by Rudio in Zürich. Rudio had well presented elliptic functions according to Weierstrass's lectures. He wrote that the last part of "Mathematische Annalen" contains a paper by Lindemann on the transcendence of π.

A letter of April 17, 1883, shows that Selivanov was in Heidelberg and was poring over Kronecker's course which he had attended in Berlin. He wrote [RM 2]:

> "Kronecker's lectures on the theory of algebraic equations have given me a great deal of work. There's much I haven't understood properly so far. I've sent the ministry my paper 'On Abelian equations of the 3rd degree, after Kronecker's lectures' as a supplement to my report. Now I'm going to write "On Abelian equations in general". This is very difficult... My conversations with Kronecker in Berlin should help me to end this work."

Selivanov writes of his friendship with Carl Runge; they attended Kronecker's and Weierstrass's lectures together and together thought over some problems. Thus, they de-

veloped a technique to find out whether a polynomial with integer coefficients could be expanded into multipliers with integer coefficients. They were going to publish their paper in the Crelle's journal. Selivanov also wrote about the lectures Weierstrass had given:

> "This semester he gave his lectures excellently, so clearly and easy to follow". He met Fuchs, whom he described as "a very nice gentleman". [RM 3]

Selivanov wrote his next letter on April 23 in response to a letter from Kovalevskaya: "I was very happy yesterday to receive your letter." Sofya Kovalevskaya had asked him to send her "Schwarz's formulas", handwritten notes on Weierstrass's lectures on the theory of Abelian functions and those of Kronecker's "On Abelian equations of the 3rd degree". Selivanov promised to do so. In connection with the memoir by Poincaré that Kovalevskaya had recommended, Selivanov wrote that he hadn't seen it and didn't know when he would be able to read it because he was so busy with his work. The following day he was going to leave Heidelberg, around which "there are vistas of the plain where the river Neckar meanders like a snake" [RM 4].

On June 7, 1883, Selivanov wrote from Berlin about his efforts to get copies of the lectures Kovalevskaya had requested, but he hadn't found a good copyist. Selivanov became close friends with Minkowski, whom he was liking more and more. And again Selivanov wrote concerning his joint work with Runge [RM 5]:

> "We want to present something perfectly exquisite, like a painter's picture. We have reached the point when we can prove in 20 minutes that the function
> $$x^5 - 10x^4 - 32x^3 + 7x^2 - 500x - 120$$

cannot be expanded."

Then Selivanov wrote that Runge was able to show that the formulas for cos *amnu* can be deduced from the addition theorem for cos *amu*. This drew Kronecker's attention to the fact that in the formula

$$z = \sqrt{k} \sin am\,(u+v)$$
$$= \frac{x\sqrt{1-ay^2+y^4}+y\sqrt{1-ax^2+x^4}}{1-x^2y^2},$$

where $x=\sqrt{k}\sin amu$, $y=\sqrt{k}\sin amv$ and $\alpha=k+1/k$, the numerator and denominator contain common multipliers that cannot be eliminated: "... they are, as it were, ideal multipliers. Kronecker is reporting this today in the Academy of Sciences." [RM 6]

In his letter of August 23, 1883, from the resort Norseebad-Juist, Selivanov wrote that he is resting. He looked forward for news from Kovalevskaya about the VIIth Congress of Russian Natural Scientists and Physicians, which was to be held in Odessa from September 11 to 21, 1883, and where Kovalevskaya was to present a paper. Selivanov asked about the Russian mathematicians Sleshinsky and Preobrazhensky, and about Vyshinskaya (it appears that this was Mariya I. Vyshinskaya, a student at the Bestuzhev Higher Courses for Woman, who participated in the student revolutionary movement and who later became a teacher in Kostroma and Tver).

On September 13, 1883, Selivanov wrote from the Harz Mountains. He regretted that he had had to leave the island of Juist and swimming in the sea, because his doctor had sent him to the Harz.

The 10 pages of Selivanov's last letter in 1883, dated November 24, were sent from Berlin. By that time Kovalevskaya had been in Stockholm from November 16 and had just begun to become familiar with it. Selivanov starts his letter with

"Dear Sofya Vasilievna,
"Your letter made us all happy."

Apparently, Kovalevskaya had sent a letter to a group of young mathematicians, to whom she gave several lectures on Abelian functions. These were her first public lectures. Then Selivanov continued [RM 7]:

"Your dreams have come true, and your work will be full of joy and success. I quite understand your delight in passing your knowledge to others. It's a pity though that you will not present the theory of the transformation of θ functions to us."

Selivanov related his own affairs in detail. He was preparing to present his paper "On the solution of equations of the 3rd degree" at Kronecker's seminar and was follow-

ing the ideas of Abel in it. He then discussed his paper minutely over four and a half pages. Selivanov then conveyed some interesting bits of news. Together with Runge, Molk, and Hensel, he visited Kronecker's wife on Thursday. Evidently, making visits to scientists' wives was a custom of the time. Selivanov told Kronecker about Kovalevskaya and he was very glad. Weierstrass had caught a cold in his throat and wanted to go to the South. Runge, Hensel, Minkowski, and Kneser sent Kovalevskaya their "awful" regards. Recently Kneser presented a short report in the Mathematical Society on Poincaré's "Sur les fonctions de deux variables", of which Kovalevskaya had told them [RM 8].

In early 1884, on January 28, Selivanov sent her a nine page letter. It is written in German because he was writing with Runge. The letter was a belated reply to Kovalevskaya's letter. Selivanov wrote [RM 9]:

> "Your news was a great joy for us. You are going to give lectures in three days' time. I wish you great, really great success. Your consideration of the power series of many variables is very interesting. Runge wants to write to you about it..."

Then Selivanov wrote about his own work: he was studying a paper in the three-volume collection of Lagrange's work where the origin of the substitution theory could be seen. Selivanov was interested in the study of conditions for which two algebraic equations

$$P(x) = 0, \qquad Q(x) = 0$$

would have two or more common roots. He expounded his thoughts on the matter. Selivanov was to leave within six weeks for Petersburg, where he had to write a dissertation within a month. He would then set out for Penza to visit his grandparents. After a rest with them he was to return to Petersburg to publish his dissertation.

Kovalevskaya spent the summer holidays of 1885 in Russia. On July 26, Selivanov wrote to her from Penza. Answering her enquiry about the address of the inventor Golubitsky, Selivanov said [RM 13]:

"If you wish to see the Golubitskys, it is very easy. You can go by the Moscow-Kursk railroad to Ivanovo (it is not far), then take post horses and ride 21 versts to the city of Tarusa (a district centre in the Kaluga province), then it is 2 versts from there to the village of Pachevo, where the Golubitskys live. They will be very happy to see you if you go."

Pavel Mikhailovich Golubitsky was a talented inventor of telephone equipment [242]. After Sofya Kovalevskaya had just been established as a Professor at Stockholm University, Golubitsky sent her a bill for 4000 roubles. In a letter to Aleksandr Kovalevsky she called Golubitsky a "robber", but the bill was probably an old debt Vladimir Kovalevsky had incurred. Nikolai M. Maslov, who knew Golubitsky, had written in explanation that Golubitsky really needed money for his inventions.

Kovalevskaya's attitude to Golubitsky changed after she visited him in his village, possibly to pay the debt. Golubitsky told Kovalevskaya about Konstantin Eduardovich Tsiolkovsky, who lived in Kaluga and whose work was being neglected in official circles. Kovalevskaya told Golubitsky that she wanted to meet the "eccentric teacher" who thought of interplanetary travel. Because Tsiolkovsky was quite deaf and therefore shy he declined an invitation to travel to meet Kovalevskaya, but later he said that her interest in him was encouraging.

Kovalevskaya was forever interested in physics and engineering. She studied physics in Palibino when she was a child, and took physics lessons during her visits to Petersburg.

I have already mentioned that Kovalevskaya and Yuliya Lermontova had worked on the application of the "candle" invented by P. N. Yablochkov. There is a sheet of paper with Kovalevskaya's handwriting and a diagram of an electrostatic machine.*

The last two letters from Selivanov were written from Petersburg. On December 11, 1885, he wrote to say that he and Markov had visited Mittag-Leffler, who had just arrived in Petersburg and was staying in the Hôtel de

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* *AAN*, f. 603, op. 1, No. 51.

France. Selivanov wrote of his happiness: some three weeks earlier he had started giving lectures and it finally dawned on him "what a great delight it is". He was presenting the theory of how to solve equations algebraically and there were 18 people in his latest audience. He hoped to be in Stockholm in April, but on April 24, 1886, wrote that his plans had to be changed because he had to invigilate an examination in algebra on May 26. He wanted to go in the summer to the congress of mathematicians in Christiania, but he lacked the money [RM 15]:

> "I would willingly walk with you over Norway. I like walking awfully, especially in your company. There is one more pleasure I share with you: ice-skating. Runge has inculcated in me the love for skating."

Selivanov recounted his progress: he gave 24 lectures during the winter although the number of auditors decreased from 15 to five, but they were the most persistent, and he hoped they would attend his lectures next semester. Selivanov added that he had got his fee for the last semester. "Do you know how much? Only 3 roubles and 72 kopecks" [RM 15]. Apparently, it was not the fee that was essential then for Selivanov, but the chance to give lectures.

The other Russian mathematician Kovalevskaya met abroad was Aleksandr V. Vasiliev, the son of a well-known sinologist Vasily Pavlovich Vasiliev. Aleksandr Vasiliev attended Kronecker's lectures in 1884 and received a Russian doctorate in mathematics for his dissertation "The Theory of Separation of the Roots of Joint Equations". The dissertation was, according to Vasiliev himself, a commentary on a memoir by Kronecker [243-245]. He became a Professor at Kazan University, and founded the physico-mathematical section of the Kazan Society of Natural Scientists becoming its first chairman. The section later became the Kazan Physico-mathematical Society in 1890. In 1907 he moved to Petersburg and chaired the Mathematical Society there.

There is only one letter from Vasiliev to Kovalevskaya in the archives. It is undated and was possibly written in 1885 in response to the letter of Kovalevskaya in connection with the coming 70th anniversary of Weierstrass. Vasiliev recounted the news of life in Kazan, writing that

their Mathematical Society had been reinforced by Professor V. V. Preobrazhensky, and that V. P. Maksimovich was soon going to defend his dissertation on differential equations. Preobrazhensky and Maksimovich were talented mathematicians native to Kazan and they played a great role in the activities of Kazan University.

Maksimovich also met Kovalevskaya abroad, in Paris. Later he sent her his paper "On the philosophy of imaginary values" and asked for Kovalevskaya's opinion [RM 23].

Many Russian mathematicians were interested in Kovalevskaya's scientific progress and many people, both familiar and unfamiliar, wrote to her. Some mathematicians addressed her as one of the editors of *Acta Mathematica*.

A. P. Starkov, a mathematician from Odessa, wrote Kovalevskaya in a letter dated March 22, 1884, of his mathematical work and sent her a paper for *Acta Mathematica*. Kovalevskaya declined the paper and explained why very diplomatically. In response to Kovalevskaya's question about "blood-red dawns", Starkov cited F. N. Shvetsov, who observed the phenomenon on November 5, 1883. The bright colour of the dawns was supposed to be due to the ash that had been spread into the atmosphere following the eruption of volcano Krakatoa. According to Kovalevskaya, the polar explorer Nordenskjöld was very interested in the phenomenon. Starkov promised to ask the "newly fledged" Doctors Kononovich (an astronomer) and Klossovsky (a meteorologist) for "the most learned data" on the subject [RM 10].

Spiritism was the rage in the 1880's in many circles in Europe. Many enlightened people were aghast by it. Anna Jaclard published a story in the journal *Severny vestnik* (1886) called "Memoirs of a Spiritist", where she revealed the charlatanism of spiritists [246].

During this period Sofya Kovalevskaya received a letter from one Semen Tsvet, who spoke out against spiritism from the viewpoint of elementary mathematics, one may say. The point was that some people had made use of the abstract mathematical notion of the fourth dimension to substantiate their ideas of the "other world" (mathematics can consider however large even infinite number of "dimensions"). Tsvet writes:

"7 Geneva
4 Oct. 86

"I allow myself to hope that you, Sofya Vasilievna, will not consider obtrusive my friendly letter to you as an old and good acquaintance. Although, as I learn with a heart-felt gladness from newspaper news, you continue to progress in your favourite science, I, an old mathematician, value you not only as a figure in mathematics, but for the whole of your exquisitely fine being. I am glad when I have a mathematical pretext to exchange a few words with you."

Having concluded that it was inappropriate to make judgements about the properties of space on the basis of algebraic formulas, he asked Kovalevskaya, "What is your opinion on these subtleties, oh, wise nymph Egeria*?" At the end of the letter he asked her to write to him of her health, life, and plans for the future, and then told her about himself [RM 16]:

"I've settled with my poor children in Geneva. If you leave your cold Varangian North to warm yourself in the South, don't forget to come and visit me. Cordially and devotedly Yours, S. Tsvet.

"Has Karbasnikov brought you my book "Progress and Poverty?"

Semen Tsvet's son was the Russian scientist Mikhail Tsvet [247]. He dedicated one of his papers to his father with the inscription, "To a thinker and activist". In 1861 Semen Tsvet set out as the scientific secretary for a round-the-world voyage, but he was put ashore in London for "free-thinking" and protesting against the corporal punishment of the sailors.

Many persons were interested in Kovalevskaya's mathematical works, but since they were published in foreign journals, non-professionals could not get them.

* Egeria was the wife of King Numa Pompilius (in other words, she was an adviser).

Germany

The famous Helmholtz and Kirchhoff were among those whose lectures Kovalevskaya attended in Heidelberg.

Hermann Ludwig Ferdinand Helmholtz was the son of a teacher in Potsdam. On graduating from the Higher School of Military Physicians, he worked there for a time as a military doctor. In 1847 he presented a paper "Uber die Erhaltung der Kraft" in the Berlin Physical Society (founded in 1845) [248]. He was the first to give a mathematical treatment of the energy conservation law and pointed out its universality.

Helmholtz is the author of fundamental works of theoretical physics, hydrodynamics, and physiology [250]. In 1858 he founded the theory of eddy motion, and in 1868 the theory of discontinuous motion of liquid [251]. By the time Kovalevskaya attended his Heidelberg lectures Helmholtz had already published books on physiological optics and the physiological theory of music. Later, in 1885, Kovalevskaya attended a reception given by King Oscar II and told him of Helmholtz's theory of overtones "which seemed to be of interest to him" [SK 40].

In 1871 Helmholtz became a Professor at Berlin University and Kovalevskaya visited him. Possibly, it was at Helmholtz's that she met Werner Siemens, who was a friend of Helmholtz. Kovalevskaya came to know Siemens very well. Believing Siemens to be a genius of an inventor, she described him in one of her letters to Mittag-Leffler (in connection with the play "The Struggle for Happiness"). She told him in another letter that Helmholtz's daughter had married to Siemens's son.

Gustav Robert Kirchhoff was also famous [252]. He gave lectures first in Heidelberg (from 1854), then in Berlin (from 1875). He hurt his leg in 1863, and later his eyesight became poor. He stopped his experiments and began to pay more attention to his theoretical investigations.

Kirchhoff's name is embedded in the theory of electricity (Kirchhoff's current and voltage laws). He also studied mechanics [253], developing the theory of deformation and the theory of liquid flow. Kirchhoff introduced the concept of a blackbody and formulated the basic law of thermal radiation. In 1859 Kirchhoff and Bunsen began studying

spectral analysis; the two of them discovered cesium and rubidium. When she was preparing for her lectures, Kovalevskaya used books by Kirchhoff, which she borrowed from the fine library of Mittag-Leffler.

Leo Koenigsberger was one professor who recognized the young Sonya Kovalevskaya to be a gifted and diligent student. Weierstrass asked Koenigsberger for a reference about Kovalevskaya. Leo Koenigsberger changed his place of work many times: he was a Professor at Greifswald University in 1864-1869, at the Higher Technical School in Dresden from 1875, and at Vienna University from 1877. He was a Professor at Heidelberg University for two periods: 1869-1875 and 1884-1913, and Kovalevskaya attended his lectures in Heidelberg.

Koenigsberger published works on the theory of functions, and on differential equations and mechanics. In 1874 his two-volume course of the theory of elliptic functions was published, followed by books on differential equations and the principles of mechanics.

During her years of study with Weierstrass, Kovalevskaya had no social life, and we do not know whether she even communicated with other mathematicians except H. A. Schwarz, whom she met in Zürich. Later, in 1885-1892, Schwarz worked in Göttingen, and then in Berlin.

When Kovalevskaya took up mathematics again, she met Schwarz in Berlin on many occasions. Whenever he talked with other mathematicians, Schwarz always referred to Kovalevskaya with enthusiasm.

The name of Herman Amandus Schwarz is known in the theory of analytic functions: Schwarz's symbol, Schwarz-reflection principle, Schwarz-Christoffel transformations which conformally map the interior of a given polygon onto the portion of the complex plain above the real axis, etc.

Schwarz was especially interested in the theory of minimal surfaces [254]; indeed the whole first volume of the two-volume collection of his works deals with this topic. In particular, he studied the minimal surfaces bounded by three-dimensional quadrilaterals (the four edges of a tetrahedron) or by polyhedra. He would experiment with soap films (Plateou surfaces) stretched over space contours. There are many fine drawings of minimal surfaces in his book.

There is a long letter from Schwarz to Kovalevskaya in the Archive of G. Mittag-Leffler, in which he considered an auxiliary differential equation that had appeared in his study of minimal surfaces.

Schwarz was engaged in the following problem: find which of the surfaces bounded by a given curve has the least area.

Here is the beginning of Schwarz's letter:

"Göttingen
December 25, 1884

"Dear Madam,

"Since the time I had the happiness to pass a few hours with you, many months have gone... How happy you must be that..., owing to your scientific achievements, you have won a position that is envied by many men.

"Our teacher has written telling me that you will be staying in Berlin till mid-January. I would very much like to meet you again and I hope I'll be able to see you when I come to Berlin for two or three days at the beginning of next year. One topic I need to talk to you about is a scientific problem whose difficulties I cannot cope with, but which you should be able to overcome. It is special partial differential equations that I encountered while studying the second variation of the area of a portion of a minimal surface..." [FM]

Schwarz had found it necessary to integrate the equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + p(x, y)\, u = 0,$$

where $p(x, y)$ is a given positive analytic function of x, y.

Of the German mathematical community in Berlin, Kovalevskaya was closest to Leopold Kronecker. According to Constance Reid, he was a small man, no more than five feet tall. He had successfully managed his agricultural business, and having provided for his family he withdrew at the age of 30 to devote the rest of his life to his beloved mathematics [255, p. 39]. He was elected a Member of the Berlin Academy of Sciences (1861) and gave lectures in Berlin University on algebra and the number theory. It was only when he was 60 years old that he acquired the official position of Professor, when Kummer retired in 1883.

A young French mathematician Jules Molk wrote to Kovalevskaya in 1882:

"Oh, Madam, what an excellent science mathematics is! And Mr. Kronecker is correct when he says that it exalts man over himself!" [85].

Sofya Kovalevskaya and her three-year-old daughter stayed with the hospitable Kroneckers, Leopold and his wife Fanny, over the Christmas of 1881. At the beginning of 1882, Kovalevskaya sent Fanny Kronecker a present, a collection of photographed Riviera sights, and Fanny thanked her in a letter [85]. Kovalevskaya sent a very interesting gift to Kronecker himself, a portrait of Galois that had been found by Joseph Perott.

Kronecker conveyed his gratitude in a letter of July 17, 1883:

"Dear Madam,

"I am touched and happy that my dream has come true through your mediation. As you can easily suppose, it was extremely interesting for me to have an idea of the appearance of the man to whose almost miraculous algebraic intuition we are obliged for the fundamental concept that I have called the Galois principle. Ever since you sent me your present I have been intending to visit you personally to express my gratitude but I am always distracted by urgent business, I did not want to wait any more and decided to thank you first in writing. One of these days I shall be able to visit you myself

Sincerely respectful and
devotedly yours, Kronecker.

"P.S. I am enclosing some of my new works." [FM 28]

The second letter from Kronecker (of January 5, 1884) was preceded by a letter from Fanny Kronecker, in which she recalls Sofya and her child, "who was, as it were, an incarnation of the sacred legend" [85, p. 13].

Kronecker joined his wife in her regards and New Year wishes and reminded Kovalevskaya of her promise to send her works to Kronecker's journal. He adds:

"Our friend Mittag-Leffler cannot be opposed to this, because our two journals have agreed to a loyal exchange of articles; I have long since begun a paper on Abelian equations for *Acta* and another one about the real roots of algebraic equations. It is only an eye ailment that prevents me from going on with them. I want with all my heart to finish the latter work, in which I intend to present my views with respect to modern functions theory. I believe it is groundless as I told you, dear Madam, during the summer. My current lectures concern definite integrals and have fortified me in this. Since the point is only the victory of truth, I think our friend Mittag-Leffler will gladly accept my considerations instead of Cantor's blind assertions to the opposite..." [78, p. 124]

Kronecker's papers did not appear in *Acta Mathematica*; most probably he did not even send them knowing that Mittag-Leffler had sided with Cantor over the set theory and had opened his journal wide for Cantor.

Kovalevskaya and Mittag-Leffler discussed some mathematical problems and published works in their correspondence. They were elated with many of them. Naturally, the two friends, who were firmly bonded by their common work, also described other people and their actions, both positive and negative. Often they discussed the vanity that is inherent in many scientists. More than once Kovalevskaya recounted the "bump of vanity" that many of the people they knew well had, and quite often they meant Leopold Kronecker, who seems to have exceeded all his contemporaries in this regard.

The excellent relations that once existed between Kronecker and Mittag-Leffler were broken. It began with the Oscar II prize. Hermite, Mittag-Leffler, and Weierstrass but not Kronecker, were appointed the judges of the competition. He felt offended and Mittag-Leffler had to explain that Weierstrass had been preferred because of his greater age. Besides, Kroneker was discontended with the last of the four problems, the one proposed by Weierstrass, who had in mind an analysis of automorphic functions. Kronecker had thought it was an algebraic problem, while Weierstrass contended that so far as Kronecker was concerned

"his claim to be the only expert and the key figure in algebra has become a well-advanced disease." [125, p. 267]

Kronecker showed his offence in a quite unexpected way. He had promised to recommend Signe, Mittag-Leffler's wife, to a famous German doctor called Hegar. However, he then made up his mind to refuse to do so and in the Summer of 1885 he wrote telling Mittag-Leffler of his decision. Mittag-Leffler wrote to Kovalevskaya that Kronecker

"began that he could not do him the friendly favour I had asked him, and he told me to get the information I wanted from Hegar myself ..." [ML 41]

Mittag-Leffler widely cited Kronecker's letter:

"... accordingly, I was disillusioned because you asked my advice on many points... but concealed from me at least one matter about which you would have informed me, at least to some degree in the same way you communited it to both Weierstrass and Hermite, if, one, your attitude to me had really been as good as it seemed from your letters and personal remarks, or if, two, you had taken into account the position that I have long held in the mathematical community, or if, three, and this is most essential, the decision had been taken by competent persons. There is also no doubt that there is no mathematician today who is in any degree as competent as I to put forward or adjudicate on an algebraic problem, bearing in mind that my whole life's work has been on this topic... You will see the harm you have done to the cause by not telling me. The fact that you have hurt me personally I can stand. The "advertisement" which is now scaring mathematicians away, will not last long. I have always been disgusted by it, and besides I believe it is better for me to spend my time on mathematical studies rather than on advertising my mathematics... I am sending you some of my latest finished work, other works' will follow soon." [ML 41]

Then Kronecker threatened to complain to King Oscar II:

"As to the prize competition, I shall address to your king directly. If it is true that you wrote to me earlier

on his behalf, he will find it quite natural if I address him myself. I'll proceed from my more significant competence in algebraic studies, which I have demonstrated in a number of my works and especially in my anniversary collection. The fact that a commission, neither of whose members is familiar with this fundamental work is putting forward an algebraic problem and shall later adjudicate the work done for it is an unprecedented anomaly. Your king will know from me what you have concealed from him, that 25 years ago, when I entered the Academy, I proved that what is explicitly the starting point for problem No. 4 is impossible. Moreover, your King should be aware of the true situation in mathematics, so that his good will can really bring about something worthwhile." [ML 41]

Kovalevskaya wrote to Mittag-Leffler that when she read Kronecker's letter she couldn't help laughing.

"I cannot imagine anything so comical as this letter from Kronecker. Starting from his refusal to give a letter of recommendation to a doctor... for poor Signe, for whom Mrs. Kronecker has so much sympathy, and ending up with a threat to complain to our poor King and present him with the true situation in mathematics is so outstanding that the letter is a masterpiece."*

Before long, Kronecker realized that he had gone too far and took to writing overtures to Mittag-Leffler.

Kovalevskaya just laughed at Kronecker's vagaries; Mittag-Leffler was cross with him and told Kovalevskaya that despite his great respect to Kronecker as one of his teachers, he felt that he had to struggle against Kronecker's deleterious influence on mathematics.

The effect of Kronecker's attacks on Weierstrass and Cantor was more tragic. Kronecker began to harangue the basic concepts of modern mathematics, especially in the theory of real numbers. Real numbers had been investigated by Bolzano, Weierstrass, and later Cantor. So Kronecker rose up against their work, claiming that mathematics should be based solely on the integer. He promised to "arith-

* *AAN*, f. 603, op. 1, No. 17, pis'mo (letter) XIII.

metize" mathematics (either himself or with the aid of his disciples) and eliminate from it all 'non-constructive' concepts [255a, p. 26]. Kronecker's ideas were not recognized. However, he not only criticized the work, he also made personal attacks on the mathematicians whose ideas he did not approve. Weierstrass took his attacks very badly and even wanted to retire and leave Berlin. As to Cantor, Constance Reid says that

> "The high-strung, sensitive Cantor, as a result of Kronecker's attacks on the theory of sets, had broken down completely and had had to seek asylum in a mental institution." [255a, pp. 26-27]

But Weierstrass did not leave Berlin. He wrote to Kovalevskaya about Kronecker:

> "I deeply regret that such a gifted person, possessing such indisputable scientific merits, can also be so envious and petty in his vanity." [125 p. 267]

Georg Cantor's letters to Mittag-Leffler are full of complaint about Kronecker [256].

Cantor's letters to Kovalevskaya have already been mentioned in connection with Weierstrass's anniversary. I want to cite here a remarkable letter of Cantor to Kovalevskaya that is dated December 7, 1884. Having told her that he had sent his *Principles* (unpublished) to Mittag-Leffler for *Acta Mathematica*, Cantor wrote about their contents:

> "The first sections deal only with the types of simple ordered sets, but there are types of double, triple, n-tuple and even ω-tuple, etc. ordered sets that seem to be able to shed much light on both old and new problems of arithmetic and cosmology." [125, p. 123]

Cantor's idea of applying his theory to cosmology is intriguing but not clear. He developed a theory of the types of infinite sets, and wrote elatedly about its philosophical significance:

> "When though the *finite* types possess an inexpressible fascination for anyone who is capable of perceiving the eternal truths—and these are the origin of the number

theory—the infinite sets can satisfy this scientific indulgence to a greater degree." (*Ibid.*)

Gilbert said about Cantor's theory of sets:

> "It is, I think, the finest product of mathematical genius, and one of the supreme achievements of purely intellectual human activity." [255a, p. 176]

During her first year in Stockholm, Kovalevskaya had a lively correspondence with a young German mathematician called Carl Runge. The name of Runge is well-known to those interested in approximate computations (the Runge-Kutta method of integrating differential equations). At the beginning of his scientific career, he studied the theory of analytical functions, in particular functions with essentially singular points.

Kovalevskaya met Runge in 1883 in Berlin, when he was attending the lectures of Weierstrass and Kronecker. In the summer of 1883, Runge wrote an enthusiastic letter about Sofya Kovalevskaya from Berlin to his mother in his native city of Bremen:

> "On Saturday we had a very interesting party at her flat. The company consisted of Mrs. Kovalevskaya and four young mathematicians, and we talked as we usually do. She is about 30 years old, her face is delicate, thoughtful, a little sad, and quite charming when she smiles. It was strange for me to talk of mathematics with a lady and to be able to discourse with complete freedom. She knows the subject well. I knew this especially when she asked me about my work by the excellent questions she put. Before, I had imagined her to be sharp-nosed, old-looking, and with spectacles, but I was amazed to find that a scientific education can match such a perfect femininity." [133, p. 43]

According to Iris Runge, her father was then a gay, handsome young man who liked ice-skating. Margarita von Bülow once asked him if she could portray him in a story to be called "Adonis on Skates", but she wanted to make him a bad character, which was not true to life.

Sofya Kovalevskaya held seminars with a group of young Berlin mathematicians, and both Runge and Selivanov were among her auditors.

The first of the available letters, of January 28, 1884, is in two parts: (1) a portion written by Selivanov in German jointly with Runge and (2) a letter from Runge himself.

Runge considered a number of mathematical problems. His part began:

"Dear Frau,

"I was going to write to you for a long time. I often recall our meetings in the summer and think it would be better if you could be here so that we could tell each other interesting pieces of news. I'm no good at writing. I read with interest your letter to Selivanov of December 12. Both examples are very fine. I've never thought of that before." [R 1]

Runge did not agree with one point in Kovalevskaya's argument, one concerning a common domain of convergence of two series:

$$\frac{1}{1-xy} = 1 + xy + x^2y^2 + \dots,$$

$$\frac{1}{2-(x-2)} + \frac{1}{2-(y-2)}$$
$$= 1 + \frac{x-2}{2} + \frac{y-2}{2} + \frac{(x-2)^2}{4} + \frac{(y-2)^2}{4} + \dots$$

He put his thoughts into examples and then asked Kovalevskaya whether she was familiar with a proof, probably due to Cauchy, for the existence of solution for a differential equation. Given an equation

$$dy/dx = R(x, y),$$

it is necessary to find its solution when $y = y_0$ at $x = x_0$.

Runge divided the interval (x_0, x) into subintervals by the points $x_0, x_1, \dots, x_n, \dots$ and wrote the equations

$$y_1 - y_0 = R(x_0, y_0)(x_1 - x_0),$$
$$y_2 - y_1 = R(x_1, y_1)(x_2 - x_1),$$
$$\dots\dots\dots\dots\dots\dots$$
$$y_n - y_{n-1} = R(x_{n-1}, y_{n-1})(x_n - x_{n-1}).$$

He held that it was possible to prove that $\lim\limits_{n\to\infty} y_n$ exists for small enough $x_n - x_{n-1}$ and presented a function of x, y that complied with the differential equation with $R(x, y)$ continuous in the neighbourhood of

(x_0, y_0) and possessing first derivatives. The proof was also applicable to non-analytic functions $R(x, y)$ and to more general cases [R 1]. Runge added that he thought it was feasible to calculate the integral.

Runge then had little experience in teaching, and he wrote

> "I'm not very happy with my lectures. I clearly feel that my presentation is not perfect, and sometimes I'm sorry for the auditors. I'd also like to have closer contact with my audience, because often I do not know whether my students manage to follow me, where my presentation should be more detailed and where I can make it shorter. But in general I'm very enthusiastic about giving lectures. There're many things that have become clear to me for the first time." [R 1]

Runge said he was glad Kovalevskaya had promised to visit Berlin during the Easter holidays and asked her to send what was written in Swedish newspapers about her. "We shall understand Swedish," he said and added, "We shan't show it to Kronecker." [R 2]

The second letter from Runge, dated February 11, 1884, included a detailed proof of the existence theorem, using the same method as in the first letter, for a set of equations

$$\frac{dx_\nu}{dt} = R_\nu(x_1, x_2, \ldots, x_\mu), \qquad \nu = 1, 2, \ldots, \mu.$$

Kovalevskaya wrote to Runge on February 18 and posed him several questions. In a letter of March 10, Runge wrote of the existence theorem for a set of equations where R_ν $(x_1, x_2, \ldots, x_\mu)$ is an analytic function of complex variables. He ended the letter by saying that he had to answer more questions and asked in turn, "Can you tell me about the theorem of the true (wahre) radius of convergence for a differential equation?" He is also interested in Kovalevskaya's studies of potential functions [R 3]. He probably meant [9].

On April 2, 1884, Runge answered some of Kovalevskaya's mathematical questions and converged his regrets:

> "What a complicated thing our correspondence is! What a grievous substitute for a heart-to-heart talk!"

He related some of the news from Berlin: in the summer he was to lecture on analytic mechanics four times a week and give a special course (privatissimum) on the algebraic solution of equations once a week; Koenigsberger had been invited to Heidelberg and would leave Vienna soon; Fuchs would sooner or later return, depending on Koenigsberger's arrival. Runge wanted to go to Stockholm, but he couldn't at the moment. At the end, Runge exclaims:

> "If only I could read what you've so thoroughly crossed out in Your letter! I see a question mark and it pains me that I'm unable to answer." [R 4]

Kovalevskaya was in Berlin during the Summer of 1884. In a message he wrote on July 8, 1884 which he attached to a manuscript of his paper, Runge said that it was Kovalevskaya's friendly remarks that encouraged him to write down the results of his research on single-valued functions. He was sending them for her to judge and asked her to cross out anything that isn't clear. He wrote the message because he didn't think he would find Kovalevskaya at home. He counted on meeting her the next day.

A message of June 14 accompanies another of Runge's papers. He shared his considerations concerning something in his preceding studies and ended with

> "Tomorrow I'm going to do nothing but swim, row, and play tennis. And this is what a privat-docent of mathematics desires." [R 6]

A week later, Runge wrote that he was pleased with the proof Kovalevskaya's student had presented for a function for which the addition theorem is valid, but that he could give a simpler proof that in a finite domain such a function can have only algebraic features. Runge expounded his proof, but he thought he did not do it too well; however, Kovalevskaya, with her quick mind, would understand. At the end of the letter, he returned to an inequality of hers, against which he had argued in one of the preceding letters. He said he could still not agree with it even

> "if you commit me to the tortures of Spanish inquisition, although I have no desire for that." [R 7]

Sofya Kovalevskaya was still in Berlin on July 5, and Runge wrote:

> "Dear Madam, I just could not wait till Tuesday to tell you of what I seem to have found. Because you know me for the vanity that is clearly drawn on my palm, I cannot change your opinion about me simply by boasting a bit less."

The problem was to find an entire transcendent function of $u_1, u_2, \ldots, u_\rho$ with a given zero domain, i.e. a domain consisting of all the points where the power series of $u_1, u_2, \ldots, u_\rho$ reduces to zero.

Runge considered the problem in the minutest detail and remarked that he has never written such a long letter; he said that he should have given some of the reasoning more clearly and accurately so that there would be no catch; he would be glad if Kovalevskaya could talk the problem over with Weierstrass.

In the Autumn of 1884, Runge went on holiday to Suffolk, England, to the native town of his mother. He wrote on August 21 that he had got the Swedish *Illustrated Newspaper* (Illustrerad tidning) and that it exceeded its German counterpart both in the quality of Kovalevskaya's picture and the exactness of her biography. He wrote that his scientific efforts were currently zero, his brain being incapable of work. However, he was not at all sorry and was pleased with his "vegetable life". The house he was staying at (Harland house) looked out on to the sea, and tents for the family had been pitched on the beach. He and his brothers played tennis and swam in the morning, then played tennis or went walking or bicycling after lunch. He only read a newspaper and George Eliot's "The Mill on the Floss", but the end of the letter was lost. In the last sentence, Runge wrote about Mittag-Leffler's theorem for the expansion of a single-valued function into series [R 8].

On September 3, 1884, Runge wrote from Eastbourne, en route to Stockholm, saying that he would leave London on September 5 on the steamer *Allegro* and would arrive in Stockholm on September 10. He wrote:

> "My stay in England was very healthy for me. Only the principle that you should leave the dinner-table before

you are completely full comforts me that the fun is over. Now serious life beckons in the shape of Abelian functions, essentially singular points, uniformly convergent power series, and the like. As to the Abelian functions, I hope to learn as much about them as possible in Stockholm. I have only recently made friends with the theta-series and I hope it will become a lasting frienship." [R 9]

Sofya Kovalevskaya was glad that Runge was soon to arrive and wrote to her friends Gösta and Signe Mittag-Leffler, who were holidaying in South Sweden. She hoped that they would like Runge for she thought him an interesting person and an enthusiast in mathematics. Runge did arrive and made a good impression on Mittag-Leffler.

On his arrival in Berlin, Runge described his trip in detail in a letter dated October 11, 1884. He had travelled in the same railway carriage with Julia Kjellberg, a good acquaintance of Kovalevskaya.

"In Malmö, as in Berlin, one can have great pleasure from a good meal and a fine town hall,"

in Copenhagen they went to see Christianberg castle; a strong fire had not long before burnt all the wooden parts. They went sightseeing in Lübeck, saw the town hall and other buildings, and arrived in Berlin in time to be fascinated by "Julius Caesar", a play performed by a cast from Meiningen. Before Fraülein Kjellberg left Berlin, they went for a time to the "aquarium" and talked about decadence, the fourth dimension and politics. Runge somehow still felt he was in Sweden, but he was soon immersed in his ideas. He wrote:

"It was very fine in Stockholm; it was something I have taken close to heart, and I hope it will have a good influence; and I met good and clever people. I have to thank you for all of that." [R 11] .

Runge worked a great deal in Berlin, and wrote about it to Kovalevskaya on October 25. He had edited a paper on the expansion of the roots of equations, then worked on a method to explain integer functions, simplified it greatly and introduced a needed categorization, and considered a very attractive example.

Runge recounted how he had met an English mathematician a couple of years before who was in despair about a place in Todhunter's book he could not grasp. Runge explained the point, advised him to study the excellent investigations of Weierstrass and attend Weierstrass's initial lectures on analytic functions. Runge explained how the elections to Reichstag were coming up, but that he was not going to go to the polls, because none of the three candidates suited him.

The next five letters (October 31, November 1, 5, 7, and 17) are related to corrections of Kovalevskaya's paper on the refraction of light in a crystalline medium which had been published in *Acta mathematica*. Runge had endeavoured to look it through and found many slips, inaccuracies, and even mistakes in the calculations, and he recommended Kovalevskaya verify the paper thoroughly. He said he could have done it himself, but was too busy preparing for his lectures.

In a message dated January 11, 1885, when Kovalevskaya stayed in Berlin, Runge wrote that the following day, Monday, he would visit Kovalevskaya at 10.45 to take her to his lecture. He hoped she had received her opera glasses that were left in his jacket.

There is an unfinished letter from Kovalevskaya to Runge and a fragment from Runge's letter in the Archive of Mittag-Leffler possessed by the Archive of the USSR Academy of Sciences. They may have only been rough copies. One is written in response to a letter from Runge dated February 11, 1884, where he considered the set of differential equations

$$\frac{dx_\nu}{dt} = R_\nu(x_1, x_2, \ldots, x_r) \quad (\nu = 1, 2, \ldots, r), \qquad (*)$$

Kovalevskaya wrote:

"Dear Mr. Runge,

"Thank you very much for your last letter. You've given me a proof of the existence of integrals for differential equations both when the R_λ are analytic functions and when they are any functions (but meeting certain conditions). The proof is very fine indeed. Yesterday I was able to give the proof to my auditors at a seminar, and they were very impressed as well.

"I'm very interested in the remark you made towards the end of your letter with regard to special solutions of differential equations, although I'm not quite sure whether I understood you correctly.

"Can you really show that there are cases when a non-analytic function can be a special solution of the analytic differential equation (*)? I can easily imagine that this may happen if the R_λ are functions with lacunary domains but can it occur for algebraic differential equations? Until now at least I was sure that the particular solutions of algebraic differential equations are nothing else but regular solutions of other differential equations of lower degree, and therefore cannot give anything substantially new. Thus we needn't rack on brains over this. Weierstrass also always proceeded from this, and it seems to result from the following considerations.

"Any algebraic set of differential equations can be replaced by another set that is linear in its derivatives, i.e., it can be replaced by a set of the following equations:

$$P_{1\lambda}\frac{dx_1}{dt}+P_{2\lambda}\frac{dx_2}{dt}+\ldots+P_{\nu\lambda}\frac{dx}{dt}\,P_\lambda,$$
$$\lambda=1,\ 2,\ \ldots,\ \nu,$$

"where $P_{k\lambda}$ and P_λ are entire functions of t. There may be two cases. The determinant

$$D=\begin{vmatrix} P_{11} & \cdots & P_{1\nu} \\ \cdot & \cdot\ \cdot\ \cdot & \cdot \\ P_{\nu 1} & \cdots & P_{\nu\nu} \end{vmatrix}$$

"may either be zero or not.

"In the first case we should distinguish whether or not every determinant D_μ that can be obtained from D by substituting $P_{1\mu}$ ..., $P_{\nu\mu}$ with the right-side parts of P_1, ..., P_ν is zero.

"If this is true, it can be shown directly that the set of differential equations in question is insufficient for a complete determination of x_1, x_2, ..., x_ν as functions of t. In the second case the equations $D_1 = D_2 = \ldots = D_\nu = 0$ give various relationships for x_1, $x_2, \ldots$, x_ν. Taking into account these equations, we can eliminate as many x_λ as possible; for the rest ones we can obtain a set

of differential equations of the same kind, for which, however, the determinant is not zero.

"Now if D is nonzero, then in order to get the particular solutions for which $D = 0$, it is necessary to take *any* $\nu - 1$ of the given equations and join them with the equation $D = 0$. All *regular* solutions must be sought for from this new set, and we attempt to find among them such that the *last*, νth, rejected equation of the original set is satisfied.

"This is the way to go on; however, this will never bring about a *non-analytic* solution of this set of equations. Now had I right to understand that point in your letter as an implication that you can really prove the existence of a non-analytic solution for an algebraic differential equation?

"Please, write and tell me what you know about this. If your investigation is not yet ripe enough, I'll naturally talk to no one about it." [R 22]

And here is the fragment of Runge's letter:

"The equation $x^2 = 2ay - a^2$ is a family of parabolas for all real a, and the straight lines $x = \pm y$ are the envelopes of the family. It is the solution of the differential equation

$$\frac{dy}{dx} = \frac{x}{a} = \frac{\omega}{y + \sqrt{y^2 - x^2}}$$

and $x \pm y = 0$ are its particular solutions. To stay within the same terms as in my last letters, I took

$$x = t, \ \frac{dx}{dt} = 1, \ \frac{dy}{dt} = \frac{x}{y + \sqrt{y^2 - x^2}}.$$

The function $x/(y + \sqrt{y_2 - x^2})$ has a value in each point $x_0 = \pm y_0$ (except the point $x = 0$, $y = 0$); however, it cannot be expanded into series in powers of $x - x_0$, $y - y_0$ in the vicinity of this point. These points define the boundary of domain τ, for which $x/(y + \sqrt{y^2 - x^2})$ is regular.

"But there is no lacunary domian as this cannot occur for algebraic functions.

"Now let us take one of the two particular solutions, for instance $x = t$, $y = t$; naturally, it is determined

for every t, but these points (x, y) are on the boundary of domain τ. The question of whether analytic differential equations can have non-analytic solutions, however, here ..." [R 23].

This is where the fragment ends. This correspondence outlines what Kovalevskaya's mathematical interests were and her good relations with the mathematicians educated in Berlin. It is clear that she quickly grasped Runge's method to prove the existence theorem for equations and sets of equations that are general, and immediately began to apply it in her teaching. What Runge said about particular solutions was very provocative for Kovalevskaya and made her bring out her own ideas in the same vein; unfortunately, only a fragment of what she wrote is available.

Gustav Hansemann, a physicist without an official position, was also a friend of Kovalevskaya. He was a well-to-do person, the son of Prussian politician David Hansemann, who was a leader of the business community in the Rhineland. In 1848 D. Hansemann became the minister of finance in the Prussian bourgeous liberal government and he headed the government for a period.

Gustav Hansemann was not a mathematician, but he maintained close relations with German mathematicians, was in the know and wrote about them to Kovalevskaya. He published three papers (with Kirchhoff, who was probably his teacher) on the electric and thermal properties of metals. By the time Kovalevskaya met Hansemann, he was already quite old.

Hansemann was famous for his generosity, and entertained the "youth", i.e. people younger than himself such as Kovalevskaya. There was a special skating-rink near his house, and Hansemann taught Sofya how to skate there. He often invited her to the theatre, though sometimes she declined when, as she said, her "pedant grandfather" took over from her "gipsy grandmother" and she had to pore over her mathematics. Once, when Hansemann invited Weierstrass as well to the theatre the latter said that he had had no time to check his computations for his lecture on the following day. Kovalevskaya quickly made the needed transformations, and the three of them went off to the theatre.

In the Archive of Mittag-Leffler, there are five letters from Hansemann to Kovalevskaya dated 1885 and one dated 1886. Kovalevskaya's letters to Hansemann were published by M. Bunsen [184].

One of the first letters of Kovalevskaya to Hansemann is a short message of July 11, 1884, which she sent when she was staying in Berlin on holiday. She wrote [184]:

"Dear Sir,

"One really must have great will-power to refuse your kind invitation, but I absolutely must work over what I want to present to Weierstrass on Sunday. Anyway, I am very grateful to you and comfort myself with the hope that we shall have time during the winter to see "The Poor Student.

With cordial regards,
Yours, Sofya Kovalevskaya".

Sofya was in Berlin during the winter holidays of 1884-1885. When she returned to Stockholm, she wrote to Hansemann on January 25, 1885. He received the letter on the 30 January, after he had begun to worry about her trip. He replied on February 2:

"You want me not to forget my friend, Sonya Kovalevskaya. Oh, my dear Frau Sonya, every day I want to talk to you, I really miss our conversations, to which I grew accustomed for the time of your stay here. You should know how much I need a friend whom I could trust completely and with whom I could talk about anything that comes to the mind, and you have become this nice, good and clever friend to me; and I hope you will remain as such." [85, p. 27]

Hansemann recounted now he and 25 other people had been invited to go skating on a large rink near Potsdam by Mrs. Du Bois-Reymond, the wife of Émile Du Bois-Reymond. Then Hansemann wrote:

"Last week I went only once to the theatre, to the Residenztheater with Weierstrass. They first performed an awfully dull and stupid French play, but then there was a nice German comedy, "The Rider" by Paul, so we were quite rewarded for having been bored. And then,

there was a fascinating actress, with whom naturally we were both delighted." [85, p. 28]

After Kovalevskaya had been allowed to give lectures on both mathematics and mechanics, she wrote jokingly to Hansemann on November 9, 1885:

"Proceeding from the idea that once you become a Professor, you can just as easily become a Professor twice, I have obtained, another Professorship... My formula reads now: Frau Sonya = (Professor)^2." [64, p. 292]

Hansemann responded to the joke, starting his letter of December 15, 1885:

"My dear friend and (Professor)^2,

"Your last letter has brought me double joy because it was twice as long as usual... The fact that you now give lectures on analytic mechanics as well, and because of that appeared an equality: Frau Sonya is equal to a Professor squared, what with your fervour to the sacred cause of mathematics, really struck me. Besides, it was Weierstrass, who is very proud of his pupil, that told me during his anniversary celebration. The celebration was excellent. The luncheon at "Hôtel de Roma", flavoured with witty and serious speeches, was a great pleasure to everyone present, then there were rejoicings that lasted till the small hours in the morning, and Weierstrass, gay, hale and hearty, took part in all of it. A week later there was a small party at his home, together with both his ladies... A week ago Weierstrass left Berlin for Lake Geneva." [85, p. 38]

This friendly correspondende between Kovalevskaya and Hansemann continued until she died. The last message was sent by Sofya Kovalevskaya during her fatal illness:

"She [Sonya] conveys her many regards, but please don't say anything of her illness to Professor Weierstrass, so as not to worry him." [184]

France

Charles Hermite played a tremendous role in the mathematical community of his time. Felix Klein writes that such

was his personality, that he could maintain a vast correspondence and

> "for many years was one of the vital centres of the entire mathematical world". He endeavoured "to raise mathematics over the one-sided (einseitig) nationalism that was gradually developing among the younger French generation." [134]

Sofya Kovalevskaya first went to Paris from Berlin in early 1882 and Weierstrass had advised her to meet Hermite.

We have available 15 letters from Hermite to Kovalevskaya. The first one, dated January 27, 1882, was sent to Kovalevskaya when she was in Stuttgart. It is clear that Kovalevskaya enjoyed Hermite's complete confidence and had been immediately introduced to the family affairs of the Bertrands and the Hermites (Charles Hermite had married Joseph Bertrand's sister). Hermite mentioned that there had been a long period of discord between the families which Sofya Kovalevskaya had put to an end [77, p. 645].

Hermite wrote in the same letter about a group of French mathematicians who were intimately related to the German science. [77, p. 654]:

> "Our common teacher is M. Weierstrass, and our lectures at the Sorbonne and Polytechnical School are largely aimed at the presentation of his works and great discoveries. And you, dear Madam, are a link of sympathy between me and the great geometrician."

In his next letter, of April 21, 1882, Hermite told Kovalevskaya that the President of France had conferred on Weierstrass the title chevalier of the Legion of Honour. In order to achieve this, Hermite had had to tear himself away from his algebra and start a round of talks with high-ranking officials, which was a hard thing for him to do:

> "You know that I'm a savage who shuns people; a wild beast who never leaves his lair." [77]

However, Hermite was not entirely satisfied with the result of his efforts, he would have liked

> "the great geometrician to have been made an "officier", like Mr. Helmholtz and Mr. Kirchhoff, because his

genius is at least their equal and he occupies the same prominent position in science as they do." [77, p. 657]

Weierstrass learned how warm the relationship between Kovalevskaya and Hermite was from Hermite himself. Weierstrass related this in his letter of June 14, 1882, and added:

"He [Hermite] wrote with enormous enthusiasm and listed the problems you touched upon in your talk." [125, p. 231]

Hermite was interested in algebra, analysis, number theory, and the theory of differential equations. He made an exceptional discovery in 1873 when he proved that the number e was transcendental. Proceeding from inequalities which he had established that limit e^{ax} between rational fractions he proved that

$$N + e^a N_1 + e^b N_2 + \dots + e^h N_n = 0$$

cannot be valid if a, b, ..., h and N, N_1, N_2, ..., N_n are integers. Suppose $a=1$, $b=2$, ..., $h=n$, we can see that e cannot be a root of a polynomial with integer coefficients, and therefore e is transcendental [258].

In the same letter Weierstrass shared with Kovalevskaya

"the provocative and very mathematical news that Professor Lindemann of Freiburg has just proved *that* π *is a transcendental number* [see [259]] by using (and this will be very interesting for Hermite) a generalization of the main theorem Hermite used when he proves e is transcendental. A clear corollary of the general theorem is that if two real or complex values a and b are related by the equation $b=e^a$, then they cannot be algebraic, except when $a=0$, $b=1$." [125, p. 236]

Lindemann's generalization of Hermite's theorem consisted in that Lindemann considered N, N_1, N_2, ..., a, b, ... to be any algebraic numbers. Then the transcendence of π follows from the equation

$$e^{\pi i}+1=0,$$

which cannot be valid if πi is an algebraic number.

Weierstrass began to think over Lindemann's theorem

and wrote in his next letter to Kovalevskaya, dated July 15, 1882, that Lindemann's results about π were correct, but that at the beginning his work was

> "based on an *incorrectly* comprehended theorem and still hasn't been proved strictly." [125, p. 237]

Using Hermite's assumptions "in his elegant work on exponential functions", Weierstrass found a strict and straightforward proof of Lindemann's theorem.

The theorems proving the transcendence of e and π have attracted mathematicians for a long time. Hermite was delighted by elegant results, and was glad when a mathematician continued his ideas. An interesting insight into Hermite's attitude can be seen from a letter written in 1893 by G. Minkowski to D. Hilbert, who had just found a new exquisite proof for the transcendence of e and π:

> "An hour ago I received your note on e and π ... and I cannot do other than to express to you right away my sincere heartfelt astonishment... I can picture the exhilaration of Hermite upon reading your paper and, as I know the old gentleman, it won't surprise me if he should shorty inform you of his joy that he is still permitted to experience this." [225a, p. 41]

During the first period of the correspondence between Hermite and Kovalevskaya, in 1884, Hermite was interested in Kovalevskaya's lecturing. He wrote on January 27, 1884:

> "The precise theory of partial differential equations that originates from the principles of Weierstrass and that you, Madam, are presenting at Stockholm University, is a very important and intricate matter. You can also do an enormous service to the students of the University if you give them lectures on what they cannot find anywhere else but in Jacobi's lectures." [77, p. 666]

And in a letter of March 8, 1884, after relating to Kovalevskaya some of his considerations concerning the meromorphic solutions of Laplace's equation and about reducing hyperelliptic integrals, Hermite said:

> "These details are unworthy of you, Madam, and I

take the liberty of communicating them to you only because I know your kindness and also that you are devoting yourself to the laborious effort of preparing your lectures." [77, p. 656]

In 1888, the main topic of the correspondence was Kovalevskaya's problem for the Prix Bordin. She had already sent her results to the Paris Academy of Sciences, but she was not satisfied with their presentation, and she wrote to Hermite about it. He answered her on June 11, 1888, that she could forward a new version in the autumn since the Academicians were on vacation and the commission for the Prix Bordin would not start work until the October. Hermite informed her that two more memoirs had been submitted for the competition, one from Paris and the other from Brest but he did not know who filed them.

Then he said:

"While I have not the honour of being a judge of the competition, I shall, Madam, be one of your readers and I thirst to learn the fine and substantial results you have arrived at in this famous problem, which I find most intriguing. It will be a pleasure for me to pick ears of grain from the field you have reaped. I am already dreaming of investigating the particular cases where your hyperelliptic integrals can be reduced to elliptic functions, like the examples provided by Jacobi." [77, p. 675]

Kovalevskaya was in Paris in the Summer of 1888. A letter dated June 19 from Hermite to Kovalevskaya contains an invitation for dinner to which Picard, his wife, and children were expected (Mrs. Picard was Hermite's daughter). Mrs. Hermite hadn't invited Kovalevskaya before, and Kovalevskaya had felt slighty offended. Now she was to be invited into the family circle.

When Weierstrass learned that Sofya had made friends with Hermite, he advised her to meet some of the other French mathematicians too; he felt that especially the junior generation which included P. Appell, É. Picard, and H. Poincaré, would be more interesting for Kovalevskaya. He wrote:

"In my opinion, Poincaré is the most mathematically able. He should not fritter away his exceptional talent and give his studies time to ripen. The theorems on al-

gebraic equations with two variables and on linear differential equations he gave to *Comptes Rendus* are most impressive." [125, p. 231]

Hermite also wrote to Kovalevskaya about these three mathematicians:

"Mr. Picard works to his utmost and publishes some remarkable papers, as do both Mr. Appell and Mr. Poincaré, and I'm very bitterly reproached that I praise them too much". [77, p. 656]

Probably, Hermite had in mind Weierstrass's opinion concerning Hermite's favourites, an opinion Weierstrass had broached to several people other than Kovalevskaya. However, he wrote to her on April 11, 1882:

"Have you paid any attention to the latest works by Poincaré? At any rate, he is a major mathematical talent." But Weierstrass regreted that "the Academy is too enticing an aim for young French researchers. To present a really valuable paper to *Comptes Rendus* every week is utterly impossible. Even the talented Picard squanders his gift like this, and Hermite encourages this restless race for an outward success too much." [125, p. 320]

Picard, Appell, and Poincaré were very prolific authors [260], [261]. By the end of his life, Poincaré had published 561 works. All three became Members of the Paris Academy of Sciences and in 1908 Poincaré was elected to the French Academy (one of the 40 "immortals") for his work on the philosophy of science.

When Kovalevskaya met Poincaré in 1882 he hadn't published his work on celestial mechanics on the equilibrium shapes of rotating liquids, the studies that made him famous. But he had published a number of articles on Fuchsian functions, algebraic equations, and linear differential equations with algebraic coefficients. In November 1881, the first part of his famous memoir *On the Curves Defined by Differential Equations* was printed; the second part appeared in August 1882 [262]. For a time, Kovalevskaya was engrossed in these memoirs, and when in 1883 her position in Stockholm was finally secured, one of the courses she proposed was one on these investigations of Poincaré. When-

ever Kovalevskaya was in Paris, she would visit Poincaré and would be sorry if he was out of town because she considered her talks with him very interesting.

Émile Picard was Hermite's favourite student, and he became his son-in-law in 1881. According to his biographer E. Lebon [263], Picard was a tough and independent man with excellent memory. He rarely spent a whole day on mathematics, for he read much on history, philosophy, art, and archeology. He travelled a lot, and had been to Egypt, the USA, and to Norway for the centenary of Niels Henrik Abel. He wrote many books, including some on the philosophy of science, on the history of 19th century mathematics [267] (in which he mentioned Sofya Kovalevskaya) and a four-volume course of mathematical analysis. In 1877 Picard published his first papers, on the theory of surfaces, and in 1879 he began publishing his articles on entire functions, which contained his famous theorems.

In his first letter to Kovalevskaya, dated October 19, 1880, Mittag-Leffler asked:

> "Are you familiar with the very remarkable memoirs of Mr. Picard 'On Entire Function'? [[264], [265]] ... He proved the fundamental theorem that *an entire function* $g(z)$—in the sense of Weierstrass—which does not yield a or b, where a and b are finite definite numbers, *must be a constant.* His proof is far from being elementary and essentially assumes that K'/K is a function of k. Weierstrass is very eager to have an elementary proof.

Then Mittag-Leffler wrote that he had not succeeded in finding such a proof, but that he could immediately obtain the result that

> "... if there is a [Weierstrassian] entire function which does not assume either the value zero or unity, then there is another entire function which cannot yield any of the values 1, 1/2, 1/3, 1/4, ..., 1/n, ..." [ML 1]

Mittag-Leffler added that if there was an elementary proof of Picard's theorem, he was sure that Sofya Kovalevskaya with her "absolutely extraordinary perception", would find it. However, we do not know whether Kovalevskaya attempted to prove the theorem.

In the same letter Mittag-Leffler told her about his ri-

valry with Picard concerning the theory of linear differential equations. Following the appearance of Hermite's work on the integration of Lamé's equation, which contains doubly periodic functions in its coefficients [266], a host of mathematicians, including Picard, Fuchs, and Mittag-Leffler, were trying to generalize the result. Mittag-Leffler wrote:

> "Recently, I've been tackling the problem of integrating a linear differential equation of nth degree, whose coefficients are doubly periodic functions. In fact, I can now integrate equations not only for which the integrals are single-valued, but also when they are algebraic functions of single-valued functions. The integrals can be presented in a form that is completely analogous to the one Mr. Hermite indicated for Lamé's equations.
>
> "Prof. Weierstrass has informed me that Mr. Picard has already submitted a memoir on the same subject to a journal, so I have decided to wait until it's published before I publish mine. Besides, there are several very challenging algebraic problems that are related to the integration of the above equation and whose solution is possibly beyond my reach. I'd also rather have a clearer picture before I publish my investigations." [ML 1]

Kovalevskaya was amazed and this shows through in her letter to Mittag-Leffler dated January 8, 1881:

> "Sir, how could you be so late with the publication of your research on the integration of linear differential equations with doubly periodic coefficients? Having in mind the work of Mr. Picard*, I infinitely regret the fact, the more so because I perfectly remember that you had finished this work, at least for the most part, last year, that is the time you arrived in Petersburg." [SK 2]

Mittag-Leffler published his paper [269] after Picard [268] had brought out his memoir.

Paul Appell, a talented mathematician, was commonly mentioned by Hermite to Kovalevskaya. In a letter of March 24, 1884, Hermite described Appell saying:

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* Kovalevskaya still followed mathematical journals when she was in Moscow.

> "Mr. Appell is a charming young man, extraordinarily kind and gentle, and liked very much by everybody. I'm no exception and recently had the great pleasure of talking with him about the work that drew your attention." [77, p. 678]

Later Appell became widely known amongst mathematicians because of his five-volume course of mechanics. But when he was younger, he was more interested in mathematical analysis.

But back to the letter. Hermite had received a message from Kovalevskaya, a résumé of her work on the refraction of light, for presentation in *Comptes Rendus*. The mathematician Moris Lévy wanted to know the contents of the message, but Hermite had asked him to be patient and wait till the journal could be published. However, Hermite added:

> "It was more difficult for me to suppress my wish to inform Mr. Appell of the very provocative solution of the equation $\Delta\varphi=0$, which you obtained with the aid of the integral
>
> $$\int_0^\infty e^{-s(x^2+y^2+z^2)}\,\theta\,(s_u,\ s_v,\ s_w;\ \varkappa_{11},\ \varkappa_{22},\dots)\,\frac{ds}{\sqrt{s}}\,.$$
>
> "I confined myself to conveying to him your approval of his memoir, and this he received with gratitude. I hope, Madam, you will give me your permission to convey to him your result too as it will not be sterile in his skilful hands." [77, p. 678]

Hermite showed Kovalevskaya's integral to Appell, who was at the time busy finding solutions for Laplace's equation with three independent variables. Appell put his considerations concerning the integral into a letter to Hermite, which Hermite forwarded to Kovalevskaya [85, p. 22].

Gaston Darboux was older than Hermite's other favourites. Picard, also a student of Darboux, later recollected that students of the École normale supérieure preferred Gaston Darboux to all other Professors. Darboux won over young people by the authority of his vast expertise. His course on theoretical mechanics excited wide interest, for Darboux presented it elegantly, methodically, and

very clearly. His course on analytic geometry, which he gave "by playfully passing from one theory to another" also aroused great interest [270]. Darboux was famous for his four-volume course on the theory of surfaces. He published a number of works on analytic and celestial mechanics.

I have already mentioned above Darboux's work on the existence theorems for partial differential equations, and that Weierstrass had worried that Sonya might be overta ken by her rival.

Kovalevskaya had met Darboux because of her interest in the *École normale supérieure* for girls, where he taught. Hermite wrote about this school to Kovalevskaya in his letter of January 7, 1884:

> "I suppose you know about the great movement that has been occurring in France for several years, the movement to provide scientific education for girls. The present *École normale* for them was established in *Auteuil*, and the Ministry of Public Education invited Mr. Darboix and Mr. Tannery to teach mathematics there." [77, p. 662]

Hermite related how in July a commission of examiners, which included Appell, worked at the school, and

> "the members of the Commission were enormously impressed by the volume of knowledge which the girls had shown and especially by their presentation (a most essential quality in a teacher), particularly since they were not expected to have been so talented in the mathematical sciences." (*Ibid.*)

Hermite was just about to present Kovalevskaya's résumé on the refraction of light in *Comptes Rendus* and so he added:

> "Your discoveries in the grand and difficult problems of analysis will therefore not only be applauded by mathematicians and geometricians. What is more your success will inspire Mr. Darboux and Mr. Tannery's young pupils, whose sympathy and love you will receive." [77, p. 666]

In his next letter, dated January 27, 1884, which was in reply to a request by Kovalevskaya to visit the school

for girls in *Auteuil* (*Sèvres*), Hermite said he would inform Darboix with pleasure and added:

> "Madam, you enjoy the particular sympathies of Mr. Darboux, who was successfully working on partial differential equations and whose memoir on the particular solutions to them was awarded a prize by the Academy of Sciences." (*Ibid.*)

Hermite gave his opinion on education for women:

> "How fine it would be, if in an age when living conditions are becoming harder for everybody, and when many women are desperate, young girls with a vocation for science could persue a life that is honourable and well provided for, that is the life of a teacher... Madam, you know better than I do that we have to win over the public opinion in this problem, and only following a complete success it will sanction the support of the French authorities. The vulgar Molièrian common sense so far controls our unrefined, shallow public, which can applaud the place in a famous play when a character says that a man feeds on soup and not on fine speeches. Madam, you have contributed to this cause more than Miss Sophie Germain, both due to your superior talent and because you appeared at a favourable period." (*Ibid.*)

In his next letter, dated February 13, 1884, Hermite wrote again about the École normale for girls.

> "The director of the school is Mme. Jules Favre, the widow of a famous lawyer and statesman, while Mme. Bortniker, a Russian, supervises the science lessons... and succeeds in this duty. The young students work with enthusiasm and the school is well run. You will be able to judge yourself when you honour the school with your presence." [77, p. 669]

But Hermite was less happy about what future the young girls that were going to graduate from the school would obtain:

> "Unfortunately, this future, which I earlier imagined would be well provided for by the founders of the school, is very unreliable. Naturally, there will be some positions,

for example, as teachers in the girls' lyceums that are being organized in the provinces, but what hardship is in store for those who will get this privilege. A vacant position may occur in a city far from her family, which a young girl should not leave. Hence for the most of them, the hopes that have been cultivated in them may be dashed in bitter frustration." (*Ibid.*)

Hermite appears to have tended to pessimism in general. Thus, Kovalevskaya mentioned in a letter to Mittag-Leffler that Hermite was always in fear of a war or revolution. He was probably depressed by the situation in France during the period of 1870 and 1871.

During the summer holidays of 1876, Kovalevskaya went to Paris to present her rotation problem to French mathematicians, and she visited the École normale for girls. She wrote to Mittag-Leffler on June 26, 1886:

"Yesterday, I had a day of success: Mr. Tannery called on me at 8 a.m. to take me to the École normale in Sèvres. Mme. Jules Favre and Messieurs Darboux, and Appell were waiting for me there. The young ladies were sitting for their examinations in my presence." [SK 116]

Kovalevskaya did not give any information about the school or what her impressions had been. It had really been a day of success: for her report of what she had achieved in the rotation problem had made a great impression. After the examinations, the mathematicians were invited to lunch at Joseph Bertrand's villa Viroflay in Sèvre. Many mathematicians were there. Kovalevskaya wrote:

" I was showered with compliments. Bertrand gave me a manuscript written by Gauss." [SK 116]

In 1888, Darboux was one of the judges and the rapporteur when the Prix Bordin was conferred on Kovalevskaya by the Paris Academy of Sciences. The other judges were Moris Lévy, Henri Résal, Emile Sarrau, and Edouard Phillips.

Joseph Bertrand was widely known by his classical course of mathematical analysis [271], which was translated into Russian and published in two large volumes [227]. He played an essential role in the Paris Academy of Sciences as its permanent secretary, and whenever Kovalevskaya happened to be in Paris, she talked to him on behalf of

Mittag-Leffler. In a letter of June 22, 1886, Mittag-Leffler wrote to Kovalevskaya that he received a courteous letter from Bertrand together with a paper for *Acta Mathematica*, Mittag-Leffler wanted him to bring the journal to the notice of the Institute of France and to give a favourable account of it. He asked Kovalevskaya to ask Bertrand to do so and added

> "Win him over for yourself, for me, and above all for *Acta*." [ML 50]

Bertrand promised Kovalevskaya to do everything possible to spread *Acta*, but he was sure that in the final analysis its success depended on Hermite.

Two years earlier, in 1884, Joseph Bertrand's brother Alexandre, a Member of the Academy of Inscriptions was the person in France who distributed *Acta Mathematica*, Hermite wrote to Kovalevskaya on March 8, 1884:

> "Mr. Alexandre Bertrand... strives to obtain subscriptions for *Acta* through the Ministry of Public Education, and now not for the faculty libraries, because almost all of them have already subscribed, but for the libraries of lyceums, which are far more numerous." [77, p. 675]

According to Kovalevskaya, Joseph Bertrand was uncommonly obliging to her. She wrote to Mittag-Leffler on June 26, 1886:

> "Just imagine what he [Bertrand] has thought up that the gentlemen [of the Academy] should gather to propose a topic for the grand academic prize of 1888. It came to Bertrand's mind to propose the problem of the rotation of a heavy rigid body. Consequently, I'll have some chance to win this prize." [SK 116]

Hermite, Bertrand, Jordan, and Darboux, the members of commission for selecting the topic, made Sofya Kovalevskaya expound in detail the results of her work, found it very interesting and said it would have a good chance of winning.

Speaking of Hermite and his attitude to Kovalevskaya, I have to mention his letter to Chebyshev, dated May 21, 1890, requesting Chebyshev to work for her return to her homeland (cited from [220, v. V]):

"My dear colleague and friend,
...Knowing your kindness, I hope that you will be able to recall to the St. Petersburg Academy of Sciences Mrs. Kovalevskaya, whose talent arouses admiration in all mathematicians and who, in her Stockholm exile, keeps in her heart longs for her native Russia. I have learned from her of the role you played in obtaining her election to corresponding membership of the Academy; at the same time she confessed to me the difficulties she faces in living abroad. I am therefore taking the liberty to request your support in helping her, as much as possible, to get out of this situation.

"I apologize for my pleading if it is immodest, I send from the bottom of my heart my best wishes for respect to your health, your happiness, and I use this occasion to assure you anew, my dear colleague and friend, of my true feelings of the most sincere and devoted admiration for you.

Hermite."

This letter from Hermite could not bring the desired effect for there was no position for Kovalevskaya in tsarist Russia.

I have already mentioned Joseph Perott, the revolutionary and mathematician. He was a very gifted man and knew some 20 languages, including Chinese and Japanese. He was awarded the Douglas Hyge prize for his research on the Gaelic spoken by the Scottish Highlanders. Perott worked at universities in France and Germany.

In 1890 Perott moved from France to the USA and started teaching mathematics at Clack University in Worcester, Massachusetts.

The letters from Perott to Kovalevskaya include five tables with numbers, apparently the solutions of some algebraic equations.

When Perott had to flee from St. Petersburg he left all his belongings behind, other than *Disquisitiones Arithmeticae* by Gauss. He had been delighted to study number theory by it.

On one occasion he criticized an article by the "brave sailor" do Jonguières, and he wrote to Kovalevskaya on March 15, 1883:

"I insist on my criticism because my arithmetical flair is much more developed now, and I consider all these despicable theorems, such as the one by de Jonquières, to be an insult to my princess (whom I found, at last, after long and futile wandering) for she is no less than the Goddess of Arithmetic." (P 13]

Perott believed de Jonquières to be an honest man though a dilettante. Towards the end of the letter, Perott wrote in English (though most was in French) that if he insisted too much in his attitude to de Jonquières, he would jeopardize "his right to love his princess".

Perott was interested in the history of number theory. He had tried to find in Spain *Arithmetic* by Juan Ortega published in 1534 as he felt it might have been available in a library in Madrid. Perott was attracted by the equation $x^2-Dy^2=1$ which Ortega had considered and later, in 1657, Pierre de Fermat. Perott believed that Ortega had read a work by Chiron of Alexandria (100 B.C.), and thus it was probable that the work was still preserved somewhere in Spain. Perott requested his friends to try and find it there.

Joseph Perott is known to have published eleven papers. They deal with algebra and number theory.

Sweden

Other than Mittag-Leffler, Kovalevskaya had not met any Swedish mathematicians until she settled in Sweden. The first she met and later worked a great deal with were Ivar Otto Bendixson and Gustaf Hjalmar Eneström. By the time Sofya Kovalevskaya started working in Stockholm University, Bendixson had already been teaching there, nevertheless he attended her lectures. Kovalevskaya met the Bendixsons directly she arrived in Stockholm.

At the beginning of his scientific career, Bendixson worked on set theory after Cantor and on some problems in higher algebra. Later he wrote papers on the theory of ordinary and partial differential equations. His research on the theory of curves defined by differential equations (he proceeded from the work of Poincarè [262]) was the most well known [272].

Judging from two messages which he wrote to Kovalevskaya, Bendixson acted as someone like a monitor for her auditors. His message dated May 29, 1885, informed Kovalevskaya that they would come to the lecture on the following day, a Saturday, but that there would be a holiday on the Monday, and hence the lecture would probably be cancelled [85, p. 48]. Evidently, these were Kovalevskaya's lectures on "The theory of algebraic functions according to Weierstrass". A little earlier, on May 25, Bendixson had sent Kovalevskaya back her lectures with an accompanying message saying that he had not had much to correct (he, probably, meant Kovalevskaya's Swedish). These seemed to be lectures on the theory of partial differential equations, which Bendixson attended in the autumn of 1884. He makes a few minor remarks [85, p. 48]. Bendixson was engaged in the qualitative theory of differential equations [272].

I mentioned that Eneström was the secretary of *Acta Mathematica*. He didn't publish many mathematical papers, but he curried out a comprehensive bibliographic activity, for which he received a doctorate from Lund University in 1918. He published articles on the history of mathematics, and translated some Russian literature into Swedish. Thus, in a letter to Kovalevskaya dated January, 10, 1886 Eneström enclosed a poem by N. A. Dobrolyubov for her critical consideration [85]. He also noted that he was going to translate some poems by I. S. Nikitin and write a critical article on both poets.

Sofya Kovalevskaya was a close friend of the Swedish astronomer Hugo Gyldén. She would meet him at faculty sessions of Stockholm University and visited him at home, where her daughter lived later. I have already mentioned the friendly attitude of Gyldén to Kovalevskaya, but sometimes he was jokingly sarcastic. Thus, he dubbed the group of five mathematicians (Weierstrass, Poincaré, Hermite, Mittag-Leffler and Kovalevskaya) the "mutual admiration society". But all of them merited admiration.

Gyldén was born and studied in Helsingfors. At the beginning of his career, he worked in the Pulkovo observatory near Petersburg. When he moved to Stockholm in 1871, he became an astronomer of the Swedish Academy of Sciences and the director of the observatory at Stockholm University. He was regarded highly as a scientist and cal-

led "a king of astronomers". Gyldén was interested in theoretical celestial mechanics, and the three bodies problem in particular. During Kovalevskaya's time the observatory was located at the end of the main street in Stockholm in an old house, and consisted of a refractor telescope. Gyldén, his family, and his assistants all lived in the same house. This was the house at which Sofya Kovalevskaya attended a party on one of the last days of her life.

When Gyldén died, his student Karl Bohlin became the director of the observatory. Bohlin too had attended Kovalevskaya's lectures. He wrote a paper on Gyldén as a scientist in *Acta Mathematica* [273] and published papers on stability of dynamic systems.

Sofya Kovalevskaya was also in constant communication with a young mathematician called Phragmén. He attended her lectures and for a time was the secretary of *Acta Mathematica*. He did research on the theory of functions of real and complex variables. When Kovalevskaya was on a leave of absence during the spring semester of 1889, Phragmén gave her lectures.

Sofya Kovalevskaya maintained friendly relations with Anders Lindstedt. He too investigated the three bodies problem, and Poincaré cited the investigations of Swedish mathematicians Gyldén, Bohlin and Lindstedt in some of the sections of his work on the subject.

One of Kovalevskaya's auditors was Ivar Fredholm, who later worked out the integral equations that bear his name.

Chapter 7

LITERARY AND SOCIAL ACTIVITY

One of the first reviews of Kovalevskaya's literary work was written by the Danish critic Georg Brandes [214]. It concerned her book *Ur ryska lifvet: Systrarna Rajevski* (*From Russian Life. The Sisters Raevsky*). It was the title her Swedish friends had suggested instead of *Memories of Childhood* and they recommended she wrote it in the third person and not the first and disguised the names for the Swedish edition. The book was published in Swedish in 1889, and after reminding the reader that the author was a famous mathematician, Brandes wrote:

> "And now, owing to this newly published book the readers can meet this interesting woman as a live and feeling being. The novella form is obviously only a mask. Tanja Rajevski is no one else but Sonya Kovalevskaya, and the book is a finely written fragment of an autobiography, and contains a description of the childhood years of Kovalevskaya in Russia.
>
> "The action takes place first at her parents' estate of Palibino in the Vitebsk province, not far from the Pos lish border, and then in Petersburg. The writer reveal- a sharp ability for self-observation and to portray realistically her environment. The book describes for us the life of a conservative Russian gentry family in its estate as it was a generation ago. Our interest in the story increases when Dostoevsky becomes one of the characters. At the end, the writer is only thirteen or fourteen years old and you close the book with an intense expectation that it is to be continued, and we hope that the sequel will be not long in coming." [214a, pp. 241-242]

Memories of Childhood [46] were admired by Russian writers, as well as by the Russian public. The historian M. I. Semevsky, who knew the Korvin-Krukovskys during their time in Palibino, remarked that the description of that life given by Kovalevskaya was correct and truthful [93].

Kovalevskaya also wrote a novella called *A Nihilist Girl*. It was published in Geneva in 1892 with an introduction by Maksim Kovalevsky, and then reprinted by various publishers. Another version was published in Sweden in 1893 in a collection titled *Vera Vorontsova*, and a year later G. Brandes wrote an essay about it. I would like to cite just the beginning of this essay: the rest mainly narrates the story which had struck the critic's imagination.

> "The focus of the collection *Vera Vorontsova* is the novella that gave the book its title and was Sofya Kovalevskaya's last work. These a hundred and fifty pages have made striking impression on me to say the least. The presentation is not particularly skillful but the story is told in an extremely simple, old-fashioned manner, and the story was written in a language the main characters never used nor was it the writer's mother tongue. What grips the reader is the plot itself, a story that has been taken from life by a stroke of genius, and delineated clearly and boldly; it takes possession of the reader with its arrangement, unpretentious but authentic." [214a, p. 255]

A Nihilist Girl was banned in tsarist Russia, and only published in the Soviet Union in 1928 [61]. The first Russian edition was printed in Geneva in 1892 by Maksim Kovalevsky. It is a story of a young girl who strives to be of use to the people by participating in revolutionary circles. She marries a complete stranger, a condemned revolutionary, in an attempt to ease his hard life. Kovalevskaya emphasized the ideas behind the girl's action:

> "Vera found socialism was the only way of solving every problem." [67, p. 143]

The prototype for the heroine was Vera Sergeevna Goncharova, a niece of N. N. Goncharova, Pushkin's wife. She had once appealed to Kovalevskaya, who was in contact

with revolutionaries, to introduce her into revolutionary circles.

In a letter dated October 7, 1890, to Maria Mendelson-Zaleska, Kovalevskaya related that she was writing a novella about Chernyshevsky:

> "Now I'm coming to the end of another novella that I hope will interest you. My guiding theme is the story of Chernyshevsky, but I've changed the name to be free with the details, and also because I'd like to write it so it will be exciting and interesting for the common public as well. I'm going to finish it in a few days, and if you wish to translate it into French, I'll send you the manuscript." [64, p. 311].

Part of this manuscript, which was believed to have been lost, was found by L. A. Vorontsova [215, p. 18] in the Archives of the USSR Academy of Sciences. It consists of 32 large pages that are a copy of the original with some of the words missing. This fragment was published in [67] under the title *The Nihilist*, because according to Ellen Key this was the title Kovalevskaya had intended.

However, even though but a fragment, the manuscript is valuable because it shows up Kovalevskaya's political views. We can now positively assert that of the movements then current Kovalevskaya preferred the revolutionary-democratic one, headed by Chernyshevsky. Kovalevskaya not only described Chernyshevsky (Mikhail Chernov) and his wife Olga (Marusya), but also wrote about some of the activists involved with the magazine *Sovremennik* (The Contemporary), such as Nekrasov, and Sleptsov, and the circle around them, particularly the women who were fighting for equal rights, for example, Nadezhda Suslova, Korsini, Yakovleva, and Panaeva.

The novella contains important new material on Chernyshevsky, his colleagues, and his family. Kovalevskaya never had the chance to meet Chernyshevsky, but she is known to have advised his son, Aleksandr to study mathematics [64, p. 515].

The novella begins with the words "5 pounds of grapes..." [67, p. 157-181], some of the food Marusya had ordered for the guests that were to meet at the Chernovs' apartment following the publication of the latest issue of *Sovremennik*.

Chernov and his family are at the peak of their happiness, but in stark contrast to the celebration, the secret police arrive and arrest Chernov.

Ellen Key has related some of Kovalevskaya's thoughts about the novella she was thinking of writing:

"The unknown Chernyshevsky suddenly became famous among young people owing to his revolutionary novel "What is to be done?" He is saluted at a lively party as the hope and the leader of youth. He returns to his small garret, where he lives with his beautiful young wife. By the time he returns home, she is asleep. He walks over to the window and looks down over the flickering lights of the sleeping city of St. Petersburg. He silently speaks to that vast and spalling city which shelters violence, poverty, injustice, and oppression. Nevertheless, he felt he would win it over, he would pour his spirit into it, and step by step, everybody would think as he thinks and as all the youth were already thinking. He recalls in particular a young inspired girl who treated him with the warmest sympathy; he begins to dream but leaves his dreams and goes to kiss his wife to wake her up and tell her of his triumph—and at this moment a sharp knock on the door is heard. He answers the door to the secret police who have arrived to arrest him." [67, p. 522]

Kovalevskaya left two other books unfinished. She had in fact written the introduction and part of the first chapter of a novel called *Vae victis* (Woe to the Vanquished), published in Russia in 1893 [55] and in Sweden in 1890. Ellen Key wrote in the preface to the second Swedish edition of Kovalevskaya's literary work:

"The introduction to *Vae victis* appeared in the magazine *Norna* (1890), and unfortunately this is all that exist as a large novel in which Sofya Kovalevskaya had intended to depict a tumultuous spring outburst of young Russia and combine it with the love stories of her character." [53]

Anne-Charlotte Leffler described the introduction to this novel in more detail:

> "It is a poetical account of the struggle of nature as it awakens to a new life in spring after a long winter's sleep. However, it is not the song of praise to spring that it is common to many other descriptions of spring; it is conversely a song about the calm untroubled winter, while spring is presented as a violent sensuous force that ignites a mass of hopes, none of which are realized." [96, p. 276]

The unfinished first chapter is the story of a girl who had fallen in love with an ailing man and who then had to part with him because he had to go to the south for treatment. Concerning the rest of the novel, Anne-Charlotte wrote:

> "In part, the novel was to illustrate Sofya's inner life. Few women enjoyed the honour and success as she achieved, and yet she intended to sing of the vanquished and not the victors, because despite her success she herself sympathized with the losers in life and never with the winners. This commiseration for the sufferings of others was her most salient feature. This was not the merciful commiseration for suffering of a Christian; no, she literally suffered for others, she took to heart their suffering as strongly as if they were her own, and her attitude was not that of a superior being who wants to console, but that of someone in distress at the cruelty of life." [96, p. 277]

There is a biographically interesting excerpt from an unfinished story *A Fragment of Romance on the Riviera*. A girl had taken a fast train to Nice to have a holiday after her studies at the Bestuzhev Courses. Suddenly, the door of the compartment opened,

> "and a tall stout man pushed into the compartment out of breath; the first thing he did was to step on the feet of the two or three ladies there and therefore cause a commotion." [67, pp. 211-220]

He, one Zvantsev, turned out to be an acquaintance of the girl, in fact, a reflection of Maksim Kovalevsky.

Kovalevskaya's essays (there are five of them) bear witness to her wide and diverse interests. How each of these essays appeared is curious.

When Saltykov-Shchedrin died, Kovalevskaya wrote an essay about him in 1889 that naturally could not be published in Russian because everyone was afraid to express openly its sympathy for the great satirist.* But it was also difficult to publish such material elsewhere. In 1889 Kovalevskaya wrote to Mittag-Leffler:

"As to my attempts at literature, I have had no success so far. Mr. Lemaître returned me my scenario** (which I'm sending you) informing me that all his colleagues to whom he showed it found the idea exceptionally ingenious, but that 'it is not for the theatre'... *La Revue bleue* also rejected my article about Shchedrin. A friend of mine, Mrs. Herzen [Olga, the daughter of A. I. Herzen] and Monod's daughter-in-law, knows Mr. Rambeau, editor of this review, very well and went to speak to him on my behalf, but he told her that too much had been recently published about Russian writers, and he didn't even want to look through my article." [SK 351]

Kovalevskaya saw the brunt of Schedrin's creative effort as concerning the decay of the ruling class in the period of serfdom. Kovalevskaya believed Shchedrin was less popular abroad than other Russian writers because he was a satirist. She wrote,

"Tears are the same everywhere, but everyone laughs in its own way." [72]

Besides, tsarist censorship made Shchedrin, "encode" his ideas. Therefore, what a Russian reader may be able to decipher would remain incomprehensible for a non-Russian reader.

Kovalevskaya made a commentary on Saltykov-Shchedrin's work and started with his story *Sore point.* She also described his novel *The Golovlevs.* In her opinion, it could be subtitled "the natural and social story of a family" for it portrayed

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* Kovalevskaya's essay about Saltykov-Shchedrin was published in Sweden. In the Soviet Union, S. Ya. Shtraikh published it in 1934 [72].

** Probably, this was the play *When There Will Be No Death,* and the action takes place at Pasteur's Institute; the play has not yet been found.

"the moral decline and gradual wreck of three generations of landowners, a wreck that was determined by the laws of heredity and accumulated influence of unwholesome and demoralizing factors." [72]

She wrote that *The History of a City*

"is, in fact, a disordered and clamorous history of the Russian empire; it is a major work and will never fail to be interesting for generations to come." [72]

She praised Saltykov-Shchedrin saying that

"... his name will remain in history not only as the greatest lampoonist Russia has ever known, but also as a great citizen who granted no mercy or rest to the oppressors of ideas." [67, p. 229]

Recollections of George Eliot [67, p. 230-243] is an interesting psychological essay. Kovalevskaya met Mary Evans (whose pen-name was George Eliot) on several occasions. Kovalevskaya was struck by some circumstances of Evan's life. When they met first, Kovalevskaya was only 19, and she thought Evans staggeringly ugly, but she was so charming and possessed such a mellow resounding voice that she quickly mesmerized the listener and made one forget her outward appearance. She could listen too and gave her guests a chance to talk creating a pleasant and agreeable atmosphere.

Kovalevskaya visited Evans again eleven years later. After her first husband died, Evans married an admirer, Mr. Cross. He was forty while she was fifty nine. Contrary to Kovalevskaya's expectations, the relations between man and wife were in no way uneasy and everything was natural and uncomplicated.

Kovalevskaya asked Evans why in a number of her novels, just when an intricate knot of events had been tied in the lives of the characters, the unexpected dénouement of death followed. What would the actions of the characters have been if there had not been a death? After a while, Evans answered.

"Havent't you noticed how often this occurs in life? I cannot give up my conviction that death is more logical than it is commonly thought... It has already happen-

ed so many times to me that my trust in death has given me the courage to live." [67, p. 243]

George Eliot passed away suddenly two weeks later after only three days of illness. Kovalevskaya wrote:

> "By joining her life with that of a man twice younger that she was she resolved to a very hazardous experiment. This very minute both were happy, but could their happiness last long?" [67, p. 244]

Kovalevskaya wrote a remarkable essay called *Three Days in a Peasant University in Sweden*, which is evidence of her considerable interest in public education.

In the Summer of 1886, Sofya Kovalevskaya and Anne-Charlotte Leffler went travelling in Sweden. They arrived in Dalecarlia, in Central Sweden, and visited an open higher school in Tern. Kovalevskaya became interested in peasant "universities" and accepted an invitation from a rector to visit such a university.

She wrote the essay in the Summer of 1890, whilst travelling in Switzerland, and published it in the magazine *Severny Vestnik* [67, pp. 244-266].

The article describes the visit as taking place during the beginning of two-week spring break taken by the Rigsdag, the Swedish parliament, she had boarded a train many of whose passangers were deputies, indeed the train was called the "Rigsdag train". Kovalevskaya outlined some of the workings of the Swedish parliament, and how the peasant party held a prominent position in it. She vividly described some of the party members, one of whom had only been educated in a common school; but was nevertheless an influential person.

The article commenced with an account of the railway station in Stockholm and the motion of the train: "The locomotive begins to puff and wave its heavy wings, and the train starts..." [67, p. 244].

At a small stop called Sale, a small gig was waiting for Kovalevskaya, the driver was a pupil from the local school. The gig was sent for the celerity by the school rector Holmberg. She was heartily welcomed by the rector Holmberg, his wife, and several young girls, their relatives. Together with the pupils of the school they lived as a single family.

In the evenings, some of the pupils (for there was just no place for all of them) would have dinner with the rector and spend their spare time reading aloud or singing. The rest of the pupils would wrestle or do physical exercises.

Holmberg outlined the history of the peasant schools in the Scandinavian countries. The idea had been put forward in Denmark, in about 1850, by the philosopher and theologist Grundwig. He had been motivated by purely religious ideals:

> "An uneducated person cannot be true Christian, therefore the people in a Christian state must not be kept in obscurity and ignorance... There is a need to develop their powers of reason, expand their outlook, reveal to them their history in its true (Christian) aspect, so that the influence of pseudoscience will be kept at bay." [67, p. 250]

Grundwig's school was an enormous success, and it had many successors. At the beginning, the schools were supported by private donations and so provided an education that was specifically directed to raise the people's moral standards. Then the schools were granted subsidies from the state and the curriculum was expanded to meet the increased requirements. In the 1890s, there were more than 40 schools for adult peasants in Denmark.

In Sweden and Norway, the people's universities began for other reasons which were due to the change in their constitutions in 1866. The rights of peasants were expanded, while those of gentry were diminished in Sweden, and those of the gentry, even the very concept, were abolished in Norway. Free of charge elementary education was introduced on a universal basis. Numerous novels about the life of the people appeared (though Kovalevskaya considered them too idealized). The schools for adults were first organized on the lines of the Danish ones, but soon they took another turn. In Norway, the schools took

> "far greater care over the general and political education of peasants than over their religious education." [67, p. 255]

In Sweden, the first higher schools for the common people were established by the state at its own expense, because

the powerful peasant party had included the development of the schools into its program. Holmberg believed the future of Sweden lay in the hands of peasants. However, a peasant's life had its own features. Although fully occupied in summer, he had much spare time during the long winter months. In these periods of enforced idleness and loneliness, brute instincts may be awakened in coarse and undeveloped people living far from each other. As any lawyer can tell you, so Kovalevskaya wrote, there is no crime too abominable for it not to occur every year among the population of the skerries surrounding Stockholm. The state needed schools where people could gather knowledge and interests "capable of giving colour and content to their lives" [67, p. 256].

The orientation of the peasant "universities" was different from that of the industrial or agricultural schools, from which the specialists graduated, and from that of the high schools and old Universities in Uppsala and Lund, which were the breeding-grounds for the growing numbers of priests, lawyers, and officials.

The goal the peasant universities pursued Kovalevskaya explained, was such that

> "without taking the peasants away from the soil and without making of them some sort of machine they have to awake in peasants their human identity, give them at least a general idea of the treasures man has accumulated in the arts and sciences, and introduce them to the intellectual delights that are accessible to the educated in society." [67, p. 256]

The people's universities did not award diplomas or confer special rights, but still they attracted young people. Hence Sweden, which had a population of four million, had 25 peasant universities. The stories of those who went home from the schools were such as to ignite in other young people a desire to learn. They learned with enormous zeal, and showed they had remarkable memories for names, years, and events in history and geography, and easily solved problems in arithmetic, even for bill discounting. Holmberg told her

> "You wouldn't believe how receptive these wholesome

young heads, which have not been exhausted by learning by rote for many years, are to knowledge." [67, p. 260]

However, they often complained of headaches during the first two or three weeks, but Holmberg believed this was because of the lack of motion. That was why he had introduced physical exercises, games, and running.

Kovalevskaya suggested she would read something to the students. They listened with interest to two of the stories by Lev Tolstoy: *Let out the Fire, Don't Put it Out* (she was frequently interrupted by bursts of laughter as she read this story) and *How Much Soil Does a Man Need*?, the philosophy of this story remained beyond her listeners. Kovalevskaya was showered with questions about Russia and its peasants. At night, she thought about her homeland and her essay ended with a question:

"Will I ever be able to talk about Sweden to young peasants in a small backwater Russian village like I spoke today about Russia in Sweden?" [67, p. 267]

The Kovalevskaya's abilities as a publicist are seen in full in two articles that appeared in 1888 in the newspaper *Russkie vedomosti* (Russian Gazette). They covered her visists to two French hospitals, *La Charité* and *La Salpetriére*. Dr. Luys had conducted a hypnotism *séance* in *La Charité*, and Dr. Charcot had given a lecture on hypnotism in La Salpetriére [67, pp. 275-281].

Kovalevskaya had been disappointed by the visits, for the *séances* had been performed on penniless patients who had played into the hands of their doctors. Her precise observation had allowed Kovalevskaya to notice various details of the behaviour of both the doctors and patients.

There are seven sketches for work Kovalevskaya had thought about and which would probably have been larger. One of the sketches, "Swedish Impressions" was published after her death [55]. It was published again [67] with additions that had been found in the Archives of the USSR Academy of Sciences* together with the other six sketches, which were published for the first time, and which had also been found in the Archives.

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* *AAN*, f. 768, op. 1, No. 8.

"Swedish Impressions" is Kovalevskaya's review of the status of Swedish society. She pointed out that Sweden was one of the freest of the European countries both politically and socially, nor was there another country (except perhaps for England) where "public opinion" played such a predominant role.

> "Sweden has never been under the yoke of a foreign state, there has never been anything like serfdom, none of her kings has ever been a tyrant, such as Ivan the Terrible or Louis XI, nor even has she suffered from religious persecutions as cruel and implacable as those in the other countries of Western Europe."

That is why the Swedes had developed

> "a reasonable, logical temper that does not tolerate discord between words and deeds nor do they stop at half deeds." [67, p. 289]

It is not easy to convince a Swede to do anything, but if you succeed, he will not stop half-way and will cast his conviction into a concrete form. Sweden cannot boast great natural resources, and this has a special influence on the daily life of her people.

Kovalevskaya showed that there was no economic or social reform that could not be carried out in Sweden

> "once a sufficient number of people were convinced that the reform was needed and desirable." [67, p. 291]

She supported this idea by the example of Stockholm University. Only a few years after it had been founded it was in the possession of several million kronor and a sizable area of land, all of which resulted from private donations. At that time some two hundred thousand people lived in Stockholm, and though there were many well-to-do families, there were no very rich people like in the United States or some of the countries of Europe. Thus several thousand kronor was an appreciable sum for anyone. Kovalevskaya compared this with what had happened in Petersburg, when of lot a effort and subscription from all over Russia resulted in miserable sum of money for the Women's Medical Courses, of cause that had been so much talked about.

There are other two fragments of works by Kovalevskaya

concerning Sweden, that is *A Drama in a Swedish Family* and *Ivar Monson* [67, pp. 303-307]. The action in the first fragment has not been developed at all, therefore how the drama was to unfold remains unknown, but it is very interesting in terms of literature and its perception of Swedish daily life. The second fragment is the story of a fine, healthy young man called Ivar Monson whose fiancée leaves him for another man. In trying to forget his grief, he gets involved with a group of socialists. It would be interesting to know how Kovalevskaya had intended to develop the theme, that is the activity of a socialist group in Swedish society.

A short story called *At the Exhibition* [67, pp. 282-288] gives many details of a preview of the World Exhibition of 1889 in Paris, to which Sofya Kovalevskaya, her daughter and Yuliya Lermontova had been invited. She vividly presented the various types of visitors and the organizers of the exhibiton.

A fragment called *Cupid at the Fair* [67, pp. 295-297] describes a fair in a suburb of Paris that is held annually in May and June. However, the story has not been developed and the Cupid of the title is nuclear because the fragment is too short.

Difficult, strained relations between a man and wife, L and O, are described in a short *Excerpt from a Romance* [67, p. 307]; this is really a fragment of a story and again one cannot make out how the relations evolve and how to explain them.

A fragment called *The Lady of Put* will be discussed when I tell about Maksim Kovalevsky, for the prototype of the lady was Maxim's mother.

Poems were always a special part of Kovalevskaya's creative effort. She wrote verses from her childhood until the end of her life, although she never published them and they were never so intended. But they are essential if we wish to undestrand Kovalevskaya, her moods, her feelings, and intellectual activity.

Together with Anne-Charlotte Sofya wrote a drama called *The Struggle for Happiness* [67, pp. 380-484] (first published in Swedish as *Kampen för Lyckan* in 1887 and in Russian in 1892). The idea behind the drama, its plot and characters were Kovalevskaya's. She would tell Anne-Char-

lotte how the action developed and Anne-Charlotte would then go to her room and write it out.

The drama really consists of two plays: *How It Was*, in which the characters do not follow the call of their hearts but comply with the established customs and prejudices of their millieu, and *How It Might Have Been*, they do things the other way round.

The first play concerns a talented young engineer Karl who was working on an invention that would make labour at a factory easier but which could also cut the number of jobs. The factory is part of the inheritance of a young girl called Alisa. When Karl's father dies, Karl has to maintain the family. Although Karl and Alisa love each other, he does not dare to propose to her while she is forced by her relatives into marrying her cousin Hjalmar and keep the family estate. But Hjalmar is in love with another girl, and after much torment he commits suicide.

In the second play, Alisa finds the courage to leave and divorce Hjalmar. She then allocates her capital so as to develop the factory, and promises the agitated workers, who are afraid they will thus lose their jobs, that she will not dismiss anyone and even makes them share-holders in the company so that they share in the profit.

Although the solution of the labour problem was naive, the drama is still significant as one of the first plays in which workers were portrayed on the stage. It was staged in Sweden, and the second part was put on in Moscow at the Korsh Theatre [216].

Kovalevskaya's comments on the leit-motiv of the drama are interesting. She once asked Anne-Charlotte,

> "Who has not in the course of their life wanted to repent a precipitate action, and who has not more than once desired to start their life again?" [96, p. 215]

Kovalevskaya tried to explain the phenomena of life scientifically, and attempted to compare daily life with Poincaré differential curves. She felt there must be critical points where the curves intersect, and then the additional conditions needed to choose which curve then to follow must be considered. Thus in the life of people, Kovalevskaya thought, their actions are pre-determined, but there may be moments when different options appear, and that life depends on which is chosen.

Kovalevskaya modelled the heroine Alisa on herself, while the other characters had features of the people from the scientific world around her.

Gösta Mittag-Leffler was not happy with the venture Sofya and Anne-Charlotte had plunged themselves into. It was engaging both women totally and was interfering with Kovalevskaya's mathematical work. When he was given a chance to read the first part he severely criticized it. Sofya replied in a letter:

April 1887

"Dear Sir,

"Yesterday I received the manuscript [of the first play] and I am very thankful for the detailed comments you have made. First of all I have to confess that both Anne-Charlotte and I could not resist the temptation to read them before she comes to the end of the second play. At present, she is so exhilarated with our new offspring that, in my opinion, a short cold critical shower is no danger. Besides, I have noticed that any event, however frightful is never as frightful as the expectation of it. Anne-Charlotte knew that you do not approve of our play, and it was much better she knew immediately all the gruesome remarks you had to make. Indeed, it is utterly impossible for me to know that something I am interested in is but two steps away, and which all I have to do is to reach my hand to take it, and yet not do so out of caution.

"I would not be sincere if I did not confess to you that I am very bitter that our poor little firstling has found so little sympathy in you. However, I have to admit that I must agree with you in many respects, and some of your comments seem very just. For a minute I was very surprised that you thought so little of Karl, but after thinking about it more, I found your reaction quite natural. The reason, I believe, is that we have lived in such different environments. I have seen so many "Karls" and been able to study them so closely that I cannot fail to believe him to be a realistic character. You forget that he is a genius, not an interesting person. He is neither an author, nor a poet, nor a publicist. His brain is a fine working tool, but imagination (in the common

sense of the word) he lacks entirely. To personify Karl, I have simply to remember images of people such as Kirchhoff (the inventor of spectral analysis), Huxley, Hermite (the famous mathematician), or Werner Siemens, that is, people who all accept as geniuses, but who, in their private lives, think, reason, and act like Karl. You reprove us that Martha is a caricature, and consider it unnatural that Karl swallows all her banalities. But I can divulge to you a secret: the prototype for Martha is a widow who was well-known and drew the admiration of a circle of Berlin scientists. She was the widow of a very talented mathematician, and she fooled another gifted mathematician. But I can assure you that our Martha still far from resembles the original, and we lacked the courage to reproduce all that widow's ambiguous and naive utterances which flowed continuously from the lips of this charming Professor's widow and which were taken by her learned admirers at face value." [286]

Towards the end of the life, Kovalevskaya was full of ideas for literary work. She told Anne-Charlotte about her projects in a letter dated April, 1889:

"At the present time, I'm working over *Vae victis*, i.e., *Woe to the Vanquished*, as I think you remember. I have in mind another long story—*Les revenants* [Apparitions], which interests me very much. I'd like you very much to give me permission to handle our common offspring *When There Will Be No More Death*. This is my favourite among our brain-children, and I'm mulling over it a lot. I've found a remarkable framework for the book: the Institute of Pasteur, with all of whose institutions I happened to be acquainted; they seem to me to have been specially created to be staged." [190, p. 160]

Anne-Charlotte wrote that Kovalevskaya wanted to serve posthumously her sister Anyuta Jaclard, who had written a play, but had not quite readied it for the stage. However, the play had met the approval of some literary critics and Kovalevskaya shared with Anne-Charlotte her plan to finish the play. Anne-Charlotte has reported that the play has "such an intense, disconsolate mood with such a lucid indigenous colour" [96, p. 279] that it was worth polishing up for the stage in Sweden.

Sofya Kovalevskaya began her literary activity with her theatre notices during the winter season of 1876-1877. We have mentioned that she contributed them to the newspaper *Novoe Vremya*. Kovalevskaya has written that "I tried my literary abilities in this newspaper as a theatre reviewer" [64, p. 144]. The same is testified by E. F. Litvinova [91, p. 46]. At present, we know of ten notices written by Kovalevskaya. Three were discovered by S. Ya. Shtraikh [64, p. 483], one was found by L. A. Vorontsova, and three more by E. S. Smirnova-Chikina. The latter also attributes to Kovalevskaya two other unsigned notices and another signed S.K.-va (which is, probably, a misprint).

Kovalevskaya's theatre notices boldly criticized both foreign and Russian plays, including the latter ones by A. N. Ostrovsky. Kovalevskaya fastidiously evaluated the plot and the actors, praising the acting of Fedotova, Strepetova and Savina; sometimes she would aptly and briefly criticize poor performance by other actors.

During the 1870s, the Alexandrinsky Theatre in Petersburg was in moral decline, its productions of Russian classics were offhand, progressive ideas were being neglected, its repetoire was too full of melodramas and shallow comedies. Kovalevskaya was not indifferent to this, as her notices reveal.

Her first notice described G. N. Fedotova's benefit performance in Ostrovsky's play *Easy Money*. It begins

> "A proof that often the stage success of a mediocre threadbare play much depends on the actors is given by Ostrovsky's comedy *Easy Money*. Although it has had absolutely no success on our stage, it played to capacity crowds twice in Pavlovsk. This was entirely due to the skilful acting of Mrs. Fedotova and the general, concerted, and gifted performance of all the other actors." [27]

The essay ends up with the same idea in other words:

> "This harmonious performance which was intelligent to the last detail, was precisely what was needed to animate such a meagre and inconsistent play as *Easy Money* which was written by the person who had created *The Storm* and *Poor Bride*.

Kovalevskaya also severely criticized another of Ostrovsky's play *Truth is Good but Happiness is Better* [32]. She did not deny the merit of the comedy inherent in Ostrovsky's plays and felt the language to be good, with many witty expressions and funny scenes. However, she thought the play dull and many scenes long-winded. In her evaluation of the plays Ostrovsky wrote in the 1870's, Kovalevskaya held opinion most contemporary critics held too.

Conversely, Kovalevskaya greatly commended an earlier drama by Ostrovsky, *Vasilisa Melentieva* [33]. She called it magnificent and believed the drama would both fascinate the audience and become engraved in the memory of anyone who saw it. Her view of the performance of the actress A. M. Dyuzhikova, who played the role of Vasilisa, was that although she had acted better than everyone else "she could not for all her appearance and physical powers" meet the demands set by the role for which an actress must have "apart from beauty, ineffable charm and feline elegance that are more dangerous and more rare than beauty".

Kovalevskaya classified a play by A. F. Pisemsky called *Bitter Destiny* [31] which had by then been staged for fifteen years as belonging to the "denunciatory" genre. The characters were a landowner possessing serfs, a bribed official, a cowed wife, and a philosophizing hero; and Kovalevskaya was amazed that such a play could have been staged for so long. Having described the plot, she then commented on the performance of the principal heroine, admirably played by P. I. Strepetova, and in comparison with whom all the other characters seemed "stilled, lifeless, and unnatural". Kovalevskaya added

> "As to a Russian peasant woman, to whom Nekrasov has said, 'Destiny three arduous lots has given', etc., I do not think it possible for one to be played more sincerelly or more simply than did Mrs. Strepetova."

Kovalevskaya was in two minds about Aleksei K. Tolstoy's play *Posadnik*, which he had left unfinished and which was only published after the author had died in 1875. On one hand, she was a nihilist and democrat and so could not hold with "the stamp of routine philosophizing" that ran through all Aleksei Tolstoy's work.

On the other hand, Kovalevskaya believed the play

possessed notable merits. The action takes place in Novgorod, at the apex of the city's prosperity and freedom. The leit-motiv of the play is patriotism for his city and the love for freedom the hero Gleb Mironych displays. He was "posadnik", or governor of the city-state elected by the citizens, and he was ready to sacrifice everything he had for Novgorod. Kovalevskaya wrote:

> "This poweful, grand personality, which could only develop in the old free city of Novgorod is admirably outlined by Count Tolstoy."

She felt the other characters had only been sketched

> "but from what we see in the drama as it was begun, we can only believe that but for its sudden end, it would have been much brighter."

Kovalevskaya did not believe the actors' performances good:

> "Mr. Leonidov spoilt, as much as was possible, the excellent role of posadnik by his monotonous howling."

And then:

> "Savina's dress was very pretty..., but it was obvious that she was not good enough for the role." [36]

Though in another notice: Kovalevskaya wrote,

> "Mrs. Savina was charming, and Mr. Sazonov played seriously and intelligently, as is proper in a comedy." [35]

Kovalevskaya only criticized Savina, then a young actress, slightly because she acknowledged her doubtless merit. In one notice she mentioned the actress Stremlyanova, Savina's younger sister, as being talented but overshadowed by Savina; Kovalevskaya recommended Stremlyanova to pick another genre different from her sister's.

The theatre performance always gave Kovalevskaya the opportunity to discuss more general problems, to champion democratic principles, and to discourse on Russian life or the Russian character. Kovalevskaya commented on the style of Russian actors and compared it with that of foreign ones. She noted that Russian actors were genuinely sincere,

but that they lacked "plasticity" when they performed in French plays. If the villain is punished, a foreign audience is satisfied, but a Russian one will argue, "But what then? What is the outcome, and why such strong measures?" [30]

In her essay on *Posadnik*, Kovalevskaya discussed historical plays in general and defended them even though it is impossible to recreate the spirit of the epoch.

In 1876, Vladimir Kovalevsky wrote an article for *Novoe Vremya* [64, p. 339] in which he appealed to help the fraternal Slavonic peoples fighting for freedom against the Turkish yoke. He proposed that a Slavonic committee be organized to raise funds among the population at large. Shtraikh believes that Kovalevskaya participated in actually writing the appeal [64, p. 5].

It is interesting that Kovalevskaya's literary work is largely autobiographical and reflects her social views. She had always been quick to respond to social events since her childhood. In *A Nihilist Girl* one character, no doubt, Kovalevskaya herself, is seen listening to peasants discussing the coming abolition of serfdom and learns much about the actual life of people. In her youth, Sofya was strongly influenced by her sister Anyuta who had been fascinated by nihilist ideas. Strannolyubsky had inculcated in Anyuta the enlightened outlook of "the people of the sixties." Vladimir Kovalevsky had once visited Garibaldi, was familiar with many revolutionaries, and had assisted them. He had then introduced Sofya Kovalevskaya.

When Kovalevskaya was in the Paris Commune, she was not an active participant, but her sympathies were on the side of the Communards. Later she recounted to Anne-Charlotte Leffler how difficult it had been to reach Paris: when it was under siege they had gone on foot part of the way, and then rowed a boat along the Seine risking being shot at and killed. In Paris, Sofya had with the other women (some of whom were Russians) attended the wounded in hospital. Kovalevskaya wanted at one time to describe the experience.

"... while the shells were falling, and as more new wounded were carried to the hospital, the girls would whisper to each other sharing their recollections of their past lives, lives in sharp contrast to the present." [96, p. 121]

Who influenced Sofya Kovalevskaya in the early 1880's, when she was in Paris, and then in Berlin?

When she was in Paris in the early 1880's Kovalevskaya met Petr Lavrov. He was an old friend of the Korvin-Krukovskys, and had as a Professor of mathematics visited the family. Then she had emigrated becoming a revolutionary, and an advocate of terrorism in the struggle against Russian autocracy. In 1881, Lavrov met the German social democrat Georg Vollmar, who had until the late 1880's, been the editor of the magazine "Der Sozial-Demokrat." Kovalevskaya met both Vollmar and a Polish revolutionary called Maria Jankowska at Lavrov's. Lavrov also knew Jaclard (Kovalevskaya's brother-in-law) and the other activists of the Paris of Commune, such as Brousse, Malon, and Lissagaray.

Before long, Sofya Kovalevskaya had become good friends with Vollmar, and they wrote to each other between 1882 and 1883 [217]. Vollmar then had the reputation being a staunch revolutionary, but later he became an opportunist.

On April 2, 1882, Kovalevskaya wrote to Vollmar in connection with the First International (International Workingmen's Association), which by then had ceased functioning.

> "Don't you think that the time has again come to call such an institution into being, only one more strictly organized and with more definite goals? I have become firmly convinced of this by observing our Russian emigrants, whose inactivity ruins them. Nonetheless, don't you think that these emigrants with their indisputable energy could render valuable assistance (under good guidance) here in Western Europe too? I have frequently indulged in fantasies about this recently, although I'm afraid that you, an active fighter, will find my fantasies impractical!" [64, p. 262]

In a letter of May 4, 1882, Kovalevslaya told Vollmar about her mediations as to whether to be engaged in science, which was only of interest to a narrow group of people, when it was sufficient to open one's eyes for

> "the indignation at injustice to be seen all around to become so enormous that every other interest pales before the great economic battle that is unfolding before us,

> and the temptation to step into the ranks of the fighters will become very strong."

She then told him that until recently she had dedicated herself entirely to science and that during the Paris Commune she was too young and too much in love with her science to be aware of what was going on.

> "Although I thought myself a socialist (in principle though with certain reservations), I have to confess that the resolution of the social problems seemed to me to be too far away, unclear, and undeserving of the attention of a serious scientist, who has more important things to do." [64, p. 263]

However, Kovalevskaya changed her mind after she had met socialists of different nationalities during her five-month stay in Paris.

> "The problems of theoretical socialism and reflections on the methods of practical struggle occupied my mind so powerfully and persistently that I could hardly make myself think of my work, which is so far away from the realities of life. At times, I could rid myself of the excruciating realization that what I was giving my every thought and ability to could only interest a few people, while each of us should devote the best we have to the masses." [64, p. 264]

It is often thought that Kovalevskaya was a utopian socialist. I do not think this was true. She sided with the social democrats of the time, for at least, Vollmar, a leader of the German social-democratic party, considered her to be one of "theirs". He wrote once that Kovalevskaya "was a convinced socialist", and during the terrible period when the "extraordinary" laws were in operation her path

> "led her not uncommonly from the official society that celebrated her to the persecuted social democrats she was friendly with". [185, p. 845]

A natural question concerns the life Kovalevskaya would have chosen had she not known Mittag-Leffler. We know that besides Vollmar, who influenced her greatly, she knew other socialists well, such as Maria Jankowska and Joseph Perott.

Maria Jankowska, neé Zaleska, and Mendelson by her second husband, was the daughter of a wealthy landowner in the province of Kiev [218], a Pole by origin. Her father had ordered her when sixteen to marry Wladislaw Jankowski, a rich Pole from Khodorov in the Kanev district of Kiev province. She had been greatly influenced by L. Waryński, an active Polish socialist who in 1882 founded "Proletariat", the first revolutionary party of the Polish working class. In 1883 he was arrested and died six years later in the Shlisselburg fortress.

Through Waryński, Maria Jankowska had become familiar with socialism, and fascinated begun to read Proudhon and Steward Mill, and then gather information about the Paris Commune. Eventually she took a train to Geneva to look for the "President of International". On meeting a member of International, she told him that she would like to help the workers' movement. "How, in this lace and silk?" she was asked. She blushed and felt miserable, but she was accepted into International and plunged into the Polish social movement, contributing large sums of money.

In 1880 Jankowska took part in the illegal meetings of the Warsaw socialists, and the tsarist police began to search for her. In August 1881 she was writing socialist propaganda, and by September 12 she was arrested with Stanislaw Mendelson. She was transferred to Poznań, where she was imprisoned pending investigation, and on February 20, 1882 sentenced to three months in prison inclusive of the term she had served before the trial. There was only one month more to serve, but the court transferred her to the Tsarist authorities. Her relatives helped release her, and she went abroad. In 1889 (several years after her first husband died), she married Mendelson, who later helped found the Polish socialist party. Maria worked for the Polish magazines *Rassvet* (The Dawn) and *Borba klassov* (The Struggle of Classes) using the pen-name of Leonovich. She was one of the participants in the Paris talks between the parties "Proletariat" and "Narodnaya volya" ("People's Freedom").

In the late 1880's and early 1890's, Maria took up the position of a right opportunist and together with Mendelson helped organize the Polish nationalistic and reformistic socialist party. Both she and Mendelson later retired from politics, and returning to her homeland she died in 1909.

Kovalevskaya first met Jankowska in 1882 in Paris at the home of P. Lavrov, and they soon became friends. Jankowska's recollections of Sofya Kovalevskaya are very interesting. She made one striking remark:

> "Each of Sofya's numerous friends preserved in his or her memory a different image, because to each she presented herself in a completely different light. But there was not the least falseness in this; it was just that her rich nature gave to the person who interested her at that moment exactly what seemed to her to suit that person. Her interest in the given personality was exclusive; it did not permit her to involve herself with others at the same time" [218, p. 136] and [284, p. 164].

These reminiscences contain several letters from Kovalevskaya. Reading them superficially, it is easy to think them the flippant correspondence between two society ladies. They discuss the "blue bird", which implied love, and in one of her letters Kovalevskaya wrote:

> "As to my private life, you just cannot imagine how dull and uninteresting it is. Of the birds, the best I can boast is an owl. Indeed, an owl is a nice and gentle creature, and one should not neglect it." [64, p. 279]

When on another occasion, Jankowska asked her about her love affairs, Kovalevskaya answered that in Sweden all young men are born married, and her admirers are all venerably old: the three of them totalling more than 200 years in age. One wrote poems to her, calling her a Prometheus bringing light to Stockholm's Lake Melar. This was the English mathematician Sylvester who also wrote a sonnet praising a lady who had sung at concerts in Oxford, and another lady, "whose star over Melar is shining" [78, p. 161].

But there is also a serious note in the correspondence, and a very sad one: Jankowska sent news of the arrests of friends Kovalevskaya knew and sympathized with. As we know, Kovalevskaya had been sympathetic to Poles since her childhood, and she took sides with the Polish revolutionaries with all her heart and was ready to help them. Thus, she gave her passport to Zinaida, Joseph Perott's sister, a deed that required great courage.

On one occasion, Kovalevskaya was able to help Perott

himself. Living in Paris in the Autumn of 1882, she telegrammed Mittag-Leffler on October 18 in Stockholm asking:

> "Do me great favour and give Perott, on board the steamer *Norra Sverige*, one hundred francs. Letter and money follow, Sofya Kovalevskaya." [SK 7]

It turned out that Perott had got into trouble and the affair was covered in the Finnish newspaper *Kaiku*. The article was titled "Mistaken for Nihilists", and it ran,

> "A few days ago we were informed from Tornio that a young pair had been arrested on suspicion of being fugitive nihilists. They were betrayed by the boatman who had agreed to take them to Sweden. Before being arrested, the pair had been to Uleåborg... where the police retained them in a room for visitors from October 8 to 11."

Although his passport was in the name Joseph Perott and the girl had a school certificate from Petersburg, the police still suspected them of being nihilists. Their grounds were extremely strange, for

> "possibly, one reason was the beauty of the girl, who had black hair and rosy cheeks."

Following an exchange of telegrams, the authority withdrew their objections. But the *Kaiku* still believed the young couple were fugitives, a wretched pair in love, whose parents would not consent to their marriage.*

In fact, it was Perott and his sister Zinaida. (Sofya Kovalevskaya is alleged to have advised her to take up the natural sciences believing that she would make another Claude Bernard.)

The hundred francs did not last long, and on November 5, 1882, Perott wrote to Kovalevskaya:

> "Dear Madame Sophie, I have received your letter of November 1... Lack of money delays us in Stockholm. We succeeded in crossing the border only after twelve days of travel, and our clothes are a sorry sight. My sister has only one boot worth anything." [P 5]

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* *AAN*, f. 603. No. 45.

He requested Sofya Kovalevskaya to forward him further five hundred francs which would do until he received the money from "Crédit Lyonnais".

Alongside the telegram of October 18, 1882, Kovalevskaya sent Mittag-Leffler a letter explaining that Perott, a mathematician,

> "had to leave Russia urgently, the reasons being very respectable, but not political," [SK 7]

and was in the meanwhile without money. The reasons were undoubtedly political, but exactly what they were remains unknown. It is also unknown to which organization Perott had belonged, but it was possibly the same one as Jankowska.

Unfortunately, the first work of fiction about which Kovalevskaya told Anne-Charlotte has not been found. In 1877 she wrote a novella about the life of German scientists called "Der Privat-dozent". Sofya Kovalevskaya had told her that she wanted to work on her first fictional work "Der Privat-dozent" and added:

> "I think that if I rewrite it completely, I can produce something remarkable. In fact, I'm a little proud that, even being as young as I was then, I could so well comprehend certain aspects of human life. When I analyzed the feelings of E. to G., I think I described the relations between my privat-docent and his Professor really rather well [96]. And what a fine way this might be to advocate socialism! Or anyway for developing the thesis that it is a democratic but not a socialist country that would pose the greatest threat to the world!" [190, p. 145]

The second part of this passage is cited by Anne-Charlotte Leffler in the Swedish edition of the book [190] but omitted in the Russian translation [96].

I wonder how Kovalevskaya would have substantiated the latter idea. In 1888, when this conversation with Anne-Charlotte took place, Kovalevskaya held the views of socialism, but to what degree is unknown. Mittag-Leffler said that his sister had become a socialist, and that he did not object so long as it did not interfere with her literary reputation. At the same time, as we saw, Gösta feared for Kovalevskaya and entreated her not to reveal herself as a

nihilist. Evidently, Russian nihilists were universally considered as basic iconoclasts. Besides, Mittag-Leffler probably had a specific cause for worry because Georg Vollmar claimed that in 1889 Sofya Kovalevskaya participated in the First Congress of the Second International (Socialist International) as a representative of the women's league.

I have already mentioned how Kovalevskaya has shone in Swedish society, and that she had become a member of the literary clubs "Nya Idun" and "Heimdall", etc.

About 1885 Kovalevskaya was elected to the journalists club in Stockholm. Anna Branting recollected [219] that as soon as it happened, Axel Jederin, the editor of the newspaper *Svenska Dagbladet*, informed the club that he did not want to remain a member of a club according Russian nihilists. But generally, the members were tolerant of the political views of fellow members, and Anna Branting recalled that ultraconservative editors could become freethinkers at club gatherings.

Sofya Kovalevskaya was at the same time working intensely and at the highest scientific level. Her professorship required a great deal of effort as well.

We can only wonder how she could manage so many vastly different duties at the same time, for she was a scientist, editor of a scientific journal, writer, social activist, mother, and society lady. It appears that her friends tried as much as they could to hide from public view that she belonged to socialist organizations, and possibly prevented her from especially dangerous actions.

Chapter 8

LAST YEARS OF LIFE

Sofya Kovalevskaya met Maksim M. Kovalevsky in Paris through Petr L. Lavrov, most probably in 1882. The acquaintance was short.

In 1884 a curious incident made Sofya Kovalevskaya angry with Stockholm postal service. In the autumn (we know of this from a letter she wrote to Mittag-Leffler dated August 26, 1884) she went from Södertelje (the suburb where she lived in the summer) to the Stockholm central post office. She was handed a package of letters and books addressed to M. M. Kovalevsky. Naturally, Sofya refused to take the package and explained to the manager of the post office that letters addressed to Professor Kovalevskaya should be forwarded to Södertelje while this package should be kept for Professor Kovalevsky in Stockholm. However, on the following day she was handed the same unfortunate package in the suburb. "I have written to the Stockholm central post office again, but I don't know what the result will be" [SK 231]. Whether or not Maksim Kovalevsky arrived in Stockholm, Sofya Kovalevskaya did not see him.

The Swedish economist V. E. Lorén left a sum of 200 thousand kronor to be used for the development of the social sciences. The Lorén Fund Committee included the Rector of the Medical School, a well-known physician A. Key (chairman), Anne-Charlotte Leffler, and Sofya Kovalevskaya. Anne-Charlotte had been a great friend of Lorén and had supported his ideas.

In 1887 one of the lecture invitations made by the Lorén Fund was on Kovalevskaya's initiative awarded to Maksim

Kovalevsky. He had been Professor of state law at Moscow University, but that year had been dismissed.

Maksim Maksimovich Kovalevsky had already had a magisterial degree and was a university professor by the time he was 26. He soon became a very popular lecturer and is reputed to have said once to his students,

> "I have to give you lectures on state law, but since there is no law in this country, what shall I tell you?" [73, p. 185].

Having been informed of his behaviour, the Minister of Public Education I. D. Delyanov dismissed Maksim Kovalevsky from the University on June 6, 1887. Maksim Kovalevsky then went abroad for more than 15 years working and lecturing in Stockholm, Paris, Oxford, Brussels, and Chicago. On one occasion he was invited by Great Britain to be an arbitrator in an international dispute with the USA [178].

In 1901 Maksim Kovalevsky founded in Paris the "Russian Higher School for Social Sciences", which he supported with a great deal of his own effort and money. At various times he invited Georgy V. Plekhanov, K. A. Timiryazev, among others, to deliver lectures at the School.

In 1904 Maksim Kovalevsky returned to Russia to lecture and was elected a Member of Petersburg Academy of Sciences in 1914. He was also member of various other academies in many countries.

Maksim Kovalevsky became a Member of the State Council and spoke on behalf of the academic group. Anatoly F. Koni has written one account of him for he was his neighbour in the session hall of the Upper Chamber.

Maksim Kovalevsky was a materialist, and a proponent of evolution. He met Marx and Engels on more than one occassion and corresponded with them. (An essay about him as a sociologist had been published by I. G. Liorentsevich [177].)

When Maksim Maksimovich Kovalevsky died in 1916, a great crowd of students and progressive activists gathered at his funeral at the cemetery of the Alexander Nevsky monastery in Petrograd*.

* St. Petersburg became Petrograd in 1914 when the First World War began.

Maksim Kovalevsky was a wealthy landowner, his estate of Dvurechny Kut was 20 versts from Kharkov, near the railway stop of Peresechnoe. In the unfinished novel "The Lady of Put" [67, p. 297], Sofya Kovalevskaya described Maksim's mother as a beautiful and elegant widow, and Maksim himself as a student returning home on holiday. The appearance she gives him, given the age change, is similar to her description of Mikhail Mikhailovich Zvantsev in "A Fragment of Romance on the Riviera":

> "His massive head was very handsomely set on his shoulders and very original... The most beautiful were his eyes, very large even for his large face and blue with black eyelashes and black brows. Although his hair receded each year, his forehead was also handsome, and his nose remarkably straight and noble for a Russian. The lower part of his face was not so good, his cheeks were too great and his lower jaw too developed. This disadvantage was compensated by a small greyish black French beard. When he was anrgy his lower lip and jaw would suddenly jut out and his face would become quite fierce. But most of the time, all Zvantsev's friends agreed, the prevailing expression on his face was good-natured" [67, p. 214].

Let us return to the time when Maksim Kovalevsky was invited by the Lorén Fund Committee to Stockholm to deliver a lecture series. According to the rules of the Committee, a candidate had to have recommendations from two prominent scientists or scholars. Two scholars responded positively to the Committee's inquiry, one of them approving of Maksim Kovalevsky's invitation even though his outlook was very different [64, p. 396].

Maksim Kovalevsky appeared in Kovalevskaya's correspondence for the first time on February 2, 1888. Sofya sent Gösta a message:

> "Dear Gösta, will you go skating today? I'd like to go, but not earlier than half past three, to have more time for work... It seems that in Russia they also believe that Maksim Kovalevsky and I are relatives, as you will see from the telegram I received today and enclose" [SK 240].

The telegram from Petersburg read:

"University Professor Kovalevskaya Stockholm Congratulations great gold medal Petersburg Geographical Society the Kovalevsky family Maikov" [SK 240A].

Several days later (the letter is undated) Sofya wrote to Mittag-Leffler that she had heard good news from Maksim Kovalevsky: he had got his foreign passport and would be in Stockholm a few days later. He had just received the gold medal from the Petersburg Geographical Society for a new book which had received many complimentary comments in foreign journals. This was possibly Maksim Kovalevsky's "Modern Customs and Ancient Law", published in 1886 in Moscow (in Russian).

Maksim Kovalevsky arrived in Stockholm in February 1888, and Sofya was glad to meet a compatriot and she sent him a note immediately [64, p. 229]:

"Sturegatan 56, 4 trapper*

"Dear Maksim Maksimovich,

"It's a pity we do not have in Russian the word *välkommen*, for I'd like to greet you with it very much. I am very glad you have arrived and hope that you will visit me immediately. I'm going to be at home until 3 o'clock. In the evening, a few friends will gather at my place and I hope you can come, too.

Sincerely Yours,
Sofya Kovalevskaya."

Mittag-Leffler, Gyldén, Ellen Key, and Branting, the chairman of the socialist party in the Swedish Rigsdag, were among the guests at Kovalevskaya's gathering that evening. Anne-Charlotte was not in town, otherwise she would have come as well because she had already met Maksim Kovalevsky in London.

The lectures on the social sciences were delivered at the University of Stockholm by Maksim Kovalevsky and Professor Beauchet from Nancy, who was at the time studying the sources of ancient Swedish law. As a member of the Lorén Fund Committee, Sofya Kovalevskaya attended

* Four stairs, i.e. the fifth floor.

both lecture series and invited the lecturers home. She took care that they would be accepted in the scientific and literary society of Stockholm.

Some 230 people attended Maksim Kovalevsky's first lecture. Kovalevsky was greatly applauded, and very favourable comments appeared in the newspapers.

His theme was the transition of a matriarchal family community into the modern family. He discovered such secular family communities among peoples in the Caucasus, Serbia, and Bulgaria. In 1890 he published in Stockholm a book in French on the development of the family and property [179]. The dedication on the first page is "*A madame Sophie Kovalevsky*", and the same dedication, "To Mrs. Sofya Kovalevskaya", appears in the later Russian edition of this book [180].

In Stockholm Maksim Kovalevsky met the famous Swedish explorer Nordenskjöld, who was the first to cross from Europe through the Arctic Ocean to the Sea of Okhotsk, Scandinavian archeologists Hildebrand and Montelius, and other scholars, politicians, and writers.

In his memoirs of Sofya Kovalevskaya, Maksim Kovalevsky tells that he made a report on the archeology of the Caucasus at the home of Montelius and then listened to a good presentation by Sofya Kovalevskaya in Swedish.

The day after his departure from Stockholm, in March 1888, Sofya wrote to Anne-Charlotte that had Maksim stayed in Stockholm longer, she would have been unable to finish her memoir for the Prix Bordin. She wrote [64, p. 300]:

> "He is so huge and takes up so awfully much room, and not only on the sofa, but in the thoughts of the others, that it would be absolutely impossible for me to think in his presence of anything other than him."

Sofya and Maksim met again in London in the Summer of 1888, where Maksim was staying with Professor Gambarov. The three of them arranged trips to the country, visited museums and picture galleries. But Sofya anxiously wrote to Gösta that she had more work to do on her memoir for the Prix Bordin, and she had to go and stay with Weierstrass, but that she did not know when and how she could

do so. It is obvious that Maksim began to occupy her mind more and more. She wrote [SK 248];

> "I am making no plans, I feel well, and I live from one day to the next. I'd like to be like Anne-Charlotte, appearing so resolute and sure of what she wants or doesn't want. But unfortunately I am another person and at bottom I like only indefinite tints. 'An idea that is spoken is false', a Russian poet* has said, and this is what I now deeply feel."

Sofya ended the Summer 1888 in the Harz with Weierstrass. Maksim also went to the Harz and lived there for a while. He had to leave for a week, and when he returned, he "was struck with a physical change in her; her face had grown thin, her eyes had sunk, and she slimmed noticeably" [64, p. 401]. It turned out that she had worked very intensely during the week and essentially finished her work for the Prix Bordin.

According to Maksim, they met in Stockholm as true friends in the autumn. He was all the more convinced of her various gifts, her interests were deep and she was quick to perceive the new ideas. Sofya Kovalevskaya now paid more attention to her literary work.

In 1889 her book *Ur ryska lifvet. Systrarna Rajevski* [43] was published (a changed version of *Memories of Childhood*). She presented Maksim with a copy** having made the inscription:

> "To Maksim Maksimovich. In memory of an unlucky trip to Monte Carlo, after which he gave me the idea to write 'recollections', and in memory of many other things, from his faitful friend
>
> Sonja Kovalevskaya."

Although in one letter Sofya Kovalevskaya wrote that she had been encouraged to write her recollections by M. M. Kovalevsky and Professor I. I. Ivanyukov, to both of whom she told of several episodes from her childhood, this inscription reveals that she had been mainly induced to do so by Maksim Kovalevsky.

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* F. I. Tyutchev (1803-1873).

** *AAN*, f. 603 (donated by P. E. Kovalevsky).

The year 1889 started very badly for Sofya Kovalevskaya: she had fallen out with Maksim and he had left Paris for Nice. She wrote to Mittag-Leffler on January 3, 1889 [SK 320]:

"Dear Gösta,

"I feel so guilty for not writing to you these last few days. But I spent them in such a feverish condition that I completely lost my head. As a true egoist, I shall start with a discourse about myself and of what interests me most at the moment. I do not know the state of my affairs in Paris nor, above all, how everything will be settled in future.

"I am afraid that the difficulties will be greater than I suspected. I talked to Hermite, who spoke then to Bertrand, but it seems to me that Bertrand does not think I can get a position in Paris soon. I shall have first to go to one of the provinces, and this does not attract me at all.

"I can confess to you sincerely that most of all I would like to have a leave of absence for the coming semester, stay in the spring where I like and have a chance to focus and concentrate my thoughts. You cannot imagine the nervous state I am in now. If I return to Stockholm at this moment and have to work again, I shall wind up seriously ill.

"On the other hand, I am unable to take any definite decision now. You have always been my kind and faithful friend, dear Gösta, but I assure you, that if you could get me a leave of absence for the coming semester, it will be the greatest service you have ever done for me. I shall send you a certificate from a doctor, saying that I absolutely need a rest now, and do not exaggerate. I am ready to let the person who will substitute me (Phragmén) have a half of my salary.

"I know that this leave will be difficult in view of the coming end of my professorship in July, but even anticipating this danger I cannot resolve myself to return to Stockholm. I am asking you, my dear and kind Gösta, do not get cross with me for this. I assure you that at the present moment I am unable to renew my work. Now I shall not annoy you any more. I'll send

you my doctor's certificate as soon as I can. Please, answer me immediately.

"I can frankly admit that I have somewhat neglected my social obligations lately. But today Kovalevsky went to Monte Carlo for 10 days to see an ailing friend of his, and I hope to become reasonable again.

"One more request, dear Gösta. Could you go to the Grand Hotel and ask what they have done with a large trunk that should have been sent to 78 Renn street. Kovalevsky made a mistake in the number. It should be 78 and not 178 as he said. Be so kind as to ask the maître d'hôtel to find the agency through which the trunk was sent and to find out what Kovalevsky should do to recover his things.

Devotedly Yours,
Sonya.

"Tomorrow or the day after I'll write you a more interesting and more reasonable letter. Please, help me to get a leave for the next semester. It's the only way for me now that will give me the time to soothe my nerves and spare me the need to take a decision. I wish nothing better now that to get a leave."

Gösta was sorry and ready to make every effort to help Sofya overcome her unfortunate condition. On January 9, he answered [ML 66]:

"Stockholm, Jan. 9, 1889

"Dear Sonya,

"I have just received your letter. Without delay, I want to answer that, of course, I'll get you a leave of absence. If you need it, then it is clear that it should be obtained. I would not like to conceal from you that this is a new and considerable predicament, but I have now so many cares and troubles that to add one more is like adding unity to $+\infty$. However, I want you to do the following:

"1. The certificate should be from one of the most well-known doctors in Paris, best of all, for instance, from Charcot. They know him well here through Nordenson, who lived in Paris for several years. Anyway, it should

be from one of the most outstanding neuropathists. I do not doubt that you can get such a certificate.

"2. Contact the Swedish embassy in Paris— a letter to Count Löwenhaupt, the ambassador of Sweden and Norway in Paris will do—and find out whether or not there is a Swedish or Norwegian doctor in Paris at the present moment. Write to him. When you obtain the certificate from a Paris doctor, ask him to visit you and give you a certificate as well.

"3. Do not tell anybody in Sweden that you intend to take a leave of absence until I give you my permission to do so.

"I hope you will not make it impossible for me to obtain your leave by neglecting these three points. They are very easy.

"Send me the doctors' certificates as soon as you can. They should be as serious as possible.

"It was intended to meet you here with great ovation*, but this does not matter any more. You haven't got any congratulations from us because we wanted to delay them until the meeting of the thirteen that should have been held on Monday. But then Loevén fell ill, and the meeting was postponed. Yesterday I visited the king. He talked about you with great admiration.

"I'll write to you more next time. But if you want to bring your friends, especially me, some good news, write at least sometimes about your life and your circumstances.

"Little Sonya is quite well.

Yours, Gösta."

Then he supplemented the letter by a rough copy of her application to the University Board concerning her leave of absence. On January 12 Sofya Kovalevskaya answered Mittag-Leffler with a thankful and emotional letter [SK 321]:

"Saturday, January 12, Paris

"Dear Gösta,

"I've just received your nice letter. How grateful I am for your friendship. I really believe it to be the most

* As the Prix Bordin winner.

precious my life has given me. And I am so ashamed that I am doing so little to give you pleasure and show you that I treasure this friendship. Don't get cross with me for that, dear Gösta. I really have lost my head now. I get letters of congratulation from everybody, but the strange irony is that I have never in my life felt so miserable as I feel now. I am unhappy as a dog, no, I think dogs cannot be as unhappy as people can, especially women.

"I am writing to you about this, dear Gösta, so that you can see the state I am in. But please, don't tell anybody else, even Signe. You know how heavy it lies on my conscience that I am talked about. Maybe I'll become more reasonable in a while. I'll strain every effort for that. I'll start working again, get interested in practical affairs, and then I naturally will take all your instructions as a guide and do whatever you please.

"At the present moment the best I can do is to keep my melancholy to myself, so that my errors would be little known in society and nobody would talk about it much.

"I've got many invitations during the last week—to Bertrand (Count L[öwenhaupt] told me that he described this dinner in detail in his letter to Stockholm, and that he wrote to you so that you would be able to see the letter first-hand); then to Menabrea, to Count Löwenhaupt, at whose home I was in the company of Prince Eugene, etc., etc.

"But I am too desolate now to give you details about the dinners. I'll do it some other time. When I come home, I do nothing else but walk about the room. I suffer from insomnia, no appetite, my nervous system is a wreck. I am even unable now to take care of my leave. Probably, I'll resolve to this next week.

"Good-bye, my dear Gösta. Save your friendship for me. I need it very much, I assure you. Kiss Fufa for me and give my gratitude to Signe for her care of her.

Devotedly Yours,
Sonya."

Mittag-Leffler hurried to answer her on January 15 and started his letter [ML 67]:

"Dear Sonya,

"I've just received your letter of 12 Jan. I am so sincerely sorry for you. If I could only help you, there would be no man more ready to do so than I. Believe me, ... it is absolutely impossible that two people, especially if one of them is a man and the other a woman, could be in an absolutely equal mood as far as their feelings are concerned, however equal the grounds for their feelings might be."

Then he wrote that he would willingly come to Paris, but he could not leave Stockholm at the moment, and reiterated his instructions to visit good doctors for nervous disorders, recommending especially August Voisin, and forwarded his address.

He wrote about Sofya's daughter, who had been left in the care of Signe, that she was well and cheerful and that she had gone to Uppsala with them and it had given her much pleasure.

A meeting of the thirteen unanimously decided to put down in the minutes that they were delighted she had been awarded the Prix Bordin. They were going to congratulate her when she arrived in Stockholm, but her arrival had been postponed, and a telegram to Paris was too late to be sent.

On January 17, Kovalevskaya sent a calmer letter. She spoke of her talk with Bertrand about her work in Paris. Bertrand had answered that the greatest obstacle to her acquiring a position in France was that she was not French. She had to obtain French citizenship. A compromise came to her mind: it was sufficient for minor naturalization to stay six months in Paris, which she could do if she obtained a leave of absence for the coming semester. In the meanwhile she would go on with her investigations of the rotation problem, and write a good paper that would be her doctoral dissertation. She could defend her dissertation in Paris gloriously, and if she did not get a proper position in France by that time, she could return to Sweden in the autumn. But she believed that the French mathematicians would be obliged to give her a position in France.

At the end of her letter she wrote [SK 323]:

"I think and think and cannot resolve. Please, dear

Gösta, write to me as soon as possible what you think about this project. I am very much disposed to follow your advice. More than ever, I feel the need now of the guiding hand and the support of a friend. Thank you, Gösta, for your friendship. I feel it is better now not to start with efforts to get a leave for the whole semester, but to send you a health certificate that I have to spend two or three weeks more here. My nerves will get better during this period, and it will be clearer what will be the right thing to do... Do you know that Hermite was unanimously elected Vice-President of the Academy?

Devotedly and thankfully Yours.
Sofya Kovalevskaya."

Naturally, Sofya greatly distressed Mittag-Leffler when she revealed to him her plans to leave the University of Stockholm. Although he didn't want it, he still acted as a true and generous friend and aided her in her plans.

Two days later Kovalevskaya wrote that she had received his telegram and had consulted Voisin. He found that her nervous system was truly disturbed and insisted that she should be treated.

"On the whole, I feel much calmer now," she wrote, "and I am ashamed of the silly letters I wrote some days ago" [SK 324].

Step by step, she began to attend more to Mittag-Leffler's interests. In a telegram of January 24 she informed him:

"If Hermite gets an award, Bertrand should also get one." [SK 325].

Mittag-Leffler was at the time trying to get Swedish awards for the French mathematicians and thanked Kovalevskaya for the information. He wrote on January 27 that on the day before he had talked with a well-known Swedish doctor named Key and that he had advised him not to present Voisin's certificate to the University, because it was too serious. It would be better to present a certificate from the Russian physician P. I. Yakobi, who was living in Paris, but to be treated by Voisin, and then go to the mountains in the summer to relax. She should ask Voisin how much he would take for his treatment.

"Tell him the whole truth about your economic situation, and he will set his fee on this basis," wrote Gösta, "he told me himself that this is the accepted practice. Don't be concerned for him he is extremely rich." [ML 68]

But Mittag-Leffler still hoped that Kovalevskaya would stay in Stockholm and paved the way for her reelection to a new five-year term:

"I've succeeded in collecting a fund of 50,000 kronor. The interest (4%), which is 2000 kronor a year, plus a further 2000 kronor from the city authorities will go for your salary."

He expected a further increase in her income later. He intended to get reviews of her work from such authorities as Helmholtz, Sir William Thomson, and Beltrami,

"and then," he wrote ironically, "it will be a great joy for Retzius and company to see that it is not just the 'little group of Weierstrass, Hermite, and Poincaré' that regards you so highly." [ML 68]

Beltrami, Bjerknes, and Hermite wrote the necessary reviews, and Leche and Ugglas also made presentations for her nomination. Mittag-Leffler succeeded, and Sofya Kovalevskaya was elected Professor of the University of Stockholm *á vie* (for life), and she could now choose whether to stay in Stockholm or not.

In late January 1889, Mittag-Leffler sent Kovalevskaya several angry and anxious letters. She had carried out his "diplomatic" assignments poorly and late, and had put him into an awkward position. Thus on January 29, 1889, he wrote:

"Dear Sonya,

"First of all I have to scold you for not informing me what had happened in Paris from the very beginning and not answering my questions properly. If you had done so, I could easily have settled the incident involving Bertrand, but it has proved entirely impossible to do it right away by telegraph. And then, didn't I ask you to perform your social obligations with respect to Löwenhaupt and didn't I draw your attention to the importance

of this? And you did not visit him [indistinct], and that is why you were not invited to the celebration on the occasion of our king's birthday, about which half of Stockholm, and the king above all, know. No, mark my words, neither in Paris nor in any other place will you ever play the social role you covet, and for two reasons: (1) However, quick witted you can be sporadically, you are entirely deprived of the insistence needed for the attainment of what you set yourself, and very well possibly, as a goal. (2) On the other hand, you haven't got the equanimity in the face of social and external circumstances that when it is sincere leads to the success you seek.

"But enough of my moralising. If you really want to retain Bertrand's benevolence, do the following.

"1. Go and visit Löwenhaupt immediately. Tell him that you were ill and had to ask for a leave for health reasons, and then apologize for not visiting him earlier.

"2. When you get your memoir published in Paris in a few days, give a copy to be bound at Hermann's (4 Sorbonne street) into a high quality, fine, and regal cover. Get them to put the monogram of Oscar II and the king's crown on top. Then write an impressive letter to the king (send me a rough copy right away) and tell there that you had hoped to present it in person to His Majesty, noble friend, and patron of mathematics, who established the prize in mathematics and therefore linked forever his name with the history of the exact sciences, but that you had fallen ill because you were exhausted by the work. Therefore you were compelled to take a leave and stay away for a rather long time, and that is why you have taken liberty to send him your work now. When the book and the letter are ready (the letter should be subject to my prior approval), visit Löwenhaupt again and ask him if he would be so kind as to forward them.

"Don't talk with him about the awards.

"Be careful not to soil the book as you sometimes happen to do, don't scratch it, don't bend the corners, etc!!!

"3. Convey to Hermite and Bertrand (on no account to Löwenhaupt) what you will find necessary from the following conversation I've had today with a person close to the Foreign Minister.

"The king cannot make an award to Bertrand alone because he is a Member of the Institute of France. Otherwise he would have to think at least of the Academicians of Berlin. But the German government is very reluctant to allow the German scientists to accept foreign awards. The king has to have a specific reason." [ML 69]

Two days later a longer letter was sent, but only fragments are extant. Mittag-Leffler reproached Kovalevskaya that her letter and telegram "with respect to Bertrand's low morale" due to his not receiving the award came too late, when it was not opportune to ask for an award for Bertrand. He added [ML 70-71]:

"And what a childish idea to think that the king would use his jubilee, his 60th birthday, to honour the Institute of France! You know very well, firstly, that the king is not a republican, secondly, that his politics tend to the German rather than French side, and thirdly, that he is very afraid to do anything to offend Germany, and lastly, as far as awards for mathematicians are concerned, he's already done much more for France than for Germany: Hermite, Serret, Poincaré, Picard, and Appell, and only Weierstrass in Germany. This is what the king himself told me two years ago, when I asked to award Darboux, but to no avail. And then he hinted in one of our conversations that he was amazed that France had not been as courteous as he with regard to awards. This goes, as it were, against the international customs concerning awards, and it seemed to me that the King felt personally offended by this."

In the long run, the French were as courteous to Swedish scientists. Awards were given to Gyldén, Dunér, and Mittag-Leffler, though Mittag-Leffler got an award of a lower degree than Gyldén and Dunér. He attempted to decline the award, but this proved impossible.

In a letter of January 31, 1889, Mittag-Leffler enclosed a list of the works by the Paris Academy of Sciences that the Swedish Academy of Sciences did not have. Bertrand was to find the missing works (he was one of the two permanent secretaries of the French Academy of Sciences) and as soon as possible hand them over to a representative of

the Swedish embassy in Paris to be immediately forwarded to Sweden: then Mittag-Leffler could renew his efforts for Bertrand to be awarded.

In the meantime Sofya and Maksim had mended their relations, and she wrote from Nice in the very beginning of February [SK 328]:

"Dear Gösta,

"Just imagine, I am in Nice now. I resolved to make this trip so quickly I had no chance to warn you. I 've written my good-bye letters to a few of my Paris friends from here.

"It is fine now in Nice. Real summer. It is hard to picture oneself such a wonderful change after the bad weather in Paris over the last few days. You can imagine the effect it produces on me: I've never been to the South.

"I've found a small Russian colony, and a very pleasant one. I was acquainted with some of these people earlier in Petersburg. Therefore, I hope to pass the time here very well, although I don't know if I'll stay here long. It depends on whether I'll be able to go on with my work. If I can make myself work, I'll stay longer, and if not, I'll return to Paris in two weeks' time. I am overpowered now by a few small literary notes that I want to put to paper before really taking any literary work.

"I don't have to tell you how impatient I am to get a letter from you and to hear from Fufa.

"Please, convey my regards to Signe and Madam Gyldén.

Devotedly Yours,
Sonya.

"My adress: Mme. Sofya Kovalevskaya, Hotel Louvre, Nice."

Sofya took Gösta's reproaches humbly and wrote to him again in 1889 [SK 330]:

"Dear Gösta,

"I've just received your last three letters. I admit that your reproaches are well-deserved. The only excuse I can make is that I really lost my head during my stay in Paris. I'm afraid you will scold me for leaving Paris without arranging anything. Tomorrow I'll write to

Hermite and tell him what you say in your letter. I'll write Löwenhaupt when I have a copy of my work to send him.

"You're right: I'm not fit for anything. There's nothing I can arrange, I cannot keep up business relations with people, I'm unbearable, and one cannot confide in me. But I assure you that I'm crazy at the moment, I overestimated my nerves. And they have had their revenge.

"The most important thing with respect to Bertrand is the 50,000 for Weierstrass*. Everything else will be all right.

"I am really sorry for all the trouble I have brought you.

"Tell Fufa to write me. It's very bad on her part that she doesn't do this.

Devotedly Yours,
Sonya."

Sofya lived in France until the autumn of 1889. In the summer she met the Mittag-Lefflers in Paris. They had come to the international exhibition and brought Fufa with them. Yuliya V. Lermontova also went to see the Paris Exhibition.

The opening ceremony of the Paris World Exhibition was to be held on May 5, and the Eiffel tower was to be opened immediately after, on May 6. But a private viewing for an invited audience was held on May 4, and Kovalevskaya was there. In her fragment "At the Exhibition" [67] Kovalevskaya wittily ridiculed the gathering. She herself was interested at the exhibition in the serious things from the field of physics, especially electrodynamics, while Fufa and her friends were more interested in the toys and the Eiffel tower.

Sofya Kovalevskaya had become a celebrity. The whole of the civilized world knew of her, and she was written about in newspapers and magazines. Of course, Kovalevskaya enjoyed her success, but soon she began to tire of it, of the numerous parties in her honour or where she was celebrated. She dreamed of returning to her homeland.

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* The expected prize France was to make to the best mathematician in Europe.

The governor of Saratov Lieutenant-General A. K. Kosich was Kovalevskaya's cousin, and met her in Paris after a long time. He saw how lonely Kovalevskaya felt in a foreign country and how she wanted to return to Russia; he wrote a letter to the President of the Petersburg Academy of Sciences, the Grand Duke Konstantin, requesting that S. V. Kovalevskaya be enabled to return to Russia and Russian science in the capacity of an Academician. Kosich reminded the Grand Duke that Napoleon once said that

> "every government should value the acquisition of a famous mathematician, artist or, generally, outstanding person, whatever their nationality, more highly than the acquisition of the richest and most prosperous city".*

It appears that the president turned the request over to the permanent secretary of the academy, K. S. Veselovsky, who then proposed that Chebyshev write an official note about Kovalevskaya. In October 1889 Chebyshev wrote a letter, part of which has been cited above. He included more details on Kovalevskaya than were given by Kosich. In particular, it was noted that in 1885 Kovalevskaya was invited to a session of the Paris Academy of Sciences, the meeting being chaired not by the President, as Kosich had written, but by the permanent secretary J. Bertrand. Chebyshev wrote at the end of the letter:

> "At the present moment, Mrs. Kovalevskaya occupies both very important and very honourable position in Stockholm: she is a Professor of the University there. She would hardly be expected to change for a position of Professor of Mathematics of the Higher Courses, the only one accessible to her given the present statutes of education establishments and where there is a chair for higher mathematics."**

Chebyshev's letter was submitted by Veselovsky, together with his comments, to the President of the Academy of Sciences as the basis of an answer to Kosich.

Kosich's petition thus met a negative response from the permanent secretary of the Academy (October 11, 1889):

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* *AAN*, f. 6, op. 1, No. 1 (1889).
** *LOA AN*, f. 2, op. 1, No. 1 (1889) (see [183, p. 115]).

"His Imperial Highness the Most August President of the Imperial Academy of Sciences has deigned to order me to inform you that Sofya Vasilievna Kovalevskaya, who has acquired abroad a wide fame by her scientific work, enjoys no less fame among our mathematicians. The spectacular advances of our countrywoman abroad are all the more flattering to us because they are entirely due to her own great merit, since the nationalist feelings there could not serve to increase the enthusiasm for her benefit. We are especially flattered by the fact that Mrs. Kovalevskaya has received a position as Professor of Mathematics at Stockholm University. The award of a university chair to a woman could only occur if everyone had an especially high and favourable opinion of her capabilities and knowledge, and Mrs. Kovalevskaya has met this challenge by her truly remarkable lectures.

"However, since access to teaching at our universities is completely closed to women, whatever their capabilities or expertise, there is no position in our homeland for Mrs. Kovalevskaya as honourable and well-paid as that which she occupies in Stockholm. The position of a mathematics teacher at the Higher Courses for Women is much lower than a university chair; as to our other educational institutions, where women can occupy a position, the teaching of mathematics is limited to the elementary level."*

The text of the letter was prepared by Chebyshev. Obviously, it had been pointed out to him that those on high had not deemed possible to give Kovalevskaya a position in Russia, and the only thing left for him was to sweeten the refusal with diplomatic words. It is interesting that when Weierstrass learned that some French mathematicians were talking about the possibility of her teaching at the Normal school (*École normale*) for women that had been opened in Sèvres, he rose against the idea and wrote to her on June 12, 1889 [125, p. 282]:

"This position would mean a demotion for you: they would say that you felt inadequate for a university chair and therefore you would be proving that women are unfit for teaching and representing the exact sciences."

* *LOA AN*, f. 2, op. 1, No. 7 (1889), p. 2-3 (reverse).

However, I think that Sofya Kovalevskaya would not have refused a position at the Bestuzhev Higher Courses for Women in Petersburg. Although girls finishing high schools were less well trained in mathematics than boys (the schools were separate), teaching at the Bestuzhev Higher Courses for Women was considered an honourable social function, and progressive professors of the time were willing to lecture there.

Kovalevskaya had always taken a keen interest in the Bestuzhev Courses. When she arrived in Russia in May 1890, she was present at an examination for a group of girl mathematicians, a fact noted in her diary on May 7. After the examinations, the girls presented her with a photograph of the building of the Courses (on the 10th line of Vasilievsky Island, St. Petersburg). The department of Mathematics and Mechanics of Leningrad State University now stays in the same building. The inscription on the picture was:

> "To the much respected Sofya Vasilievna Kovalevskaya from the students of the Higher Courses for Women, who are sincerely grateful to her for her visit and want it to be remembered.
>
> "St. Petersburg, May 15, 1890."*

Then follow the signatures of 24 students.

During this stay in Russia, which turned out to be the last she ever made, Kovalevskaya was warmly greeted by the Russian public. She was invited to a session of the Petersburg City Council, Duma, where she was saluted by the city mayor. In her answer Kovalevskaya "voiced her delight in connection with the advance of people's education" [218]. Kovalevskaya wrote to Mendelson-Zaleska (Jankowska) on October 7, 1890 [64, p. 312]:

> "I passed the summer wonderfully, first in Russia, then in Switzerland and Italy. In Petersburg I was made to give a speech to 5000 people in answer to a toast by the chairman in my honour."

Again, as in the 1870s, Russian society was indignant that the tsarist government would not allow such a famous Russian woman to work in her homeland. The mathemati-

* From the family archives of S. V. Kovalevskaya (the daughter).

cians Chebyshev, Imshenetsky and Bunyakovsky therefore decided to gain honours for Kovalevskaya by other means.

The Academy of Sciences awarded an honorary title of Corresponding Member to Russian scientists from other cities and to foreign scientists.

When Kosich's petition was rejected by the President of the Academy of Sciences, the following application was submitted to the Physico-mathematical section of the Academy and heard on October 24, 1889:

"The undersigned have the honour to nominate for election as Corresponding Member of the Academy, in the Department of Mathematical Sciences, Doctor of Mathematics, Professor of Stockholm University, Sofya Vasilievna Kovalevskaya.

"P. Chebyshev, V. Imshenetsky, V. Bunyakovsky."*

On November 4 the Academy decided in principle "to admit persons of female sex to be elected as Corresponding Members" by 20 votes to 6. On November 7 a session of the Physico-mathematical section approved Kovalevskaya's nomination by a vote of 14 to 3. A general assembly of the Academy of Sciences confirmed the election of Kovalevskaya on December 2, 1889. Chebyshev sent her a telegram on November 8, after the session of the Physico-mathematical section [64, p. 354]:

"Our Academy of Sciences has just elected you a Corresponding Member, admitting by this an innovation which was unprecedented. I am very happy to see one of my most ardent and well-grounded wishes come true, Chebyshev."

Kovalevskaya's election was read out at a ceremonial session of the Academy of Sciences on December 29; then the diploma was sent to her. On February 11, 1890, Kovalevskaya acknowledged in a letter to Veselovsky, permanent secretary of the Academy,

"Dear Konstantin Stepanovich,

"I was not in Stockholm, and the diploma for the title of Corresponding Member of the St. Petersburg Academy

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* *AAN*, f. 2, op. 1, No. 10, p. 16.

of Sciences which had been sent to me on January 12 of this year was only delivered to me today. Allow me, dear Sir, to request you to take upon yourself the trouble of expressing to the Academy my deep and heart-felt gratitude for the high honour that it has bestowed upon me by electing me a Corresponding Member. This salutation from my dear homeland has touched me deeply and made me happy. Receive, dear Sir, the assurance of my full respect and devotion.

Sofya Kovalevskaya."*

No material award accompanied the title of Corresponding Member. In November 1889, Kovalevskaya wrote to Kosich in connection with the telegram from Chebyshev [64, p. 306]:

"You cannot imagine how happy I was to receive this telegram. So, your efforts were not in vain and have brought a result. Thank you very much from the bottom of my heart. Of course, the title of a Corresponding Member is no more than an honorable one and it does not give me the chance to return to Russia, but still I'm very glad they resolved to give it to me because now, if there is a vacancy for a full Member, they'll have no pretext to refuse me simply because I'm a woman."

When Weierstrass learned of Kovalevskaya's election he expressed his satisfaction in a letter dated February 5, 1890 [125, p. 289]:

"My dearest friend, ...

"Now receive my heart-felt congratulation in connection with the title you have been awarded by the Petersburg Academy. It is well-deserved. I was sincerely happy that the first academic honour has been given to you in Russia."

Since there was no possibility of returning to Russia, Kovalevskaya decided to work in Paris, a large scientific and cultural centre. However, she did not succeed in obtaining a professorship in Paris, in spite of the high opinion of outstanding French mathematicians such as Poincaré

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* *LOA AN*, f. 2, op. 1, No. 1 (1890), p. 61.

Elizaveta Fedorovna Korvin-Krukovskaya (née Schubert) (1874)

Vasily Vasilievich Korvin-Krukovsky (1874)

Fedor Fedorovich Schubert, Honourary Member of the Petersburg Academy of Sciences

Fedor Ivanovich Schubert, Member of the Petersburg Academy of Sciences

Margarita Frantsevna Smith

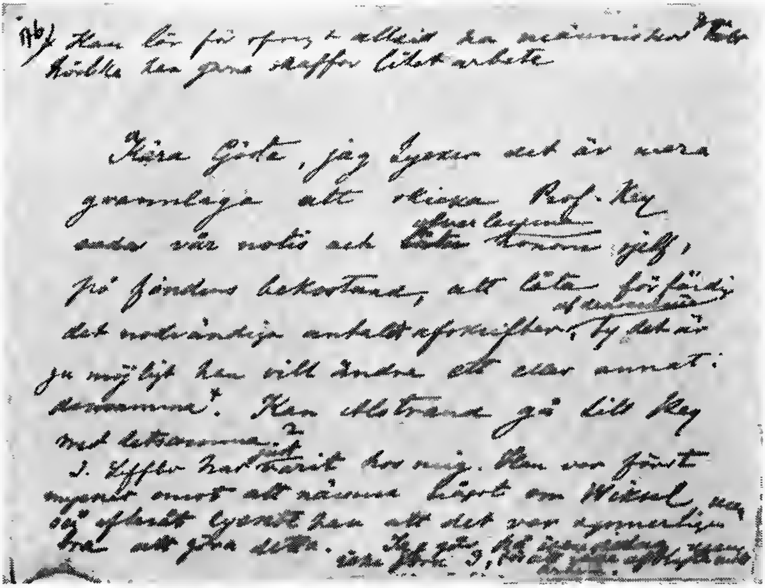
Sofya Vasilievna in 1865

Iosif Ignatievich Malevich

Aleksandr Nikolaevich Strannolyubsky

Manor-house at Palibino

Heidelberg, late 19th century



A latter from Sofya Kovalevskaya to Gösta Mittag-Leffler

The house of Signe and Gösta Mittag-Leffler in Djursholm

Sofya Kovalevskaya and Yuliya Lermontova

Anne-Charlotte Leffler and Sofya Kovalevskaya

Pafnuty L. Chebyshev

Karl Weierstrass (1870)

Vladimir O. Kovalevsky

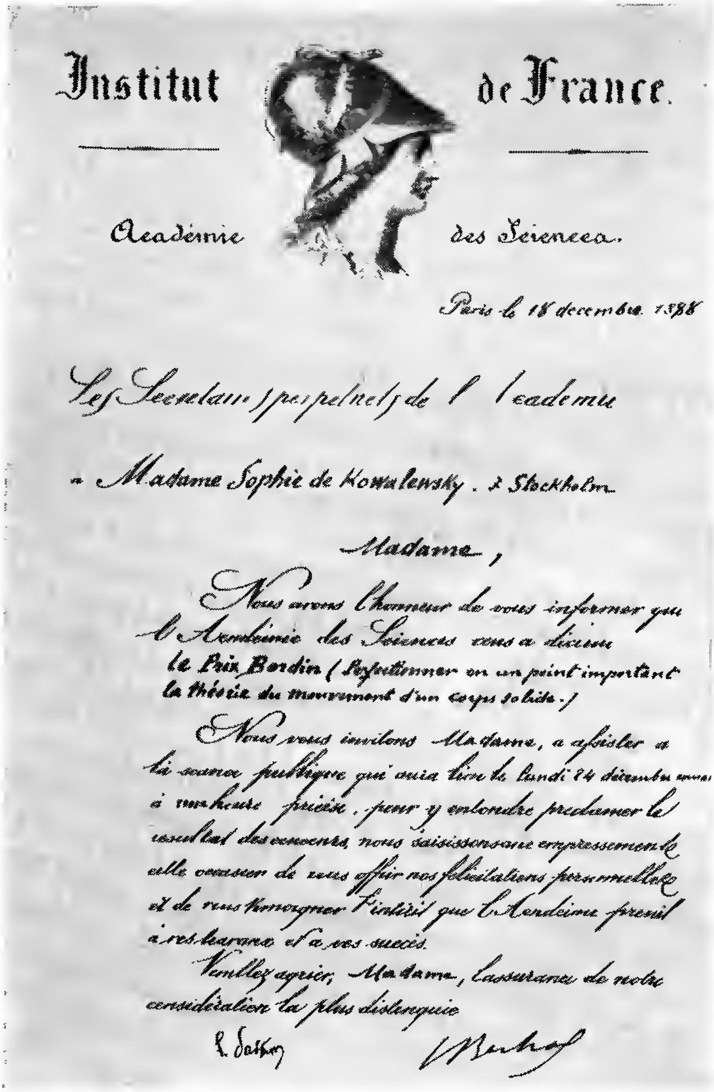
Karl Borchardt

Yuliya V. Lermontova

Sofya Vasilievna Kovalevskaya (1878)

Mariya V. Jankowska (Mendelson)

Anna V. Korvin-Krukovskaya (Jaclard)



Institut de France

Académie des Sciences.

Paris le 18 décembre 1888

Les Secrétaires perpétuels de l'Académie

à Madame Sophie de Kowalewsky. à Stockholm

Madame,

Nous avons l'honneur de vous informer que l'Académie des Sciences vous a décerné le Prix Bordin (Perfectionner en un point important la théorie du mouvement d'un corps solide.)

Nous vous invitons Madame, à assister à la séance publique qui aura lieu le lundi 24 décembre courant à une heure précise, pour y entendre proclamer le résultat des concours, nous saisissons avec empressement cette occasion de vous offrir nos félicitations personnelles et de vous témoigner l'intérêt que l'Académie prend à vos travaux et à vos succès.

Veuillez agréer, Madame, l'assurance de notre considération la plus distinguée

G. Darboux J. Bertrand

The letter from the Institute of France announcing Sofya Kovalevskaya the winner of the Prix Bordin

Sofya Kovalevskaya

Gösta Mittag-Leffler

Hugo Gyldén

Ellen Key

Sofya Kovalevskaya with her daughter Sonya

Observatory in Stockholm

Stockholm, 1877 (from D. I. Mendeleev's album)

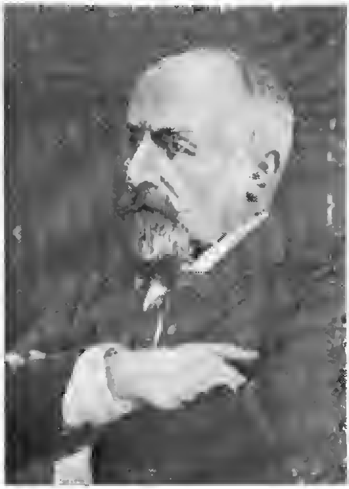
Niels Abel

Gustav Eneström

Andrei A. Markov

Maksim M. Kovalevsky

Leopold Kronecker



Georg Cantor

Hermann Schwarz

Charles Hermite

Gaston Darboux

Henri Poincaré

Emile Picard

Joseph Bertrand

Sofya Vladimirovna Kovalevskaya (Sofya Vasilievna Kovalevskaya's daughter)

S. V. Kovalevskaya's grave (in 1950)

and Hermite. The inertia and deep-seated prejudice with respect to women in science also existed outside Russia.

Even those who admired Kovalevskaya's abilities were not inclined to grant a woman the right to acquire a position in science equal to that of a man. Some were ready to make an exception for Kovalevskaya, but only so far as honours were concerned. Thus, following a petition by some French mathematicians, she was made an award by the French Ministry of Public Education. In the Summer of 1889, she wrote to Mittag-Leffler from Sèvres [SK 358]:

> "On July 14, I received a letter from the Minister of Education. He informed me that he had appointed me an 'officer of public instruction', this being the highest award from the Ministry of Education. Poincaré was also so appointed... The most amazing thing is that neither of the local newspapers included my name in the list of those awarded."

In Autumn 1889 Kovalevskaya returned to Stockholm and started her lectures. She exchanged letters with Maksim Kovalevsky.

E. P. Kovalevsky, Maksim Kovalevsky's nephew and legal executor, has written that Maksim passed over to him Sofya Kovalevskaya's letters, which were very numerous and interesting [182, p. 31]. However, these letters remain unpublished and it is not known where they are. As far as Maksim's letters are concerned, some of them deeply pained Sofya while others were so good that Sofya, having read them, was beside herself with joy and would travel to see her adored. Anne-Charlotte Leffler has born witness that Kovalevskaya would laugh and swirl around the room, exclaiming, "Oh, how happy I am! I can't stand it! I'll die! How happy I am!" [96, p. 291]

Probably in response to such a letter Sofya sent a telegram to Maksim on January 10, 1890 from Paris, where she was on holiday: "Received letter very happy leave tomorrow for Beaulieu Sophie".

In the Summer of 1890, during the holidays, Sofya went to Maksim's villa Beaulieu feeling uncertain. She wrote a post script to a letter Fufa, her daughter, had written to Yuliya Lermontova on June 18, 1890 [82, p. 58]:

"Dear Yuliya,

"I'm leaving today for the South of France, but whether for joy or for woe I don't know myself, probably for the latter.

Good-bye, my friend.
Yours, Sofa."

However, despite her misgivings, Sofya's stay with Maksim in Beaulieu proved happy. She discussed their forthcoming trip to the Caucasus, which Maksim knew well from his own travels. Gösta and Anne-Charlotte wanted to join them, but the plan never came about. Sofya and Maksim decided to go over different places in Europe, and the two of them travelled for two months and a half. Judging from Sofya's letters, the trip was very exciting. For instance, Sofya wrote to Yuliya Lermontova from Bonn on June 24 that Amsterdam was positively worth seeing, and that they had stayed there for two days before going to Cologne, where they swam in the Rhine, and that they spent a night in Bonn.

Their plan was to go by steamer up the Rhine to Mainz, to be in Heidelberg after two or three days, then to Switzerland, to Tarasp.

"At this very minute," she added, "as you can imagine, it is fun for me. God knows what is going to happen next; with my light-mindedness, I don't think of the future." [64, p. 308]

The period in Tarasp was happy for both Sofya and Maksim. Sofya wrote a very nice and tender letter in July 1890 from Tarasp to her daughter in Moscow [64, p. 309]:

"Dear sweet Fufa,

"I started writing to you on another piece of paper, but now Maksim Maksimovich has come back from a walk and brought me some note-paper with a view of Tarasp, the small place we're staying at, and I think it will be nicer for you to get a letter on this paper. It's very beautiful here. There're mountains everywhere, their tops are covered with snow, although they seem to be pink down here, because they're strewn so densely with small pink flowers. Most of the flowers here are the same as on our meadows, but they're much bigger and

finer. However, sometimes I've seen flowers that I haven't seen in Russia, for instance, alpine roses, that is, wild rhododendrons. I'll try and dry some of them for you.

"Yesterday we went in a carriage to see some lovely places. I was awfully sorry you weren't with us. For more than an hour we could see nothing around us other than the mountains covered with snow; we were so high that firs and birches cannot grow and there were only bare rocks covered with snow. When you grow up, we'll go with you and mama Yuliya to Switzerland.

"I'm very glad you have started your lessons. It's time for me to start my work as well, but I am much lazier than you are, I only talk whenever I really have to work, but every day there is an excuse to be lazy. Write to me, my dear Fufa, as often as you can. I'm enclosing 10 roubles in the letter because I'd like you to get a picture of you in Moscow and send me a copy. But if you have some other great wish, you may use the money for that. I kiss you all over.

Your mummy."

Kovalevskaya also wrote from Tarasp to Hansemann. She included the following words:

"Everywhere here is so wonderfully beautiful that I always think, despite what the poet said, that happiness is where we are." [184]

On September 29, 1890, Kovalevskaya was in Stockholm, as it is evidenced by her letter to A. N. Pypin, written in a good mood [64, p. 310]:

"I'm in Stockholm now and up to my eyes in work. Now I work hard and with pleasure because I was lazy in the summer and rested to my heart's content. I went to Switzerland and Italy (for the first time in my life), how I love the latter."

Although Kovalevskaya wrote to many people that she was "lazy", this was not, in fact, correct. She was writing an article about a peasant "university" in Sweden for the magazine *Severny Vestnik* (Northern Messenger). On her arrival in Stockholm she was pleasantly supprised to find letters from several Russian women she had never

met. They were insistent that she continue her memoirs, and Kovalevskaya then devoted "every minute free from mathematics" to writing her reminiscences, and describing her student years.

Looking through Sofya's letters, we might think that the journey to Switzerland and Italy was one of complete and unclouded happiness. However, her diary gives another impression. Her entries reveal that Sofya and Maksim had had several rows [71, p. 332].

An important thing is that Sofya's negative experience was aggravated by her being ill and very nervous. Ellen Key tells us that

> "during her last years, the spectre of death was always on her mind and she knew that she had a weak heart". [64, p. 415]

Kovalevskaya wrote to Mittag-Leffler that sometimes she had to miss a lecture because of short periods of indisposition, mainly headaches that would disappear the next day. Once, when Sofya was invited to a royal ball, she wrote that she didn't want to go to the palace at all because her migraine was so severe she could hardly keep up her head:

> "I'm really afraid I might have a heart attack at the ball" [SK 369].

Ellen Key also said that Sofya overworked herself. Sometimes she slept no more than four or five hours a day.

> "Her delicate organism stood up to such strain and kept being fresh owing to her healthy habits. She liked baths and physical exercise; she was very moderate in her meals and drink and avoided any stimulants. Even the characteristic attribute of a Russian—a cigarette—she hardly ever used. This life style meant that her nerves were rarely disordered even when she worked strenuously". [64, p. 414]

What was Maksim's attitude to Sofya? He wrote about Sofya in an article called "Recollections of a Friend" [64, p. 392]:

> "Her feeling of loneliness abroad made her seek friendship, and when there was a chance of frequent contacts

with a compatriot who was no less torn from Russian life, something close to attachment appeared in her. Sometimes she thought that this feeling was tenderness. But this did not in the least interfere with her ability to bury herself in her work any time she wanted or to spend whole nights solving her complicated mathematical problems."

Maksim appears to have believed that Sofya could not live for him alone or be involved in his interests only, in spite of all her love for him. This seemed to gall him and make him irritated with her mathematical studies.

Ellen Key described Sofya's feelings to Maksim:

"I saw her every week in the course of several years, but I met Sonya Kovalevskaya essentially only once. It happened to be at a concert where Beethoven's *Ninth Symphony* was performed. Contrary to her habit, Sonya was elegantly attired: she wore a black silk dress with lace... The object of her love, her countryman, was sitting beside her. The divine sounds of Beethoven's music flew around. The commonly nervous features of Sonya Kovalevskaya were lucid and calm. She seemed transformed. She was in love, and the music carried her away into a world of bright dreams... I only saw this expression on her face twice, the other time Sonya Kovalevskaya was dead." [149, p. 320]

During the Christmas holidays of 1890, Sofya Kovalevskaya went to the South for the last time. During the summer relations between her and Maksim had become stronger.

After arriving at Beaulieu, Sofya wrote to her daughter [82, p. 58]:

"My dear sweet Fufa,

"I arrived in Beaulieu yesterday. I travelled whole five days. It was winter all the way here. There was much more snow in Germany than in Sweden, and it was very cold. But just as I went over the Alps, everything's changed. It's warm here like in summer. I'm writing you at an open balcony door. Roses, camellias, and violets are blossoming in the garden, and there are not quite ripe oranges hanging down from the orange trees..."

Anne-Charlotte wrote that "the sun, happiness, and the fragrance of flowers" [96] ran through a letter Sofya had sent from Nice to Ellen Key.

Sofya sent an invitation from Beaulieu to Anne-Charlotte and her husband, the Duke del Pezzo, to visit her with Maksim at his villa Batava. Anne-Charlotte declined the invitation on December 21 because her trip with her husband was already too tightly scheduled. However, they had agreed before to meet in Genoa to celebrate the New Year together. But they did not meet because of a misunderstanding and wrong addresses. However, Sofya and Maksim did stay for a few days iu Genoa.

In early January 1891 Sofya wrote from Genoa to Mittag-Leffler in Petersburg, where he was at the time. But the letter only reached him on June 11, and was forwarded from Moscow. Let me quote the whole of this last letter [SK 240]:

"Dear Gösta,

"I'm writing to you from Genova where we arrived in the hope of meeting Anne-Charlotte. Unfortunately, there seems to have been a mess-up with letters and telegrams, and they probably passed Genoa without stopping, so we made our trip for nothing and celebrated the New Year very gloomily (*d'une manière fort maussade*).

"I received your nice letter and thank you very much for the news from Petersburg. Judging by what you write, hope is not entirely lost that one fine day I'll become a full Member of the Academy. If you have another chance to go to Petersburg, I'd be very grateful if you could visit the grand duke and speak to him to that effect. I think such a talk would be only advantageous for me.

"I absolutely don't know what I have to give the coming semester. Maybe, I should continue with the number theory, if only the rest of your course is just as well developed as the beginning. Please, tell me by what date I should be in Stockholm and if it would be possible to be late for a week without a scandal, because I'd like to be present at the carnival that starts here on February 2. But I have to confess that I nourish the hope I can ask for a leave of absence in April in order to be able to go to Russia, where Max may be by that time. Anyway, I wouldn't like to jeopardize my chances of obtaining a

leave of absence in April, and with this in mind it'd be better for me to be punctual now. Please, write to me, Gösta, what you think about this.

"I'm not writing to you today any more, dear Gösta, because I hope to see you before long.

"Please, convey my regards to Signe, Fritz, and aunt Leffler.

Devotedly Yours, Sonya.
Regards from Max."

The letter is full of hope for the future, but there is a frivolous wish to see the picturesque carnival. But there is the presentiment of something doleful. Sofya was cheerless because they had been to the cemetery in Genoa on the first day of the year (Campo Santo, one of the most beautiful cemeteries in the world).

Maksim Kovalevsky recollected that when Sofya had finally made up her mind to return to Stockholm, he saw her off to Cannes. Kovalevskaya went from there to Paris to see the mathematicians she was friendly with, whence she left for Berlin to meet her friends at Georg Vollmar's home. Later Vollmar wrote a heart-felt article about Sofya Kovalevskaya [185]. He described Sofya's arrival in Berlin:

> "She arrived from the sunny South, from Italy, where she had taken her leave of absence as a professor of Stockholm University, and she appeared gay and happy. She... brimmed with creativity in the arts and sciences, full of plans for the years to come. Her entire being was so abundant with life and charm, her talk was so sparkling, and her friendliness was as generous as always. The friends parted merry and smiling and hoping they would meet soon, be it in Bavaria or Scandinavia or Paris or wherever." [185, p. 845]

Maria Bunsen was nonplussed when she read Anne-Charlotte Leffler's reminiscences of Kovalevskaya. Maria Bunsen felt Anne-Charlotte had covered the end of Sofya's life and her relations with Maksim Kovalevsky too sombrely. Maria Bunsen remembered that when Maksim travelled from Nice to Stockholm after receiving a telegram about Sofya's final illness, he was en route in Berlin and told friends

they had planned to marry in June 1891 [184, p. 232].

No one knows how Sofya and Maksim were going to lead their life together. It is clear from Kovalevskaya's last letter to Mittag-Leffler that she had not lost hope she might be elected a full Member of the Academy of Sciences in Russia. But it is more probable that this nomination might have become possible only later, after 1904, when there was a breath of freedom in Russia (Maksim Kovalevsky returned to his homeland in 1904 and became a Member of the Academy in 1914). The Kovalevskys could have lived in France, where Sofya hoped to obtain a position in the *Ecole normale* for girls. Possibly, she might also have concentrated on literature for a while. She had in mind many ideas for literary work. But she had also new ideas about the rotation problem, of which she had told Poincaré and Hermite, and she knew she should return to mathematics.

In late January 1891 Sofya returned to Sweden. Unfortunately, she had caught a bad cold on the way and came to Stockholm feeling ill.

After gathering details from the recollections of Maksim Kovalevsky and Anne-Charlotte Leffler, and from letters written to Mittag-Leffler by many people, Ann Koblitz writes [284, p. 232-233]:

> "Kovalevskaya left the south of France in late January 1891. Maxim accompanied her as far as Cannes, where she caught a cold. She needed to return to Stockholm in time for the spring semester, however, so she continued her journey in spite of her illness. She stopped in Paris and then in Berlin to talk with Hermite, Picard, Weierstrass, Kronecker, and other colleagues. She apparently ignored her fast-worsening cold in her desire to crowd as much mathematics as she could into a few days.
>
> "Kovalevskaya's colleagues in Paris and Berlin did not even notice her illness, so sparkling and full of plans was she that January. She discussed her hopes for an Academy of Sciences post, her Stockholm lectures, and a memoir by the mathematician Dirichlet. No one guessed that she was feeling ill—she charmed them all with her erudition and general liveliness.
>
> "Since there was rumour of a smallpox epidemic in Copenhagen, Kovalevskaya, who was terrified of the

disease, took an indirect, complicated route to Stockholm to avoid the afflicted city. This route entailed several mid-night changes of train, and since Sofya had no Danish money with her, she had to carry her bags herself. The weather was cold and rainy, making the tedious journey across the Danish islands even more exhausting and unpleasant than it was ordinarily.

"By the time Kovalevskaya returned to Stockholm, she was seriously ill ..."

She informed Mittag-Leffler in a short message that she had arrived at 8 a.m. and asked him to visit her. She made an error that bears witness to her nervous condition, for she signed the note "Eder Gösta", i.e. "Yours, Gösta", [SK 418] instead of "Yours, Sonya". However, she gave her scheduled lecture on Friday, February 6, and in the evening went to a party at the Gyldéns' observatory. She appeared well-dressed and sprightly, of which several old Swedish women, then schoolgirls, remembered not long ago. But suddenly, half-way through the evening she felt shivery and went home. Absent-mindedly, she took a wrong horse-tram and rode in a cold car for a long time, and this worsened her condition. The next morning she felt still worse and sent Mittag-Leffler a message on the back of her visiting card:

"Dear Gösta, I don't feel well today. I've already had influenza, but still went to the Gyldéns. However, I had such a fit of shivering that I had to go home almost immediately. Later in the evening I had an attack of severe vomiting and was then feverish throughout the whole night. I have now acute pain in the back on the left side, and I feel so bad that I'd like to call a doctor, Be so kind and write a few words to your doctor so he'll visit me today and send the message with a courier. I don't know any doctor."

Then follow a few lines written by Mittag-Leffler:

"7.II. The last letter. I sent for the doctor immediately, and came myself after my lecture. Influenza. The lungs." [SK 419]

Sofya was nursed by Theresa Gyldén, her daughter Elsa, and Ellen Key. But on February 9 in the evening the doctor

told them that they could go home, leaving a nursing sister in charge. However, during the night someone knocked at the Gyldéns' door and brought a message that Professor Kovalevskaya was dying. Theresa and Elsa hurried to Sofya, but she passed away a few hours' later without regaining consciousness.

A few days after her mother had fallen ill, Fufa was to go to a fancy-dress children's party, and Sofya Kovalevskaya did not want to spoil her daughter's joy. The costume had been made. But Fufa never went to that party. On Saturday, February 7, Fufa started writing a letter to her god-mother, Yuliya Lermontova, and kept writing it for three days [79, p. 94]:

> "The doctor says there is no great danger, but she'll probably be in bed for a long time. If there's anything exciting, she'll grow worse ... No steamer leaves for Russia until Wednesday, today is Sunday, so I'll use this time to write you in the same letter ...
>
> "Monday. Mummy's a bit better, she sweated during the night, and today her fever is not so high.
>
> "Tuesday. Dear mama Yuliya. Yesterday in the evening mummy took some morphine, and I was not allowed to see her. Fru Gyldén was with mummy until 7 o'clock when she left, mummy said she's better and was so calm. She became much worse during the night. Fru Gyldén was sent for, and she came and woke me up. A bit later mummy started wheezing badly and all of a sudden she stopped breathing. I didn't notice when it happened.
>
> "I'm at the Gyldéns' home now. I want you to come very, very soon. I'm so sad ...
>
> Fufa"

Sofya Vasilievna Kovalevskaya passed away on February 10, 1891, in the full flower of her creativity. She was just forty-one years of age. Frequently, Theresa and Elsa Gyldén recalled that the last words they heard from Kovalevskaya were: "Too much happiness".

Maksim Kovalevsky was sent for by telegram on February 8:

> "Sonya seriously ill pneumonia telegraph arrival Mittag-Leffler Stockholm."

The telegram was delivered to Beualieu on February 9. Although Maksim Kovalevsky hurried he did not see Sofya alive.

A large number of the public, academicians, professors, and students attended Sofya Kovalevskaya's funeral.

Maksim Kovalevsky said at the funeral [64, p. 407]:

> "Sofya Vasilievna,
>
> "Your knowledge, your talent and your nature have always been and will be the glory of your homeland. Not without reason all scientific and literary Russia mourns you. The wreaths on your grave have been sent from all over the vast Russian empire, from Helsingfors and Tiflis, and from Kharkov and Saratov. You were not destined to work in your native country, and Sweden accepted you. Honour to this country, the friend of science! Special honour to the young University of Stockholm! However, working far from your homeland, you have kept your nationality, you stayed a faithful and devoted ally of young Russia, peaceful, fair, and free Russia, the Russia to which the future belongs. On its behalf I am saying good-bye to you for the last time."

Gösta Mittag-Leffler made a short heart-rending speech [13, p. 338]:

> "On behalf of those who work in the mathematical sciences in every country, on behalf of all your close and distant friends and students I address you with our last words of parting and gratitude. I thank you for your depth and clarity, with which you directed the intellectual life of youth. And posterity, like the current generation, will honour your name for that. Thank you for the treasure of the friendship you gave to everybody close to your heart."

It is appropriate to quote here some of Mittag-Leffler's words that appeared later, in 1893, in the pages of the journal *Acta Mathematica* [186, p. 388]:

> "She appeared to us as a harbinger of new scientific ideas. What significance she attached to them for the solution of the most essential and vital problems, and how willingly she shared her unusually rich knowledge and her ideas with each of her students!"

Telegrams, letters, and wreaths poured in from all over Europe. Obituaries were published in dozens of periodicals. N. V. Stasova, the former director of the Bestuzhev Higher Courses for Women, wrote in her "Notes" [187, p. 395]:

> "Kovalevskaya is dead! What a tragedy! She was not valued enough here!"

Weierstrass had been ill for the preceding three years, and he was so distressed at the news of the death of his disciple that his relatives and closest friends worried for his life. Among the wreaths laid on Kovalevskaya's coffin, there was a touching one of white lilies with the words "To Sonya from Weierstrass".

Those attending Kovalevskaya's funeral were given black-bordered copies of Fritz Läffler's poem "Death of Sonya Kovalevskaya. February 16, 1891. Stockholm" in Swedish:

"Death of Sonya Kovalevskaya"
(a poem by Fritz Läffler)

Själ af eld och själ of tankar,
Har Ditt luftskepp lyftat ankar
Nu att stjärnerymder plöja
Evigt, där Du för sågs dröja
Mången gång, did stadd på spaning
Öfver värlas systemets daning
Häg din tanke lyfte vingen,
När i stjärneklara kvälen.
Stråla sågs Saturnus-ringen
På den dunkel blåa pällen?

Månne ifrån högre zoner
Analytiska funktioner
Svaret nu dig finna låta
På odödlighetens gåta?

Ljusets strålar från det höga
Såg Du förr med forskarns öga
Mot kristallegrund sig bryta.
Huru ser Du nu dem flyta?

Från de ljusa himlavårldar
Ofta nog du blicken vände
Också ned till mörkets hädrad,

Till vår egen jords elände.
Där också i hoppets stunder
Såg Du mot kristallegrunder
— Utaf *kärlek* — ljus sig bryta
Och med mörkret väldet byta.

Själ af eld och själ af tankar,
Tryggast fann Du kärleks ankar.

* * *

Så farväl och tack! Ej täcke
Tungt den svenska jord det unga
Lif, som lämnas nu åt grafvens
Långa, ljufva hägn! — Så länge
Som Saturnus-ringen svänger
Sig på färd bland ljusa världar
Och än lefva män, Ditt minne
Mälas skall bland stora själars.

* * *

Here is an English rendition of the poem:

Imaginative, fiery soul, your
Boat of spirit' weighed its anchor
Now to wander through forever,
'Mid the stars, the where you ever
Tarried, so that to learn the cunning
Of the world's creation stunning.
Has Your thought gone up, designing,
When the stars were saying 'love you',
When great Saturn's ring was shining
In the dark blue sky above you?

Will the numbers' supreme spheres
Analytic functions, dears,
Open to You in condescendence
Immortality's transcendence?

Rays of light, you saw them, gazing
At them with insight amazing.
They refracted were in crystals,
Now what are they in the distance?

From the bright celestial bodies
You turned your gaze, the tenet

Being not to lose your modest
Grip with the woes of our planet.
Here you saw that, sometimes distal,
Love refraction in a crystal,
Hope that *love*, its incarnation,
Could drive gloom from domination.
Imaginative, fiery soul, your
Heart found love a reliable anchor.

* * *

So, farewel and thank you! Covered
By the Swedish soil, you'll lie here
Guarded by your grave. Your young life
Will be long, will stay so far that
Saturn's ring would round the planet
In the midst of bright rotation,
And as far as men would live, your
Image will in minds be stationed.

(*Translated from the Swedish by M. Burov*)

The poem was later published by Anne-Charlotte Leffler [190], and translated by D. Mikhalovsky for the Russian edition [96, p. 71], and quoted elsewhere [64, p. 459]. A translation by S. Vengerov was published in the magazine *Severny vestnik* (1891, No. 3/4, pp. 1-2).

Kronecker too wrote an obituary for the journal he edited then. He said that Sofya Kovalevskaya had

> "together with the memory of her exceptional talent left the remembrance of a significant and also charming personality in the hearts of all those who had the happiness to know her." [188, p. 88]

Kronecker himself did not live for long after Kovalevskaya and passed away in the same year (on December 29, 1891).

In 1896 Russian women used the money collected by the Fundraising Committee of the Bestuzhev Higher Courses for Women and other organizations to erect a monument over Kovalevskaya's grave in Stockholm. It was designed by the architect N. V. Sultanov and made of black Finnish granite.

The monument was unveiled on September 6(18), 1896. At the ceremony the Russian Consul in Stockholm, Mr. Kudryavtsev, addressing Sofya Vladimirovna Kovalevskaya (the daughter) called her

> "the daughter of the great and unforgettable Sonya Kovalevskaya, whose memory will be cherished and honoured not only in Russia, where she first saw the world, and not only in Sweden, where the world ended for her, but also in every place where people admire science, erudition, knowledge, and those who pass it on."*

Anne-Charlotte Leffler published her recollections of Sofya Kovalevskaya [190] a year after her death. The Danish writer Georg Brandes, who was considered to be a "dangerous radical", said [214, p. 241]:

> "This book is the story of a great person. It brings us closer to the person whose life was richer in its both inner and outer aspects than the life of anyone else in Scandinavia".

Anne-Charlotte Leffler quoted what others of the Russian intelligentsia said and what is a tribute to the versatile, unprejudiced and broad outlook of the truly enlightened and free-thinking Russians. She wrote that this was admitted by everyone who knew Russians, and not only Kovalevskaya, and added:

> "They are among the most advanced peoples of Europe, they are distinguished by an unusually quick ability to grasp new ideas as soon as they appear on the horizon, and they join in, with an almost unheard-of quickness of thought and with an enthusiasm and faith in their ideals that we do not encounter in any other European nationality. [96a], [96, p. 132], [190]

Those who knew Sofya Kovalevskaya remembered her appearance. She was small, her face was lively, and her eyes shone with a penetrating gaze. She spoke with a ready vivacity. She used several languages to express her ideas. She knew French, German, and English and enough Swedish to be able to communicate and lecture.

---

* From the family archives of S. Vladimirovna Kovalevskaya.

However, she repeatedly said that she was oppressed by being unable to speak her native language abroad, because she could not express the finer nuances of her ideas:

> "It is as if you're made to wear a mask all day long". she would say. "That is why each time I return to Russia I feel as if I have returned from prison, where my best thoughts were locked." [96, p. 163], [190]

Nonetheless Kovalevskya made a deep impression abroad by her colourful individuality, wide interests, bright intellect and love for humour and paradox.

Kovalevskaya's literary gifts were highly praised during her life, and her literary accomplishments are now widely recognized.

After Sofya Kovalevskaya died, A. Volynsky described in general her literary achievements for *Severny vestnik* (Northern Messenger):

> "Kovalevskaya was an established figure in science, and she was a cherished hope in Russian literature. The work she had published in three magazines, viz., *Russkaya mysl*, *Vestnik Evropy* and *Severny vestnik*, bears witness to a major talent which had, no doubt, yet to develop both in depth and width. In the recent recollections of Kovalevskaya, there are many pages which reveal her undisguised creativity, literary splendour, and vivid memory. During the last years of her life, Kovalevskaya dreamed of some serious literary activity, for which she was so prepared. With her powerful and flexible intellect Kovalevskaya could wield her wonderful gift of imagination. Her speech and writing were neither dry nor ponderous, and hence were not like other people's. Her creative imagination, lively temperament, and passionate feelings mark everything that came from her pen. Her style is animated with poetic colour and highlighted, where needed, with apt allegories and subtle artistic comments.
>
> "To Kovalevskaya, her service to mathematics and her service to arts were one: an outlook that reveals a lofty philosophical mind and sublime ideas about the goals of fiction. Naturally, Kovalevskaya would never have abandoned her scientific pursuits, but there is no doubt that has she spared some of her leisure time to purely literary work, Kovalevskaya would have soon occupied

an outstanding position among the most famous European woman writers...

"However, the shining hope of young Russia was not to be realised. Just a few months ago, Kovalevskaya spoke to Swedish peasants about Russia and wondered whether she could tell Russian peasants about Sweden in a backwater Russian village... But then came merciless death. Sofya Kovalevskya, this faithful and devoted friend of Russia has fallen beneath the peaceful shadow of eternity too soon, without saying her last word, without completing her ingenious and plentiful pursuits..." [211, p. 148]

Not very long ago, Militsa V. Nechkina wrote two inspired papers [212, 213], in which she considered Kovalevskaya literary accomplishments in relation to her social views, analyzed their evolution and clarified Kovalevskaya's historical context.

All of Kovalevskaya's literary work (except for her theatre notices) were published in 1974 as *Recollections, Novellas* [67].

Fairly recently several new articles and books about Sofya Kovalevskaya have appeared [192], [193], [284], [285], and her correspondence with Mittag-Leffler was published in Russian [286].

Interest in Kovalevskaya was renewed after the publication in the Soviet Union of the collection of her literary work [67], and the appearance of Weierstrass's letters to Kovalevskaya in both German and Russian [125].

Kovalevskaya's *Memories of Childhood* and her autobiographical sketch as translated into English by Beatrice Stillman, together with my analysis of Kovalevskaya's mathematical work, were published as *A Russian Childhood* in 1978 [68].

Ann Koblitz's *A Convergence of Lives. Sofya Kovalevskaya: Scientist, Writer, Revolutionary* was published in 1983 and is the first biography in English to present a unified historical portrait of this extraordinary woman scientist. Koblitz did the research for her book in the archives and libraries of Leningrad, Moscow, and Stockholm and incorporated previously unpublished letters, reminiscences, and diaries. In this biography Sofya Kovalevskaya emerges

as a woman whose life and commitment to science speak about issues which concern readers today.

Among other things, Yu. I. Manin, a Professor of Mathematics at the Steklov Institute, Moscow, has said of this book,

> "The author's extensive contact with the mathematical community enables her to grasp and appreciate Kovalevskaya's role as a link between the Russian and West European scientific cultures."

Other recent works include Don H. Kennedy's *Little Sparrow* [285], an entertaining and informative biography, and articles by Koblitz, Cooke and Rappaport. Professor Manin was right when he said:

> "A century has passed since the death of Sofya Kovalevskaya, yet everything her life represented is of undiminished relevance today. Her scientific achievements and her social activities, her strength of character and her talent, all strike a resonant chord in the latter part of our century."

Chapter 9

THE PROBLEM OF ROTATION OF A RIGID BODY

The rotation of a heavy rigid body about a fixed point has interested scientists for a long time. And not without reason because it is related to very common phenomena in nature and technology, such as gyroscopes, tops, and celestial bodies (discounting their translational motion). A particular case of the rotation problem is the motion of a pendulum, which using modern mathematical methods has been solved completely. However, the general problem of the rotation of an arbitrary rigid body about a fixed point is extremely complicated and we do not yet have an analytical solution.

Prior to Kovalevskaya, only two particular cases of the rotation of a rigid body had been studied:

(1) when the body has an arbitrary shape and fixed at its centre of gravity (Euler's case); and

(2) when the body is symmetrical (in the sense that its two principal moments of inertia are equal) and the centre of gravity lies on the axis of rotation (Lagrange's case).

The rotation problem has been called the "mathematical mermaid" because of the mathematical difficulties it presented.

When this problem is discussed, it is usually compared with the other enigma of theoretical mechanics, the n body problem. The problem of the motion of two gravitationally attracted bodies has been solved completely, while the case for three or more bodies has turned out to be very difficult. These two challenging problems may be said to be on the agenda of mathematicians of the 19th century.

Kovalevskaya had been interested in the rotation problem since the days she was a student. Later, she said in a letter dated 21 November 1881 to Mittag-Leffler:

"Last autumn I started a work on the integration of the partial differential equations that occur in optics of the refraction of light in a crystalline medium. I've advanced pretty well in this investigation, but I've got a weakness and have diverged to work on another problem which has run in my head almost since the start of my mathematical studies and of which I had thought other researchers had outstripped me. It concerns the solution of the general case of rotation of a heavy solid about a fixed point by means of Abelian functions."

When she first tackled the problem, her attempts were futile. Then Weierstrass's investigation of the stability conditions and the analogies with other dynamic problems, as Kovalevskaya put it,

"revived my enthusiasm and gave me the hope to solve this problem using Abelian functions whose arguments are nonlinear functions of time.

"These investigations are so attractive and exquisite that I have forgotten everything else for a while and have abandoned myself to them with as much zeal as I'm capable of... The calculations to which I have come using this technique are so difficult and complicated that so far I'm unable to tell whether I'll reach my goal. Anyway, in the course of two or three weeks, no more, I hope to find which way to go, and Mr. Weierstrass consoles me that even in the worst case I can always invert the problem and try and determine under the effect of which forces the rotation occurs whose variables can be defined in Abelian functions, though this problem is rather meagre and far from being as interesting as the one I've set myself" [SK 5].

The equations of motion of a heavy rigid body about a fixed point are a set of six differential equations whose left-hand sides are time derivatives of the desired functions, and the right-hand sides are second degree polynomials. Kovalevskaya started with a set of similar equations but with fewer variables. In her letter to Mittag-Leffler of December 29, 1884 [ML 35] she considered the set of three equations

$$\begin{aligned}dx/dt&=ax^2+by^2+cz^2+2dyz+2ezx+2fxy,\\ dy/dt&=a_1x^2+b_1y^2+c_1z^2+2d_1yz+2e_1zx+2f_1xy,\\ dz/dt&=a_2x^2+b_2y^2+c_2z^2+2d_2yz+2e_2zx+2f_2xy.\end{aligned}$$

Kovalevskaya wrote that this set can be linearly transformed into simpler sets, for instance,

$$\begin{aligned}dx/du&=x(ax+by+cz),\\ dy/du&=y(a_1x+b_1y+c_1z),\\ dz/du&=z(a_2x+b_2y+c_2z).\end{aligned}\tag{I}$$

In a special case, when $a_2=a_1=-a$, $b_2=-b_1=b$, $-c_2=c_1=c$, this set can be integrated by means of elliptic functions $\sigma\ (u)$, namely, the general integral is presented as a linear function of three ratios

$$\frac{\sigma_1(u-u_0)}{\sigma(u-u_0)},\qquad \frac{\sigma_2(u-u_0)}{\sigma(u-u_0)},\qquad \frac{\sigma_3(u-u_0)}{\sigma(u-u_0)}$$

where the three constants g_1, g_2, g_3 used to form σ are arbitrary. Kovalevskaya noted an important property of the solution obtained: it is expressed as single-valued functions of a variable u that possess no more than one essentially singular point $u=\infty$, and they only have poles of the first order for finite values of u.

As to the case of arbitrary a, b, c, ..., Kovalevskaya asked:

> "Can a set for x, y, z satisfying equations (I) possess any poles? Or only essentially singular points? In other words, is it possible to satisfy equations (I) by series of the form
>
> $$\begin{aligned}x&=x_{-m}(u-u_0)^{-m}+x^{-m+1}(u-u_0)^{-m+1}+\dots,\\ y&=y_{-m}(u-u_0)^{-m}+\dots,\\ z&=z_{-m}(u-u_0)^{-m}+\dots,\end{aligned}\tag{II}$$
>
> where m is a positive integer (or, at least, any positive number)? It is easy to see that this is possible if and only if $m=1$."

Now Kovalevskaya noticed that for arbitrary a, b, c, ... series (II) will be determined up to a factor, i.e. contain only one arbitrary constant. This shows that the general integrals of equations (I) should have other features apart from poles.

In a particular case, when $a_1b_2c=a_2bc_1$, one more coefficient of the series (II) still remains indeterminate, with the series possessing three arbitrary constants. Therefore, as in the particular case, there exists a general solution. Kovalevskaya added:

"This allows us to conclude that in this [particular] case the general integrals are also single-valued functions over the entire plane and possess only one essentially singular point $u=\infty$, and for finite values of u they only possess poles of the first order."

She hoped that a further investigation of the single-valued functions, whose existence she had proved,

"may some day clarify the properties of the more general functions

$$dx_\alpha/dt=g_\alpha(x_1, x_2, \ldots, x_n),$$

where g_α is a quadratic form in n variables." [75, p. 106]

The presented problem clearly shows the process of thinking that brought Kovalevskaya to the discovery of another case in the rotation problem.

Kovalevskaya had already obtained the basic results in her problem in 1886, the year in which the Paris Academy of Sciences announced two prizes for 1888 in the physico-mathematical sciences; a prize in mathematics, the grand prix of the mathematical sciences, consisting of a medal and 3000 francs, for an improvement of the theory of algebraic functions of two independent variables, and the Prix Bordin, also consisting of a medal and 3000 francs, for an improvement in an essential point of the theory of motion of a rigid body (see Appendix 5).

Charles Loren Bordin was a notary public who bequeathed in 1835 to the Institute of France a sum of 15 000 francs the interest from which to be distributed among the five French academies and paid as competition prizes. According to Bordin's will, the competition subjects should be in the public interest, prosperity of the mankind, the advance of sciences, or national pride.

Kovalevskaya decided to present her work for the Prix Bordin. But first she had to carry out all the vast mathematical calculations and prepare her memoir properly. She said in a letter she wrote in Summer 1888 to Mittag-Leffler:

"My head is now so full of mathematics that I can neither think nor talk of anything else. I have come to a definite result, and a very pleasant one: I mean that this case of the rotation problem can be integrated, as a matter of fact, with the aid of ultraelliptic functions. But I have yet to develop the final formulae, and I don't know whether I can manage to do so until the end of the month. I can't help informing you in more detail about my work. However, because of lack of time, I'll write very concisely, but please, try and think the problem over." [SK 273]

Let us consider this problem and write out the set of six equations in two groups defining the motion of a heavy rigid body about a fixed point [146]:

$$\begin{aligned} A\frac{dp}{dt} + (C-B)\,qr &= Mg\,(y_0\gamma'' - z_0\gamma'),\\ B\frac{dq}{dt} + (A-C)\,rp &= Mg\,(z_0\gamma - x_0\gamma''),\\ C\frac{dr}{dt} + (B-A)\,pq &= Mg\,(x_0\gamma' - y_0\gamma) \end{aligned} \tag{1}$$

and

$$\begin{aligned} \frac{d\gamma}{dt} &= r\gamma' - q\gamma'',\\ \frac{d\gamma'}{dt} &= p\gamma'' - r\gamma,\\ \frac{d\gamma''}{dt} &= q\gamma - p\gamma'. \end{aligned} \tag{2}$$

Here x, y, and z are the coordinates of an arbitrary point in the solid in a moving frame of reference related to the rotating solid, the origin being the fixed point of the solid; p, q and r are the components of the angular velocity vector of the solid; and γ, γ', and γ'' are the directional cosines of the vertical axis with respect to the moving x-, y-, and z-axes; M is the mass of the solid; (x_0, y_0, z_0) are the coordinates of the centre of gravity; and A, B, and C are the principal moments of inertia, i.e.

$$A = \int_\Omega (y^2 + z^2)\,\rho\,d\tau,$$

$$B = \int_{\Omega} (x^2 + z^2)\, \rho\, d\tau,$$

$$C = \int_{\Omega} (x^2 + y^2)\, \rho\, d\tau,$$

where Ω is the space within the solid; $d\tau = dx\, dy\, dz$; and ρ is the solid's density.

The problem is to find $p, q, r, \gamma, \gamma', \gamma''$ as time functions if their initial values $p_0, q_0, r_0, \gamma_0, \gamma'_0, \gamma''_0$ at a moment $t = t_0$ are known. The relationship

$$\gamma^2 + \gamma'^2 + \gamma''^2 = 1$$

should be valid.

It is known that the set of equations (1) and (2) has three first integrals

$$\begin{aligned} &Ap^2 + Bq^2 + Cr^2 - 2Mg(x_0\gamma + y_0\gamma' + z_0\gamma'') = C_1, \\ &Ap\gamma + Bq\gamma' + Cr\gamma'' = C_2, \\ &\gamma^2 + \gamma'^2 + \gamma''^2 = C_3 = 1 \end{aligned} \tag{3}$$

The set of equations (1) and (2) is autonomous, i.e. time is present only in the form dt; therefore, by solving (1) with respect to the derivatives and dividing each side of each equation into one of them, we obtain five equations. The last factor theory makes it possible to find another integral. Therefore, it is sufficient in addition to (3) to have one more, a fourth integral in order to obtain a complete solution of the problem.

Particular cases were known in which the fourth integral was given and algebraic.

(1) Euler's case, when $x_0 = y_0 = z_0 = 0$, i.e. the centre of gravity coincides with the fixed point. It is then not difficult to find a fourth integral

$$A^2p^2 + B^2q^2 + C^2r^2 = C_4 = l^2.$$

We can obtain a solution, of which we shall only write the relation of q to t (for the case $B > D$, where D is given below):

$$n(t - t_0) = -\int_0^{Nq} \frac{du_1}{\sqrt{(1 - u_1^2)(1 - \chi^2 u_1^2)}} = -F(Nq, \chi), \tag{4}$$

where we have

$$n=\sqrt{\frac{(A-B)(B-C)}{AC}},$$

$$N=\frac{l}{h}\sqrt{\frac{B(B-C)}{D(D-C)}},$$

$$\chi^2=\frac{(A-B)(D-C)}{(A-D)(B-C)}, \qquad D=\frac{l^2}{h}$$

(assuming $A>B>C$).

The function $q(t)$ can be found by inverting the elliptic integral (4):

$$q=-\frac{1}{N}\operatorname{sn}[n(t-t_0)],$$

where sn is the sign of the elliptic sine.

Similar relationships can be found for p and r; γ, γ', γ'' are found from

$$\gamma=\frac{Ap}{l}, \qquad \gamma'=\frac{Bq}{l}, \qquad \gamma''=\frac{Cr}{l}.$$

(2) Lagrange's case, for which $A=B$, $x_0=y_0=0$, i.e. we consider a solid with a symmetric ellipsoid of inertia whose centre of gravity is on the z-axis. Here the last of equations (1) is very simple: $C(dr/dt)=0$, whence $r=C_4$ is the new, fourth algebraic integral. Again the problem reduces to the inversion of elliptic integrals.

The case of complete kinetic symmetry ($A=B=C$) can easily be reduced to the Lagrange case by shifting the coordinates.

Kovalevskaya adopted a new approach to the rotation problem. She considered time t to be a complex variable as had Poincaré in the n-body problem (the real values of t being taken into account for each particular problem). This entailed the application of the theory of functions of a complex variable. She sought a solution by assuming that the functions p, q, r, γ, γ', γ'' have poles in the complex plane of variable t. If one of the poles is $t=t_1$, then we can seek a solution in the form of series

$$p=\tau^{-n_1}(p_0+p_1\tau+p_2\tau^2+\dots),$$

$$q=\tau^{-n_2}(q_0+q_1\tau+\dots),$$

$$r=\tau^{-n_3}(r_0+r_1\tau+\ldots),$$
$$\gamma=\tau^{-m_1}(f_0+f_1\tau+\ldots),$$
$$\gamma'=\tau^{-m_2}(g_0+g_1\tau+\ldots),$$
$$\gamma''=\tau^{-m_3}\ (h_0+h_1\tau+\ldots). \qquad (5)$$

Substituting these series into equations (1) and (2), Kovalevskaya determined the order of the possible poles:

$$m_1=m_2=m_3=2, \qquad n_1=n_2=n_3=1$$

and the existence conditions for solutions in the form (5). It turned out that they are possible both for the known two cases mentioned above and for another case which she has thus discovered, viz. when

$$A=B=2C, \qquad z_0=0.$$

Apart from the three integrals (3), the fourth algebraic integral in this case is

$$(p^2-q^2-c_0\gamma)^2+(2pq-c_0\gamma')=C_4=k^2;$$
$$\left(c_0=\frac{Mgx_0}{C}\right). \qquad (6)$$

These were the results Kovalevskaya covered concisely in her letter to Mittag-Leffler. Then she presented in the same letter some ideas concerning how to find $p(t)$, $q(t)$, ..., $\gamma(t)$, ..., and added that she had had no time to consider the last of the formulae in the letter.

"This is very frustrating because, as you can see, my work has become very interesting. The worst thing is that I'm so tired and so exhausted that I will sit and sit and think for hours on end about a simple thing that I could have solved in half an hour in other circumstances.

"I'd be very grateful if you would write to Hermite, as you proposed it, and tell him about me and my memoir. I have only another week to work on it, but I don't think I'll be able to meet the deadline...

"If the memoir is not ready by then, I'll have to put it off until the autumn because it is unlikely that I'll be able to develop my mathematical problems much in the summer. It's so exasperating to be so close to my goal and still not be able to reach it! And I'll have to find comfort in the fact that I've done a good job anyway and not grieve too much about the prize. But be so kind as to write to Hermite. However, I'll write to him myself

to give him an account of my work. But it would be better if you could write to him too...

"Anyhow, I can be consoled that there's nothing I can reproach myself for, at least with regard to the last few weeks because I have been as diligent as I possibly could." [SK 273]

Kovalevskaya wrote another letter in the summer of 1888 also describing her work on the rotation problem:

"Dear Gösta!

Today I corrected my memoir: tant bien que mal, plutôt mal que bien*. The problem has been solved completely. I've got over all the theoretical obstacles. I show that each of the six values p, q, r, γ, γ', γ'' can be rationally expressed as relations in the form $\theta_\lambda(v_1, v_2,)/\theta(v_1, v_2)$, where v_1 and v_2 are linear functions of time. That it was not easy you can see from the fact that Weierstrass, to whom I wrote both before and after I found that the problem could be solved using ultraelliptic θ-functions, and who seems to have thought it over very seriously, was unable to prove it. He wrote to tell me that he begins to think my task is impossible and that I have probably made a mistake in my considerations, i.e. that p, q, r are not single-valued functions of time. But I haven't succeeded, as a matter of fact, in finishing my calculations. They are, in fact, a purely mechanical thing and can be probably performed in less than a week by anyone who is accustomed to θ-functions. However, right now I'm so tired I can do nothing more. That is why I decided not to send the memoir to the Academy of Sciences directly, but to send it to Hermite and attach a long letter, in which I have covered in detail all the reasons for the delay in my work. I told him of some of what I think are wonderful and interesting results that I found with respect to the general case. Now Hermite has to decide what to do with my paper. I've chosen the following maxim to accompany my work:

Dis ce que tu sais,
Fais ce que tu dois,
Advienne ce qui pourra.**

---

* Bad or good, rather bad than good.

** Say what you know, do what you must, and whatever will be, will be.

"I leave for London tonight. I'll write at least a few lines to Anne-Charlotte from Copenhagen.

Devotedly Yours, Sonya." [SK 274]

After her scientific triumph in Europe and after she had been elected Corresponding Member of the Petersburg Academy of Sciences, Kovalevskaya returned to Russia in the summer of 1890. Having visited V. G. Imshenetsky, she wrote in her diary on May 18, 1890:

"Markov has claimed in public that my memoir is full of errors, but that he would show them only after the respected academicians who nominated me for membership had been kind enough to read my memoir... But after Markov had been made an extraordinary Academician, he was so condescending that he said in a private conversation, that my memoir was not as bad as it had at first seemed." [64, p. 181]

In order better to understand Markov's attacks, let us formulate the theorem.

Kovalevskaya's theorem. The equations of motion of a heavy rigid body about a fixed point in the general case do not have single-valued meromorphic solutions admitting five arbitrary constants, with the exception of the three cases mentioned above including the new case found by Kovalevskaya.

Later Markov wrote to A. M. Lyapunov (the letters we quote were undated):

"My initial claim with respect to § 1 of S. V. Kovalevskaya's memoir had only one aim: to prove that P. L. Chebyshev is not familiar with her works at all and therefore cannot evaluate them."*

In a letter Markov wrote to P. A. Nekrasov after Kovalevskaya had died, he said in connection with her work:

"Here are the words that I believe are unsubstantiated: 'It is easy to see, comparing the power indices of the first terms on the left-hand and right-hand sides of the equations discussed, that

* *LOA AN*, f. 257, op. 1, No. 20.

$$n_1=n_2=n_3=1, \qquad m_1=m_2=m_3=2\text{'}.$$

"Consequently, my objection is that one cannot draw the conclusion on the basis of comparing the power indices of the first terms.

"As you see now and have possibly heard from me before as well, I do not doubt the case S. V. Kovalevskaya found, but I doubt the uniqueness of the case."

It is a pity Markow did not convey his doubts to Kovalevskaya personally. G. G. Appelrot, P. A. Nekrasov's junior colleague, undertook the more detailed calculations for § 1 of Kovalevskaya's memoir after her untimely death. But Markov claimed that Appelrot's calculations

"*are senseless* because they are founded on the *wrong basis* and are only the substitution of one set of equations for another".

As a matter of fact, Appelrot only slightly changed Kovalevskaya's form of series (5) [194]. But Markov appealed to Lyapunov.

There are three letters of Markov to Lyapunov and two of Lyapunov's replies in the Archive of the USSR Academy of Sciences.

Markov put forth two basic objections.

The *first objection* was that one cannot by comparing the power indices draw the conclusion that the values $n_1=n_2=n_3=1$ and $m_1=m_2=m_3=2$ are the only ones possible. Indeed, by considering equations (3) in § 1 in the first of Kovalevskaya's memoirs, and given

"the principle of the greatest and least power indices, which has been known since Newton's time... we can note that each of the following six sets should contain at least two equal numbers:

$$\begin{array}{llll}
(1) & n_1+1, & n_2+n_3, & m_3,\ m_2, \\
(2) & n_2+1, & n_1+n_3, & m_1,\ m_3, \\
(3) & n_3+1, & n_1+n_2, & m_2,\ m_1, \\
(4) & m_1+1, & n_3+m_3, & n_2+m_3, \\
(5) & m_2+1, & n_1+m_3, & n_3+m_1, \\
(6) & m_3+1, & n_2+m_1, & n_1+m_2.\text{"}
\end{array}$$

In each of these six sets, S. V. Kovalevskaya assumes that not just two, but all numbers (four or three) are equal, and therefore she rejects without sufficient grounds an infinite multitude of cases, for instance

$$n_1 = n_2 = n_3 = 2, \qquad m_1 = m_2 = m_3 = 4.$$

The *second objection*. Kovalevskaya does not consider the case of multiple roots of her main determinant, although the existence of multiple roots does not prevent the possibility for a single-valued general integral to exist.

That the second objection was valid was noted by Appelrot [194], [195] and Nekrasov [196], and they found the solutions missed by Kovalevskaya. However, further investigation revealed that these new integrals were multivalued, and therefore the cases were withdrawn and did not change Kovalevskaya's theorem.

As to his first objection, Markov wrote:

> "My first point cannot be refuted, and moreover, I strongly doubt that anyone will be able to fill the gap in the foreseeable future."*

However, Lyapunov filled the gap soon afterwards. In his introduction to the article [197], Lyapunov published on the matter, he wrote that

> "while agreeing with Markov that Kovalevskaya's analysis was insufficient", "still tend to think that the problem is solved in the manner proposed by Kovalevskaya, and the solution can be reached without hindrance if only the problem is tackled somewhat differently..."
>
> "Therefore, I have decided to consider the problem from another viewpoint and try and apply to it the technique that for a long time seemed to me the most appropriate for this kind of problems." [13]

Lyapunov's article was delayed, and in the meantime Appelrot published a book called *The Problem of Rotation of a Heavy Rigid Body about a Fixed Point* [195], in which he proved Kovalevskaya's theorem on the basis of a general investigation concerning sets of nonlinear equations, which involved Lyapunov's theorems.

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* *LOA AN*, f. 257, op. 2, No. 56.

As to Lyapunov's article he proved Kovalevskaya's theorem and moreover, proved a more general theorem, namely: of all the cases when constants A, B, C, x_0, y_0, z_0 are real, and each of A, B, and C is non-zero, the functions p, q, r, γ, γ', γ'', defined by the equations (1), are single-valued for arbitrary initial values in only the three known cases. In other words, the solution cannot be a Laurent series with an infinite main part (Kovalevskaya only considered Laurent series with finite main parts).

Lyapunov's technique was to add small increments to the parameters p_0, q_0, r_0, f_0, g_0, h_0, and so vary the solution of the set of equations. Lyapunov thus obtained a set of linear equations with variable coefficients for the variations. However, if the simplest particular solutions of the given set that have the poles

$$p=a/t, \quad q=b/t, \quad r=c/t, \quad \gamma=f/t^2, \quad \gamma'=g/t^2, \quad \gamma''=h/t^2,$$

are taken as the initial solutions, then the linear set obtained is Eulerian, and the question as to whether its solutions are single-valued can be investigated in full. Lyapunov considers in particular the case of a set of real initial values which correspond to a practical physical problem.

Lyapunov's investigations, which he carried out with the skill of the great scientist he was, completed the question of single-valued general integrals in the problem of rotation of a heavy rigid body about a fixed point.

In connection with Kovalevskaya's theorem, which Lyapunov generalized, I can add that Kovalevskaya's mathematical intuition led her to the correct result. That she did not herself do the investigations as pointed out by Markov is psychologically understandable in that the probability of obtaining new cases this way is small, because by equalizing the indices in pairs, we obtain more than six equations for only six constants p_0, q_0, r_0, f_0, g_0, h_0.

Consequently, we see that Markov's attacks encouraged Lyapunov to pay more attention to Kovalevskaya's work and lead to the completion of the investigations she had begun. However, we can only feel sorry that Markov's suggestions were wrapped in a form that brought Kovalevskaya a lot of unpleasantness and it is a pity that Markov underestimated the work of the first Russian woman mathematician.

But that was Markov's nature. This can be seen by the

decision of the Moscow Mathematical Society at its session on November 17, 1892. After it had discussed a number of Markov's claims and heard Nekrasov's report:

> "The Society decided that because unfounded claims, such as those made by Prof. A. A. Markov with respect to the work of S. V. Kovalevskaya, V. G. Imshenetsky, P. V. Bugaev, and G. G. Appelrot, serve no purpose to science, and the discussion of such claims has vainly distracted the Society from its work, hereafter the Society will not accept for discussion any unfounded and sharp claims." [198, p. 845]

We have already seen that the French mathematicians admired Kovalevskaya's work. She had also other admirers, one of whom was G. G. Appelrot who dedicated his long life to the rotation problem. He said that one could see a sparkle of talent in Kovalevskaya's work on the rotation of a rigid body.

Bearing in mind the mathematical idea that guided Kovalevskaya, Professor V. V. Golubev, who incorporated some of Kovalevskaya's letters in his book (see [146]), has written in a letter to me dated December 15, 1953:

> "... in order to comprehend this idea, one should look at it from the standpoint of the scientific interests of the school of Weierstrass, which Sofya Vasilievna shared in full.
>
> "There are two circumstances that are striking when you read the work on the motion of a solid and compare it with later comments, supplements, and clarifications.
>
> "1. Nowhere in her work did S. V. Kovalevskaya show any particular delight with the new algebraic integral she had found in the case she considered. She utilizes the integral as an additional tool that allowed her to simplify the solution and that is all...
>
> "2. Nowhere in her work did S. V. Kovalevskaya seek cases with single-valued integrals, she only sought for cases with meromorphic integrals. A. A. Markov, with his desire to criticize her no matter what, found in this restriction a pretext for a severe criticism of the work. But in my opinion, it was this restriction that reveals the basic idea of the work.

"The following points seem pertinent to me.

"In 1876 Weierstrass published his studies [199] on entire and meromorphic functions; these investigations attracted the attention of scientists so much that in 1879 Picard translated them into French [200].

"Obviously, any problem (in the field of mechanics or wherever, that would lead to equations integrable in entire functions, could be considered completely solved, because the Taylor series of an integral will only give its value for a moment in time. But according to Weierstrass's theorem, meromorphic functions are ratios of entire functions; therefore, with some additional complications, the same inference is valid for equations with meromorphic integrals too. They can also be believed to have been completely solvable by means of series expansions of the entire functions whose ratios are the sought-for meromorphic integrals. And it is quite irrelevant whether these entire functions can be expressed in the studied ones.

"However, this idea could only be applied to meromorphic functions; if the integrals possess movable essentially singular points, then it is evident that they cannot be reduced to ratios of entire functions; S. V. Kovalevskaya did not deal with them.

"Consequently, S. V. Kovalevskaya was looking for cases when the equations of motion can be reduced to produce entire functions within the equation. Generally speaking, the last factor theorem isn't necessary. The last factor theorem allowed S. V. Kovalevskaya to simplify her conclusions and reduce the problem to known functions; but in theory it was possible to do without the last factor theorem. I have tried to expound these ideas in more detail in my lectures on the motion of a rigid body (Ch. II and Ch. VI)" ...

In the end of his letter, Golubev said that he considers the work of Kovalevskaya to be

"...an extraordinary application of the general ideas of the analytic theory of differential equations to problems in mechanics".

Kovalevskaya's research opened a number of new and exciting pages in the history of the rotation problem. Firstly, Kovalevskaya discovered another integrable case, for which

she found a fourth integral (in addition to the three known ones) and gave a general solution. Secondly, her results posed two mathematical questions concerning (1) the existence of single-valued solutions in the problem of rotation of a heavy rigid body about a fixed point, and (2) the existence of a fourth algebraic integral. Thirdly, Kovalevskaya's work stimulated many other investigations connected with particular solutions for the general problem, and studies of the particular solutions of Kovalevskaya's case.

As we mentioned, the problem of the single-valued solutions for arbitrary initial data was completely solved by Lyapunov.

Many scientists had contributed to the proof of the following theorem: if the ellipsoid of inertia is an ellipsoid of revolution, then a fourth algebraic integral exists in the cases of Euler, Lagrange, and Kovalevskaya. Therefore, a fourth algebraic integral in the problem of rotation of a heavy rigid body with a fixed point exists if and only if there are single-valued general solutions over the entire plane of t for p, q, r, γ, γ', γ''.

There is the question as to whether this is a mere coincidence or is due to hidden underlying reasons. Using the method of the small parameter, V. V. Kozlov showed [201] that the existence of an infinite number of multi-valued solutions prevents the appearance of a new single-valued analytic integral in the general case.

Many scientists have undertaken to simplify and polish the proof of these theorems, which may be called "nonexistence theorems", so this aspect can be considered to have been covered completely.

Further investigation firstly concerned particular solutions, i.e. solutions containing less than five arbitrary constants, or when the initial values of the sought-for functions are not arbitrary, but are interrelated. Russian scientists have contributed to this study and obtained some interesting results. These include V. A. Steklov [202], D. N. Bobylev [203], S. A. Chaplygin [204], and some others. One such case of integrability was found by W. Hess [205]. Attempts to find integrable particular cases are still being made and sometimes it happens that the "solutions" are wrong (for instance, Schiff's and Agostinelli's), i.e. they do not satisfy the initial differential equations [206].

There have been no important results since 1910, until an Italian mathematician D. Grioli produced his solution in 1947 [189]. New ways of finding integrable cases were needed. The avenues were opened up after research done by P. V. Kharlamov [209] who succeeded in reducing the set of six equations in the rotation problem to a set of two, and the investigations of E. I. Kharlamova who reduced the problem to only one integro-differential equation. Then in 1959 E. I. Kharlamova found a new case of integrability [207].

Poincaré introduced the concept of invariant relationships (sometimes termed the particular integral) for a set of equations

$$dx_i/dt = X_i(x_1, x_2, \ldots, x_n) \qquad (i = 1, 2, \ldots, n).$$

These are relationships of the form

$$f(x_1, x_2, \ldots, x_n) = 0$$

which imply the equalities

$$\sum_{i=1}^{n} X_i \frac{df}{dx_i} = 0$$

Proceeding from this definition, P. V. Kharlamov proposed a generalized concept of invariant relationships containing a number of parameters, and developed a method of obtaining exact solutions with invariant relationships. He considered a kind of generalized problem: instead of considering a gyroscope, where only the force of gravity acts, he took a gyrostat, in which there are other forces producing additional linear terms in the equations of motion.

The invariant relationships (some of them being combinations of the first integrals) are taken in the form of polynomials of the first, second, and higher degrees with respect to the sought-for functions. This allows classifying all the general and particular solutions obtained. G. V. Gorr *et al.* [206] produced a table of all these solutions, of which there turned out to be twenty. Kovalevskaya's solution is one of the most important.

Kovalevskaya's case has a far more complicated solution than either the two previously known general solutions or the subsequent cases. Therefore later researchers have a

better grasp of the obstacles in the general problem. This involvement particularly encouraged the development of the geometric interpretation of Kovalevskaya's case, although it was a formidable task.

Poinsot found a remarkable geometric interpretation for Euler's case involving one moving and one fixed axoids, the ellipsoid of inertia rolling along a horizontal plane. The geometric interpretations, which Darboux and other scientists produced for Lagrange's case are more intricate.

N. E. Zhukovsky believed demonstrative interpretations, or simulations, of the motion are essential. He argued that they make it possible to explain mathematical ideas to laymen who want to comprehend them. For Kovalevskaya's case Zhukovsky has used an interpretation of auxiliary variables s_1 and s_2 [208], but there is no generally demonstrative representation similar to the one given by Poinsot for Euler's case.

P. V. Kharlamov's research is very significant. He expounded a godograph technique and proceeded from his kinematic equations [209]. An article by Kharlamov and Mozalevskaya [210] gives a geometric interpretation of some of the motions of Kovalevskaya's gyroscope. A fixed and a moving godographs of the body's angular velocity vector are considered; numerous kinds of motion were obtained for different ranges of the problem's parameters.

There are very many publications on the problem of a rotating solid having a fixed point. Many scientists have endeavoured to find solutions, analyzing them and investigating the various properties of the possible forms of motion. They were also interested in Kovalevskaya's case, its details and particular instances, when the solution can be reduced to elementary functions. There are also many generalizations, such as the problem of a gyroscope filled with a liquid. Different devices have been proposed to reproduce the motion in Kovalevskaya's case. One such device is described by N. B. Delone:

"An example of such motion is that of a rectangular parallelepiped with dimensions $2a<2b<2c$, where $c=b\sqrt{3}$, and supported at a point $x_0=\sqrt{(b^2-a^2)/3}$ from the centre of gravity and lying on the straight line passing through the centre of gravity and parallel to the $2a$ edge.

A support of this sort would be a needle passing through the parallelepiped." [191]

The first model of Kovalevskaya's gyroscope was made by Schwarz at her request. The device consists of two identical parallel cylinders $2H$ in height with the base radii R; the distance $2b$ between the axes of the cylinders is defined by the formula $b^2=H^2-\frac{1}{4}R^2$ (with $b>R$). The fixed point is a distance a away from the centre of gravity with $a^2=\frac{1}{3}H^2-\frac{1}{4}R^2$. For these conditions $A=B=2C$ is valid [13, p. 243].

Whatever future investigations may reveal in the problem of a rotating solid, Sofya Kovalevskaya's name will forever be foremost.

CONCLUSION

Sofya Kovalevskaya published nine scientific works dealing with six different topics: the problem of rotation of a rigid body, the existence theorem for a set of partial differential equations, the problem of reduction of Abelian integrals, the shape of Saturn's ring, the refraction of light in crystalline media, and finally, the Bruns theorem from the theory of potential.

An evaluation of Kovalevskaya's work was given by the Moscow Mathematical Society, to which she was a member since 1881. On March 3, 1891, a session was held in her memory. At the session, the physicist A. G. Stoletov gave a short review of her life and achievements and remarked that he was personally acquainted with both Sofya and Vladimir Kovalevsky, had visited them at their home in Moscow, and had only the most favourable of recollections.

The famous Russian scientist N. E. Zhukovsky spoke about Kovalevskaya's work in mechanics, and in particular about the problem of rotation of a rigid body, to which, as was mentioned, Zhukovsky himself had contributed. He said:

> "In the Summer of 1889 I met Poincaré in Paris; he told me that Sofya Kovalevskaya was developing the case she had considered [in the rotation problem] and hoped to solve the problem where the centre of gravity lay on the equatorial plane of an inertial ellipsoid that was an ellipsoid of revolution. Unfortunately, her intimely death has put an end to all these hopes and left us without a compatriot who has done much to make Russia famous". [159, p. 22]

Professor of mathematics P. A. Nekrasov gave the third report on Kovalevskaya's results in pure mathematics [160].

They all highly esteemed the work of the Russian woman scientist, acknowledging her to be the equal of the most talented men mathematicians. Her deep insight into the modern methods of mathematics had enabled her to make outstanding discoveries in it.

Foreign mathematicians also paid tribute to the great Russian woman scientist. Thus, Paul Du Bois-Reymond said that

> "she excelled her women predecessors and, what can be said to her honour, she occupied one of the most prominent places among modern mathematicians". [274]

Poincaré, one of the greatest of French scientists, was a great admirer of Kovalevskaya as a mathematician. He never missed an opportunity to credit her in his work. Thus, in his paper "A self-analysis of the scientific work of Poincaré," he mentioned Kovalevskaya many times and on equal footing with Cauchy, Fuchs, Briot, and Bouquet. He wrote in section IV, which dealt with celestial mechanics:

> "I have used the method that Mrs. Kovalevskaya used in her memoir on Saturn's ring: the expansion of the periods of an elliptic function into power series of the modulus." [276, V.III, p. 643]

In his *Analytical Résumé* of his work, Poincaré wrote in connection with the solution of partial differential equations of the first degree:

> "Cauchy and Kovalevskaya have taught us how to expand into series the integrals of these equations in the neighbourhood of an ordinary point." [276, V.III, p. 581]

Lampe, the German biographer of Weierstrass, called Kovalevskaya Weierstrass's genius disciple and believed Anne-Charlotte Leffler was too harsh when she said that all Kovalevskaya's accomplishments were nothing but the development of the ideas of her great teacher. Leffler had overestimated Weierstrass's influence, possibly because Kovalevskaya herself had been too modest [131, p. 68].

To evaluate Kovalevskaya with a historical perspective, one should compare her with other mathematicians, both men and women.

Prior to Kovalevskaya, we can only indicate a couple of dozen of women scientists, throughout history.

Hypatia, daughter of Theon of Alexandria was famous in the 5th century A. D. for her learning. She lectured on philosophy and mathematics, and wrote commentaries on the work of Apollonius and Diophantus in mathematics (unfortunately, her commentaries have not survived). Kingsley, Mauthner, and Meyer have all written about her. Her life ended tragically for in 415 A. D. Hypatia, a pagan, was cut to pieces by a mob of fanatical Christians incited by priests [277].

In 1978, the *UNESCO Courier* commemorated the 300th anniversary of the first woman to receive a doctorate. She was Lucretia Kornaro of Venice and she defended her dissertation in Padua. The defence was made into a ceremonial occasion in Padua cathedral in front of a large gathering of the public. In the end, she received a diploma of Doctor of Philosophy. However, there had been other women in Italy starting in the 14th century, who had lectured in universities, sometimes substituting for their fathers.

The Marquise Émilie du Châtelet translated Newton's book *Principia* from Latin into French, and wrote her own commentary which was edited by A. C. Clairaut.

Laura Bassi gave lectures on physics in Bologna. She was uncommonly beautiful and had twelve children [277].

In 1748, an Italian woman Maria Gaetana Agnesi published her best-known book *Instituzioni analitiche ad uso della gioventu italiana* (A Course of analysis for the use of Italian youth), which was translated into French in 1775. One of the elegant third-order curves is called "the witch of Agnesi". Agnesi left the learned world early by retiring to a convent.

Hortensie Lepaute, a French woman, did astronomical calculations with Clairaut and together they compiled a table of the oscillations of pendulums of various lengths.

Hypatia, du Châtelet, Agnesi, and the other women mentioned all showed their ability to assimilate the most advanced mathematical theories of their time and to present their knowledge to others. Sophie Germain revealed a more

profound creative talent. In 1808 she won the Napoleon prize of the Paris Academy of Sciences for her research in the theory of elasticity. She also worked in number theory.

Mary Sommerville was elected an honorary member of the Royal Society of London for her work in physics and astronomy. She published an English version of *Celestial Mechanics* by Laplace.

The only woman elected (1783) to full membership of an academy (the Russian Academy and not the Academy of Sciences) in pre-revolutionary Russia was Duchess Ekaterina Romanovna Dashkova (1744—1810), the director of the St. Petersburg Academy of Sciences and president of the Russian Academy (1783-1796).

The Russian Duchess Evdokiya Ivanovna Golitsyna, who was well educated, wrote an article "An analysis of the concept of force", a part of which was published in Petersburg (1837) and the other part in Paris (1844). The great Russian poet Pushkin was charmed by Golitsyna and delighted with her power of reason. He wrote verse in her honour [278, p. 312]

Recently, much has been written about Ada Augusta Lovelace, Byron's daughter, who has been called "the first programmer". She collaborated with the inventor of the first computer Charles Babbage; it was she who introduced the notion of a loop.

Sofya Kovalevskaya was superior to her women predecessors in her talent and the importance of her results, and she was the first woman elected to corresponding membership of an Academy of Sciences.

In 1889, i.e. when Kovalevskaya was alive, Vera Iosifovna Shiff, a woman, was granted permission to teach mathematics at the Bestuzhev Higher Courses for Women. She compiled collection of problems in analytic geometry and differential and integral calculus. Remember this was the time when E. F. Litvinova, a Doctor of Mathematics, had also appied for a position at the Bestuzhev Courses and had been rejected.

In 1903, Russian newspapers carried stories about two more women Doctors of Mathematics: Nadezhda Nikolaevna Gernet and Lyubov Nikolaevna Zapolskaya.

Lyubov Zapolskaya became Professor at the Moscow

Higher Courses for Women and Nadezhda Gernet was Professor at the Bestuzhev Petersburg Higher Courses. The latter received her doctorate from Göttingen University for her work on the calculus of variations and in 1912 published a book *On the Fundamental Simplest Problem of the Calculus of Variations* [279]. Recently, researchers into the theory of control have shown renewed interest in this book (see [280], p. 698).

A major mathematician was Emmy Noether (1882-1935), Professor of Göttingen University, the founder of a new trend in algebra. She was a lonely woman, who devoted all her life to science, and is often compared with Sofya Kovalevskaya, whose private life was full of complicated feelings and her work in mathematics alternated with literary activities [282].

The election of Kovalevskaya to corresponding membership of the Petersburg Academy of Sciences opened the ways for other women. In fact, several women were elected Corresponding Members and Honorary Members of the Petersburg Academy of Sciences after Kovalevskaya. Praskoviya Sergeevna Uvarova, an archeologist, became an Honorary Member in 1894; the philologist Olga Izmailovna Sreznevskaya became a Corresponding Member in 1895: and the botanist Olga Aleksandrovna Fedchenko was elected to corresponding membership in 1906. Maria Sklodowska-Curie, the famous physicist, became a foreign Corresponding Member in 1907 (and an Honorary Member in 1926). Earlier, in 1898 Queen Elisabeth of Rumania, was elected Honorary Member. She wrote under the pen-name Carmen Sylva and collected Rumanian folklore.

Ekaterina Alekseevna Naryshkina received her doctorate in 1939. Her name is only known to a small number of mathematicians, because her work was related to the special and difficult field of theoretical seismology, namely, the science of the elastic oscillations in a solid. Her main results have been presented by S. L. Sobolev in the Russian edition of *Differential and Integral Equations in Mathematical Physics* by Frank and Mises [281].

Nina Karlovna Bari was the first woman in the Soviet Union to receive a doctorate in the physico-mathematical sciences. She was so awarded for her work in the theory of trigonometric series without defending a thesis. Lyudmila

Vsevolodovna Keldysh was one of the first to defend a doctoral thesis, her topic being "The structure of B-sets."

Many women in the Soviet Union work in science, mathematics in particular, and nobody is amazed when they win titles and awards for their research, or are elected to membership of the Academies of Sciences. The Corresponding Member of the USSR Academy of Sciences Olga A. Ladyzhenskaya, Professors Olga A. Oleinik, Nina N. Uraltseva, and many other younger Soviet women can be called outstanding mathematicians of our time.

Apart from her scientific merit, Kovalevskaya occupies an exceptional position in the history of the feminist movement. Her versatile mind, lively nature, and literary talent contributed greatly to her vast popularity.

Fame and admiration have surrounded Sofya Kovalevskaya since she was young. And this was a natural tribute to her many gifts and pioneering in science, especially in such an involved field as mathematics.

Kovalevskaya's scientific popularity can be accounted for by her good choice of problems and their excellent solution. The two most essential of her works are related to the basic issues of mathematics and mechanics. The simplicity of some of her results made it possible to include them in courses of mathematics and mechanics. Kovalevskaya's research on the rotation of a rigid body provided the momentum for many further investigations.

Crowned with just glory, Sofya V. Kovalevskaya will remain forever both in science and in the history of the Russian social movement.

Appendices

1. Sofya Vladimirovna Kovalevskaya and Her Recollections

Sofya Vasilievna Kovalevskaya was 28 years old when she gave birth to her daughter Sofya, who was nicknamed Fufa. Sofya Vasilievna was a restless and doting mother. She was always fearful that the child would be hurt or infected or neglected by her nanny. She even thought up of a special technique of swaddling the baby.

When she went abroad in late 1880, Kovalevskaya was afraid to take her two-year old daughter with her and left her in the custody of Yuliya V. Lermontova.

When Kovalevskaya went abroad again, in 1881, she took her daughter, then three, and her nanny Marya Dmitrievna. Later the girl had to move from city to city and live with different families. When in Russia, she would stay with Yuliya Lermontova or sometimes with the family of her uncle Aleksandr O. Kovalevsky. When in Sweden, she stayed most frequently with the Gyldéns, and sometimes with Anne-Charlotte or Signe and Gösta Mittag-Leffler. Once she stayed with Ellen Key.

The girl was well cared for in all the families, and Sofya Vladimirovna had the best possible reminiscences of her time with these people.

Fufa's last letter to her mother was written during the Christmas holidays (a month before Sofya Kovalevskaya died). On January 10, 1891, the girl wrote that she was going to visit Palme in Djursholm for a children's party.

> "I'm learning Latin a little; when the Gyldéns tidied up an old wardrobe, they found a grammar and another book. I'm studying every day and can understand the first pages.
>
> "My overcoat is not ready yet, but I think it'll be ready soon and that it'll be nice. You know, mummy, the shoes we bought were so nasty, they started to tear right away, and not the sole, but the leather itself.
>
> "Have you got many presents for Christmas? Now I have to go to Djursholm. Give my regards to Maksim Maksimovich. Fru Gyldén and Elsa send you their regards.
>
> Good-bye, dear mummy, Yours, Fufa.
>
> "The box with my things hasn't arrived yet."*

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* *AAN*, f. 603, op. 1, No. 27.

Two weeks later, Fufa wrote a letter to Maksim M. Kovalevsky that showed that warm feelings existed between them.

"January 24, 1981

"Dear Maksim Maksimovich,

"Thank you very much for your letter [apparently, Christmas greetings]. I'm awfully sorry that you're so ill. I hope you'll get well soon.

"I will have enormous pleasure next summer to go with you to the south of Russia. I hope you'll be healthy again. I'm still having my riding lessons, but only a few times a week. Next summer Yuliya Vsevolodovna [Lermontova] promised me a horse so I'll be able to ride.

"Next Sunday our class will perform a German play. I have a role. It'll be fun, but I'm a little afraid. Recently, Mr. Backlund from Petersburg visited us. He's brought me some books and a box of sweets from uncle Sasha [Aleksandr O. Kovalevsky]. I liked especially *To Russian Children* by Dostoevsky.

"I was in the theatre recently and saw "Frikelsbrödern".

"It was very funny.

"The Gyldéns send you their regards. Good-bye, dear Maksim Maksimovich.

Yours, Fufa". (*Ibid.*)

When Sofya Kovalevskaya died, there was a problem as to whom was to look after the girl, and she was glad to live in Sweden with the Gyldéns until she finished Swedish secondary school. Then she furthered her education in Russia, in the senior grades of a Russian high school.

Fufa studied well, liking English and physics in particular. She spoke German fluently by that time, but had to have private lessons in algebra and geometry.

The girl went to Russia during the summers and began to work hard to prepare for her exams. She entered the sixth grade of Tagantseva's high school in Petersburg.

Sofya Vladimirovna (Fufa) finished the high school in 1897 when she was 19. She intended to enter the Women's Medical Institute, but at that time only women who had reached their majority i.e. no younger than 20, were accepted. Being free, she decided to enter in the meantime the Physico-mathematical Department of the Bestuzhev Higher Courses for Women. Possibly, she wanted to try herself in mathematics. However, she did not remain with the Courses and, when she was 20, she became a student of the Medical Institute.

As far as I know, after Fufa graduated from the institute, she worked in a laboratory and did not practice medicine. When I met her (after the Second World War), she was a middle-aged woman with black brows, and somewhat severe in her appearance. She spoke Swedish, French, and German fluently. She generously allowed her parents' correspondence to be studied, especially by S. Ya. Shtraikh.

When the USSR Academy of Sciences received photocopies of the letters from the archive of Mittag-Leffler in Sweden, Sofya Vladimirovna helped decipher and translate them, mainly the Swedish letters.

She passed away in 1952 aged 74 and her ashes were buried at the Novodevichy Convent cemetery.

After Sofya Vladimirovna's death, several recollections of her life with her mother in Stockholm were found and published [82, pp. 144-154], [64, pp. 360-368]. They are now in the Archive of the USSR Academy of Sciences*. Below are some excerpts from these interesting reminiscences.

2. Daughter's Recollections

My first reminiscences of my mother are associated with some travels over a railroad, with a chest, from which alcohol burners and pots were taken to boil some milk and cook semolina. Mummy is tender but anxious, she often kisses me, then puts me to bed and gives me a thermometer. Then, as I remember, a strange man appeared; I have to let him examine my throat, which I was really reluctunt to do, and I was put to bed again and fell asleep... Apparently, this happened in 1882, when, according to mummy's recollections, she went abroad again after several years in Russia. Evidently, she took me with her.

After that I had no reminiscences of my mother, because I stayed in Odessa, with my uncle Aleksandr O. Kovalevsky, while mother lived abroad and I saw her seldom. I clearly remember my life in my uncle's family the happiest period of my early childhood.

The family included Aleksandr Onufrievich Kovalevsky himself, his wife Tatiana Kirillovna Kovalevskaya, and their three children. There was a large garden beside the house with a lot of flowers, a big pool of water, and a vineyard. The sea was not far away, and we rode there to bathe sometimes. I had my own nanny, Marya Dmitrievna, who was with me since my early childhood and whom I loved very much. My mother was somewhere abroad and father was in Moscow; he came to see me from time to time. My relations with him were very good and I looked forward to his comings.

Then something happened to my father. An alarming letter was received; the grown-ups became serious, and there were even tears in my uncle's eyes. He took me on his knees and told me with a catch in his voice that my dad was very ill and couldn't come to us, so he would help me instead of my father. I didn't understand what had happened, but I weeped too, and there was a dim comprehension of great trouble in my heart.

My cousins were very tender with me and often kissed me. Then my mother arrived. I already became estranged from her and at the beginning tried to avoid her. She wore black, was very sad, and often cried. She kissed me so passionately that I was even a little frightened.

From the bits of the conversations between the grown-ups I picked up I discovered mother wanted to leave soon and this time she took me with her. I was very sorry to part with my uncle and his family, but I was sorry at the same time for mother, who spent much time with me.

She was very tender and told me how much I had grown, and that she was very glad to take to Stockholm a daughter who was already big. She stayed for a time in Semenkovo, going for walks with the

* *AAN*, f. 603, op. 2, No. 4-7.

others and making fine embroideries, to which she was always attracted in her periods of rest from her scientific work.

She told me very much about Sweden, of the room she had arranged for me, and of friends' children. She also checked my knowledge of German and named some things in Swedish...

Then the day came for mother and I to begin to collect our things. Yuliya Vsevolodovna [Lermontova] was very sorry to part with me, and all our friends came to wish us well. Many people thought that it would have been better to leave me with Yuliya Vsevolodovna and not take me to a strange country, where mother would not have much time for me, and the environment would be strange. But mother was firm; she had become familiar with Sweden, had a good position there, and believed that she ought to bring me up herself.

I could not go to school for the first year, and I studied Swedish by making trips to the market with our servant and learning from her the names of all the things she bought.

Our apartment was in a new part of the city, not far from a large park next to a forest. Many people went there on Sundays, but the place was almost deserted on weekdays. After her lectures, mother often went there with the writer Anne-Charlotte Edgren, who was her great friend. Sometimes, they were joined by Anne-Charlotte's brother Prof. Gösta Mittag-Leffler. But more often they went to the skating-rink, and I could accompany them as soon as I had learned to skate.

During their walks, mother and Anne-Charlotte had long heart-to-heart talks, discussed the plots for Anne-Charlotte's new stories and plays, and sometimes drew up projects for people's future life, "when there'll be neither rich, nor poor, and everybody will be equal". These ideas were then in the air and fascinated the two friends very much.

I can hardly remember the first apartment to which I was brought from Russia and where we lived in Stockholm, but I clearly see before my mind's eye the other apartment to which we moved the following year and where we lived until mother died. It was on the third floor of four- or five-storeyed brick house at No. 56, Sturegatan Street, in the same part of the city as the first apartment. There was a small public garden on the other side of the street, opposite the windows of our apartment, and Lombardy poplars grew there ...

The apartment had four rooms, a kitchen, and a small room for the servant. Our housemaid Augusta had her room near the kitchen. She could receive visitors there, and I loved to sit there in the evenings when mother was not at home. We had other servants, too, but Augusta stayed with us the longest, and she was our last servant remaining until mother died.

Anne-Charlotte Edgren-Leffler said of our apartment in her recollections that it made an impression of something incidental, about to fall apart at any moment. Possibly, it seemed so to her for she was accustomed to the respectable Swedish apartments of well-to-do Swedish families... However, our apartment seemed gorgeous to me. Our living-room with its redwood furniture upholstered in red satin, furniture mother had brought from Russia, seemed magnificent to me, and I hardly noticed the defects that were striking for Anne-Charlotte. There was a tall mirror in a gilded frame on a low marble base, and two trellises with live plants (I remember ficuses on one and blooming tra-

descantias on the other). And old man called Nordenskjöld would sit in this living-room and tell us such interesting stories about his journeys around the shores of Siberia on the ship *Vega*, and we met here the young Nansen, who was just embarking on his career as an Arctic explorer. We were visited here by university Professors, such as Gyldén (an astronomer), Brögger (a geologist), Leche (a zoologist), Doctor of Medicine Medin (the Heine-Medin disease was named after him), and Mittag-Leffler, together with his sister, Ellen Key (a writer), and the editor of a newspaper (*Free-Thinker*) called Branting, who became very famous later as a representative of the social democratic party in Rigsdag, but then he was often in prison for his insulting remarks about the King.

From 1888 onwards Professor Maksim Maksimovich Kovalevsky often visited us here; he came to give lectures on sociology. We would entertain Swedish and Norwegian artists, writers, and critics, such as Brandes and Ibsen and there were many others whose names I've already forgotten.

As to my feelings to mother, I have to say that they were rather complicated and were not as intimate as those for Yuliya Lermontova. I could chat with the latter about all my experiences, withholding nothing from her and never fearing I might cause her displeasure. She loved me as I was, and did not apply any "pedagogics" or try to influence me in a way. This does not mean that she was indifferent to my drawbacks or pampered me without measure, she just made her reproofs very gently, and never touched my self-esteem. Obviously, she had her own ideal of a "daughter" and tried to see it realized by me. No doubt, she loved me but thought it improper to give vent to her tenderness, and I was almost never caressed, apart from a kiss for the night. Only when she was leaving for holidays or returning from her travels would she suddenly started to smother me with impetuous and passionate caresses, but these fits were soon over. For she still wanted to bring me up. She had read articles on education and talked about upbringing with familiar Swedish educationalists. Besides, she seemed to be under the influence of her reminiscences of my father and the errors that led to his tragic end. His dismal mood during the last years of his life, lack of will and departure from science on the one hand, and his "lightheadedness" on the other, had frightened her and made her fear these feelings would reappear in his daughter, who though resolute and strongwilled, could sometimes be just as gloomy as he had been. She wanted to make me a strong and energetic person and despaired when she did not succeed.

Ellen Key soon became one of my mother's closest friends, second only to Anne-Charlotte Edgren-Leffler. She was then a modest teacher in a private school for girls, but later she became socially prominent as an advocate of the "right of children". She wrote a book *The Age of Child*, which became very popular and was translated into Russian, among other languages. She was not a proponent, as one might think, of the social rights of women, like most of the so-called feminists, but on the contrary, she called women to go deeper in their family life and urged girls to look for "true happiness" based on a marriage with a man they really loved and without any other motive besides their feelings. Her views were mistaken by the conservative sections of society as an advocacy of "free love", she was severely

castigated in the press, had to leave teaching and stay for a time abroad, in Italy and Germany, earning her living by writing. Several years later, when the opposition subsided, she could return to Sweden. She had in the meantime saved money and could buy a small house, which was bequeathed to the Union of Working Women. Ellen Key died in 1925, honoured by everybody.

Ellen Key noticed the somewhat strange relationship between my mother and myself because of mother's excessive educational zeal and tried to convince her not to be so impatient in the upbringing of the child and not to expect results too fast. This is what I gathered from bits of conversations I heard, and later Ellen Key told me this herself.

Ellen Key wrote one of the best biographies of my mother.

3. S. V. Kovalevskaya: Application to Be Admitted to an Examination for a Magister's Degree*

"To His Excellency the Rector of St. Petersburg University Privy Councillor Petr Grigorievich Redkin from Doctor of Philosophy of Göttingen University Sofya Vasilievna Kovalevskaya.

Application

On the basis of the rules existing for Doctors of foreign universities wishing to take the examination for Magister of Mathematics at St. Petersburg University, and presenting my Doctor's diploma, I humbly beg Your Excellency for permission for me to sit the magisterial examination.

St. Petersburg, March 27, 1875

Doctor of Philosophy of Göttingen University

Sofya Kovalevskaya

My residence is 14 First Line, Vasilievsky Island."

On the same day, Magister of Mineralogy and Geognosy Vladimir Kovalevsky submitted his application to the Dean of the Physico-mathematical Faculty A. N. Beketov:

"Desiring to become a Privat-Docent at St. Petersburg University in the subject of Paleontology, I humbly beg You to instruct, as is in your power that I be permitted to give lectures on the above subject".

The following was decided at a session of the Physico-mathematical Faculty on April 25, 1875.

"2. The Application of Sofya Kovalevskaya:
postpone until next time.

"4. The day for the test lectures by V. O. Kovalevsky is set as Wednesday, May 12 [indistinct], the topic will be the evolution of the brachiopod mollusks.
Signed by: A. Beketov, A. Savich, I. Somov,

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* *LOA* f. 14, op. 3, No. 14816.

A. Korkin, Yu. Sokhotsky, A. Famintsin, and A. Butlerov."

The session of May 10, 1875 decided:

"1. Admit S. Kovalevskaya to a magisterial examination. Signatures: Dean A. Beketov, M. Okatov, A. Butlerov, F. O-v [indistinct], D. Mendeleev."

4. Courses of Lectures Given by S. Kovalevskaya at Stockholm University*

1. Theory of partial differential equations (Autumn 1884).
2. Theory of algebraic functions following Weierstrass (Spring 1885).
3. Elementary algebra (Spring 1885).
4. Theory of Abelian functions following Weierstrass (Autumn 1885 to Spring 1887).
5. Theory of potential functions (Spring 1886).
6. Theory of motion of a rigid body (Autumn 1886 and Spring 1887).
7. On the curves defined by differential equations, following Poincaré (Autumn 1887 and Spring 1888).
8. Theory of theta-functions following Weierstrass (Spring 1888).
9. Applications of the theory of elliptic functions (Autumn 1888).
10. Theory of partial differential equations (Spring 1890).
11. Theory of elliptic functions following Weierstrass (Autumn 1889).
12. Application of analysis of integer theory (Autumn 1890).

5. Programme des Prix Proposés Pour les Années 1887, 1888, 1889, 1890, 1891 et 1895.

GÉOMÉTRIE

GRAND PRIX DES SCIENCES MATHÉMATIQUES.

(Prix du Budget.)

(Question proposée pour l'année 1888.)

"*Perfectionner la théorie des fonctions algébriques de deux variables indépendantes.*"

Les Mémoires manuscrits destinés à ce concours seront reçus au Secrétariat de l'Institut jusqu'au Ier juin 1888; ils devront être accompagnés d'un pli cacheté renfermant le nom et l'adresse de l'auteur.

* This list was published by Mittag-Leffler [186].

Ce pli ne sera ouvert que si le Mémoire auquel il appartient est couronné.

Le prix sera une médaille de la valeur de *trois mille francs.*

PRIX BORDIN

(Question proposée pour l'année 1888.)

"*Perfectionner en un point important la théorie du mouvement d'un corps solide.*"

Le prix sera une médaille d'or de la valeur de *trois mille francs.*

Les Mémoires manuscrits destinés au concours seront reçus an Secrétariat de l'Institut jusqua'au Ier juin 1888; ils seront accompagnés d'un pli cahceté renfermant le nom et l'adresse de l'auteur. Ce pli ne sera ouvert que si le Mémoire auquel il appartient est couronné.

C. R., 1886, 2e *Semestre.* (T. CIII, N°. 26.)

6. Photocopies of Letters* Cited from the Archive of G. Mittag-Leffler in the Archive of the USSR Academy of Sciences

1. Letters from S. V. Kovalevskaya to G. Mittag-Leffler [SK]. Album No. 15, 1880-1883; No. 16, 1884; No. 17, 1885; No. 18-19, 1884-1885; No. 20, 1886; No. 22, 1888; No. 23, 1889; No. 24, 1890-1891.
2. Letters from G. Mittag-Leffler to S. V. Kovalevskaya [ML]. Album No. 35, 1880-1884; No. 36, 1885; No. 37, 1886; No. 38, 1888; No. 40, different years.
3. Letters from J. Perott to S. Kovalevskaya, album No. 45 [P].
4. Letters from C. Runge to S. Kovalevskaya, album No. 46 [R].
5. Letters from foreign mathematicians, album No. 49 [FM].
6. Letters from Russian mathematicians, album No. 51 [RM].

* All of these letters (and messages) were unpublished by 1984 (*AAN*, f. 603, op. 1). All the available letters (and messages) from Kovalevskaya to Mittag-Leffler and from Mittag-Leffler to Kovalevskaya were published in [286].

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* Signed *S.K.*

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S. V. KOVALEVSKAYA: CURRICULUM VITAE

1850 on January 3(15), born Sofya Vasilievna Kryukovskaya, since 1856 Korvin-Krukovskaya.
1853-1858 lived in Kaluga with parents.
1858 the family moved to Palibino, now in Pskov region.
1868 in spring, met Vladimir O. Kovalevsky.
1868 on September 15 (27), married to Vladimir O. Kovalevsky and left with him for Petersburg to study science.
1869 in April, left for Heidelberg.
1870 in October, moved to Berlin and started lessons with Weierstrass.
1874 in July, Sofya Kovalevskaya was granted the degree of Doctor of Philosophy by Göttingen University.
1878 on October 5 (17), her daughter was born.
1879 on December 28 (January 9, 1880), made a presentation at the VI-th Congress of Russian Natural Scientists and Physicians in Petersburg, the topic being "On the Reduction of Abelian Integrals of the 3rd Rank to the Elliptic Ones."
1881 on March 17 (29), elected to membership in the Moscow Mathematical Society.
1882 on July 24, elected to membership in the Paris Mathematical Society.
1883 on April 15 (27), died Vladimir O. Kovalevsky.
1883 on August 22 (September 4), reported to the VII-th Congress of Russian Natural Scientists and Physicians "On the Refraction of Light in a Crystalline Medium".
1883 on November 18, Sofya Kovalevskaya arrived in Stockholm.
1884 on February 11, gave the first lecture at Stockholm University.
1884 appointed Professor extraordinary of Stockholm University.
1888 on December 24, awarded the Prix Bordin by the Paris Academy of Sciences for her work "On the Rotation of a Rigid Body about a Fixed Point."
1889 on November 7 (19), elected Corresponding Member of the Petersburg Academy of Sciences at a session of the Physico-mathematical Section; the election approved by the general session of the Academy of Sciences on December 2 (14).
1889 awarded with the prize of Stockholm Academy of Sciences for her second memoir on the study of rotation of a rigid body "On a property of a system..."
1889 appointed Professor of Stockholm University for life (*à vie*).
1891 on February 10, Sofya Vasilievna Kovalevskaya died.

TO THE READER

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Academician STEKLOV:
Mathematician Extraordinary

by **V. S. Vladimirov,** Mem., USSR Acad. Sc.,
and **I. I. Markush,** Cand. Sc. (Phys.-Math.)

Academician V. A. Steklov (1864-1926) was a remarkable Soviet mathematician. An outstanding scientist in his own right, he was the founder of the Petersburg School of Mathematical Physics and many of his former pupils became leading Soviet scientists themselves.
He was elected Vice-President of the Academy of Sciences in 1919 and during the very difficult years of the Civil War in the country Steklov guided the Academy of Sciences both in its scientific work and in its internal affairs. He was an enthusiastic member of many commissions that were concerned with the national economy, the progress of science, and the organisation of higher education.
The book contains both Steklov's biography and simplified accounts of his main contributions to science. It draws on archival materials, his correspondence with other scientists, and the reminiscences of his contemporaries. Some of the material has hitherto remained unpublished and the book contains a full bibliography of his work.
The book will be of interest to a wide circle of readers, including those concerned with the history of Russian science and the development of scientific concepts.

Rem Khokhlov

by **V. I. Grigoryev**, D. Sc. (Phys.-Math.)

The book is about Rem Khokhlov, his childhood, youth, and later years. It is about a great scholar, public figure, administrator of scientific research and education, Rector of Moscow University, Academician, and mountain-climber. His illustrious personality was remarkable for its integrity and charm and left an indelible imprint upon what is now known as "Khokhlov's school of physics". The school's spirit remains permeated with Rem's devotion to physics and the enormous contribution he made to its development.

If we were to estimate the life of Academician Khokhlov in units of time, then his years as Rector of Moscow University, and as a scientist of international renown and a public figure, his life's work that started in car-repair shops as a teenage fitter, would seem but a fleeting instant. However, there is yet another measure. It is what he did and what is being done now by his pupils.

His attitude to people in his own country and abroad rested entirely upon his firm conviction that one had to see the good in people, understand and respect them, and trust them implicitly.

This book is intended for readers whose affinity to physics is either professional or rests upon a keen interest in the subject.

Krzhizhanovsky

by **V. Kartsev**, D. Sc. (Eng.)

The book is a biography of Gleb Krzhizhanovsky, a prominent revolutionary, a personal friend of Lenin's, an engineer, a poet, a thinker. Born in 1872, Krzhizhanovsky became one of the early leaders in the underground revolutionary movement in tsarist Russia.

When the Great October Socialist Revolution triumphed, he became one of the leaders of the national economy. He was the driving force behind the famous State Electrification Plan which started the speedy industrial development of the Soviet Union. He was the first chairman of the Central State Planning Board setting the national development goals and supervising the work on their fulfilment. He was Vice-President of the USSR Academy of Sciences in the period of its reorganization that led to its becoming the largest research institution in the world.

Krzhizhanovsky was a living legend in the Soviet Union, a man decorated with the highest national awards, a poet whose songs were sung throughout the country, a deeply respected patriarch of Soviet science and technology.

This is fascinating and highly documented story
about a fascinating person
written by another fascinating person.
Pelageya Kochina (Polubarinova-Kochina)
was born in 1899
and is a Member
of the USSR Academy of Sciences (1958),
a State Prize Winner (1946),
and a Hero of Socialist Labour (1969).
She is the head of the section of mathematical methods
at the Institute of Problems of Mechanics,
USSR Academy of Sciences.
Apart from her work on the theory of filtration,
meteorology, theory of tides,
partial differential equations, and many other topics,
Pelageya Kochina is widely
known as a historian of science and writer.
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